Electromagnetically Induced Transparency in Ensembles of Classical Oscillators

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We develop a classical model of the parametric effect of electromagnetically induced transparency (EIT) within the line of resonance absorption of an electromagnetic wave in the medium—an effect initially discovered for a quantum three-level system. On the basis of this model, the EIT effect for electromagnetic waves at frequencies of the electron-cyclotron resonance in a cold plasma is considered. Similar to the analogous quantum scheme, the EIT window in the classical model is characterized by group deceleration of the reference electron-cyclotron wave.

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(I) An interesting parametric effect in contemporary coherent and nonlinear optics is electromagnetically induced transparency (EIT) in ensembles of three-level atoms caused by interference of the quantum states of electrons. This effect manifests itself as formation of a "transparency window" within the zone of resonance absorption, which is accompanied, at the same time, by extremely slow group velocity of the reference light wave (e.g., see review [1]). Specifically, the EIT effect underlies the recent experiments on the "storage of light" described in [2,3]. In terms of the general theory of coherent radiation processes the search for a classical equivalent of EIT is evidently rather natural; this search is also stimulated by the hope to transfer the new ideas from quantum optics and electronics into classical microwave electronics and plasma physics. (It has been noted earlier (in [4,5]) that there is some similarity between electromagnetically induced transparency in quantum systems and one of the regimes of the parametric wave interaction in isotropic plasmas. However, in the "standard" (quantum) variant EIT is associated with the appearance of the transparency window within the zone of resonance absorption [1]. That is why it seems important to discuss also direct classical equivalents of the corresponding quantum effect.)

In this paper the simplest classical model of EIT is built up: a system with lumped parameters (Section II); it is shown that under EIT conditions such a classical equivalent for the effect of interference of quantum states in the three-level system is a variant of the effect of a dynamic damper in a system of two coupled electric LC circuits known in the theory of oscillations. Using this model the theory of EIT for electron-cyclotron waves in cold collisional plasma is developed in Section III. The obtained results are discussed in Section IV.

(II) Let us consider the quantum three-level system (Fig. 1) with transition eigenfrequencies ω_{31}, ω_{32}, and ω_{21}. Let this system interact with the bichromatic field:

$$\vec{E}(t) = \vec{x}_0 \,\mathrm{Re}(E_1 e^{-i\omega_1 t} + E_2 e^{-i\omega_2 t}). \tag{1}$$

The frequencies of the reference wave and the pumping wave (ω_1 and ω_2) are close to the transition frequencies ω_{31} and ω_{32}, respectively; hence, the "beat" frequency $\omega_L = \omega_1 - \omega_2$ is close to transition frequency ω_{21}. The susceptibility χ of the medium formed by such three-level atoms, linear in terms of the field E_1 and quadratic in terms of the pumping field E_2, is equal to (see [1,6,7])

$$\chi = (\omega_0/4\pi) \times \frac{\omega_{21} - \omega_L - i\gamma_L}{(\omega_{31} - \omega_1 - i\gamma_1)(\omega_{21} - \omega_L - i\gamma_L) - \Omega_R^2}. \tag{2}$$

Here $\gamma_{1,L}$ are the phenomenological constants of "transverse" relaxation of the transitions with frequencies $\omega_{31,21}$, respectively; $\Omega_R = |(d_{21}E_2)|(2\hbar)^{-1}$ is the Rabi frequency determined by the pumping amplitude, $\omega_0 = 4\pi|d_{31}|^2 N/\hbar$, where N is atom density (population of the $|3\rangle$ and $|2\rangle$ levels is assumed negligibly small), and d_{ij} is the matrix element of the projection of the dipole momentum operator on the direction of the electric field.

It is evident that at $\Omega_R^2 \gg (\gamma_1\gamma_L)$ excitation of resonance oscillations of the quantum system in the vicinity of the transition frequency ω_{31} is suppressed in the frequency band of the order of the Rabi frequency. (The detailed discussion of the frequency dependence of the polarization of the medium in the EIT regime can be found in review [1] and its references.)

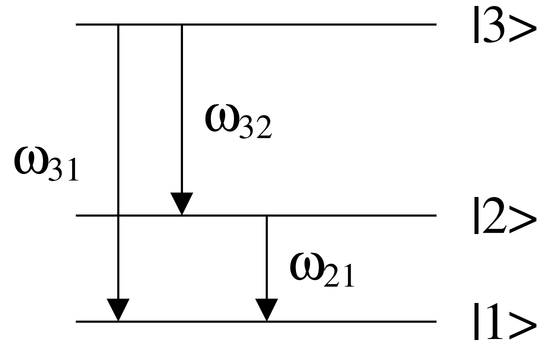

FIG. 1. The quantum three-level system.

Let us pass over to the classical equivalent of the quantum system considered above. Note that, generally speaking, the equations that describe the density matrix of the system correspond to the system, which describes excitation of the ensemble of classical coupled oscillators. For the case, when such coupling is caused by Stark effect in dc field the classical analog in the form of two LC circuits, coupled by the constant mutual inductance was proposed in [8]. In the case of HF drive considered now the coupling of quantum transitions is parametric, so that adequate classical model (see Fig. 2) represents two LC circuits (with eigenfrequencies ω_{31} and ω_{21} and relaxation constants γ_1 and γ_L), which are coupled via mutual variable inductance $M = \mathrm{Re}(\mu_2 \exp(-i\omega_2 t))$. In this case the harmonic coupling between the circuits acts as pumping E_2 in the quantum system. It is not difficult to verify that if the resonant conditions $|\omega_{1,L} - \omega_{31,21}| \ll \omega_{31,21}$ are fulfilled the frequency dependence of the impedance for the circuit depicted in Fig. 2 is completely equivalent to the formula (2) for susceptibility of the quantum system in the EIT regime up to the following substitution:

$$\Omega_R^2 \longrightarrow \omega_1 \omega_L |\mu_2|^2/(16 L_1 L_L). \tag{3}$$

When the conditions of the resonant excitation of the oscillator with frequency ω_{21} are fulfilled, oscillations of the first oscillator can be suppressed. Note that the very effect of resonance buildup of an oscillator being suppressed due to coupling with another oscillatory system is well known: it is so-called dynamic damper. However, in its standard version this effect is provided by constant coupling of two oscillators, which causes excitation of the damper at the frequency of the external force (see, e.g., [8,9]). In the case under consideration the "parametric" coupling of the oscillators results in buildup of the damper at the beat frequency: this is the main peculiarity of the discussed effect.

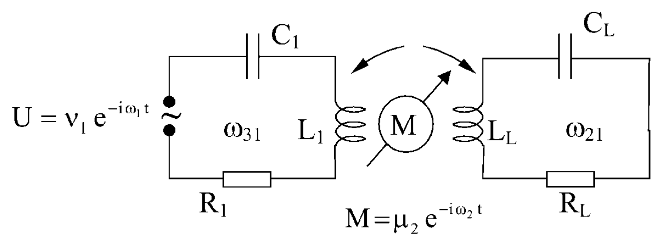

FIG. 2. The equivalent oscillatory system with lumped parameters.

(III) As an example of the EIT effect realized in a classical wave system we will consider the electron-cyclotron resonance in cold magnetized plasma. Let two circularly polarized extraordinary (X) waves propagate along a constant magnetic field $\vec{H} = \vec{z}_0 H$:

$$\vec{E}_\perp(z,t) = \mathrm{Re}[\vec{e}_+ (E_1 e^{-i\omega_1 t + ik_1 z} + E_2 e^{-i\omega_2 t + ik_2 z})], \tag{4}$$

where $\vec{e}_+ = (\vec{x}_0 + i\vec{y}_0)/\sqrt{2}$ is the polarization vector of X waves and $\vec{z}_0$, $\vec{x}_0$, and $\vec{y}_0$ are unit vectors of the Cartesian axes. Let us focus on the case where frequency ω_1 is close to electron gyrofrequency $\omega_H = eH/mc$ and the beat frequency $\omega_L = \omega_1 - \omega_2$ is close to the plasma frequency ω_p: $|\omega_1 - \omega_2| \simeq \omega_p$ (sign of difference $\omega_1 - \omega_2$ is not significant). Oscillations of the transverse and longitudinal (relative to the constant magnetic field) velocity components of the electron component are determined by the Euler equations, in which we will take into account also the hf magnetic field term in a Lorentz force:

$$\frac{\partial \vec{v}_\perp}{\partial t} + \omega_H [\vec{v}_\perp \times \vec{z}_0] + \nu \vec{v}_\perp + v_\parallel \frac{\partial \vec{v}_\perp}{\partial z} = -\frac{e}{m}\left(\vec{E}_\perp(z,t) + v_\parallel \frac{\partial}{\partial z}\int^t \vec{E}_\perp(z,t)\,dt\right), \tag{5}$$

$$\frac{\partial v_\parallel}{\partial t} + \nu v_\parallel + v_\parallel \frac{\partial v_\parallel}{\partial z} = \frac{e}{m}\left(\vec{v}_\perp \frac{\partial}{\partial z}\int^t \vec{E}_\perp(z,t)\,dt + \frac{\partial \varphi}{\partial z}\right); \tag{6}$$

here $\nu \ll \omega_H$ is the transport frequency of collisions, and φ is the electrostatic potential. Taking into account the continuity equation and the Poisson equation, and operating within the assumption that the ion density is constant and the plasma is quasineutral, one can obtain equations that describe excitation of longitudinal collective oscillations by the beat ponderomotive force:

$$\frac{\partial^2 n}{\partial t^2} + \omega_p^2 n + \nu \frac{\partial n}{\partial t} = -\frac{eN}{m}\frac{\partial}{\partial z}\left(\vec{v}_\perp \frac{\partial}{\partial z}\int^t \vec{E}_\perp(z,t)\,dt\right), \qquad v_\parallel = -\frac{1}{N}\int^z dz\left(\frac{\partial n}{\partial t}\right) \tag{7}$$

(here $\omega_p^2 = 4\pi e^2 N/m$ is the electron plasma frequency, N is the unperturbed plasma density, and n is the perturbation of electron density).

Having introduced the complex amplitudes of the corresponding values

$$\vec{v}_\perp = \mathrm{Re}\left(\vec{e}_{(+)} \sum_{j=1,2} \hat{v}_j e^{ik_j z - i\omega_j t}\right), \qquad n = \mathrm{Re}(\hat{n} e^{i(k_1 - k_2)z - i\omega_L t}), \tag{8}$$

we can pass over to the expression for the amplitude of the electric current, and then to one for the effective susceptibility of the wave (see, e.g., [10]) that propagates along the magnetic field at frequency ω_1:

$$\hat{j}_1 = -e\left(N\hat{v}_1 + \frac{1}{2}\hat{n}\hat{v}_2\right), \qquad \chi = -\frac{\hat{j}_1}{i\omega_1 E_1}. \tag{9}$$

Let us determine the values of $\hat{v}_1$ and $\hat{j}_1$ with the account for the terms, which are quadratic in terms of field E_2

and neglect small values of the order of $(\nu \xi_{\mathrm{EC}}/\omega_H)$ and $(|\omega_1 - \omega_H| \xi_{\mathrm{EC}}/\omega_H)$. Then we will obtain the following expression for the susceptibility:

$$\chi = -\frac{v}{4\pi} \times \frac{(\Delta^2 - v + i\Delta s) + 2\xi_{\mathrm{EC}}\Delta\sqrt{u}\,(\frac{2k_1\omega_1}{k_2\omega_2} - \frac{3}{2} + \Delta)}{(1 + is - \sqrt{u})(\Delta^2 - v + i\Delta s) - 2\Delta^2\sqrt{u}\,\xi_{\mathrm{EC}}}. \tag{10}$$

In (10) $\xi_{\mathrm{EC}} = (eE_2/2m|\omega_2 - \omega_H + i\nu|)^2(k_2/\omega_2)^2 = |v_2|^2/(2v_{ph})^2$ is the ratio of squares of the oscillatory and phase velocities for the pumping field, $v = (\omega_p/\omega_1)^2$, $u = (\omega_H/\omega_1)^2$, $s = (\nu/\omega_1)$, $\Delta = (\omega_L/\omega_1)$. It is evident that when the pumping is sufficiently powerful, i.e., $\xi_{\mathrm{EC}} \gg s^2/\Delta$, the standard linear absorption becomes significantly weaker in the frequency band that corresponds to the condition of $|(1 - u^{1/2})(\Delta^2 - v)| \leq (\Delta^2 \xi_{\mathrm{EC}})$ (see [10]). (Moreover, if the approximate relation $(\Delta^2 - v + i\Delta s) \approx 2\Delta[\Delta + i(s/2) - \sqrt{v}\,]$ is used, in the domain of parameters $s \gg \xi_{\mathrm{EC}} \gg s^2/\Delta$ from (10) the expression follows which corresponds *exactly* to the frequency dependence of the complex susceptibility for the "quantum" medium in the EIT regime.) However, in order to achieve absolute smallness of absorption at the wavelength, it is necessary that inequality $s \ll (\xi_{\mathrm{EC}}/v^{1/2})$ (which is, generally speaking, stronger) should be fulfilled. The group velocity $v_{gr} = \mathrm{Re}(\frac{\partial \omega_1}{\partial k_1})$ in the EIT "window" will be estimated as

$$v_{gr} \approx c(\xi_{\mathrm{EC}}/v^{1/2}). \tag{11}$$

(IV) The expression for the "effective" susceptibility of the reference EC wave [Eq. (10) obtained above] allows one to study the features of wave propagation in the EIT window. For example, if the plasma density is about 10^{13} cm^{-3} and the electron temperature, which determines the transport frequency of Coulomb collisions, is the order of 300 eV, the absorption length for a reference EC wave of frequency $(\omega_1/2\pi) = 100$ GHz is about 1 m for an energy flux 250 (kW/cm^2) of the "pumping" radiation. (All parameters used here are typical for modern gyrotrons (see [11]). The parameters of the considered plasma device (plasma density $\sim 10^{13}$ cm^{-3}, magnetic field ~ 3.5 T, the length ~ 1 m, cross section ~ 100 cm^2) also correspond to some real plasma experiments [12].) In this case, the length of linear cyclotron absorption is of the order of the wavelength. Figure 3 exemplifies formation of the transparency window within the region of linear cyclotron absorption (Fig. 3b corresponds to the above-mentioned parameters of plasma and radiation). It is seen that in the region of the "EIT window" the direction of the group velocity coincides with the direction of the wave vector, its magnitude rather well corresponds to the estimation (11): $v_{gr} \simeq 10^{-3}c$ in Fig. 3a and $v_{gr} \simeq 10^{-4}c$ in Fig. 3b.

In this paper we have demonstrated a simple classical analog of quantum EIT effect. This analog makes it possible to design various wave systems with parametrically induced transparency; we considered the typical example of such a classical system—electron cyclotron waves in magnetized plasma.

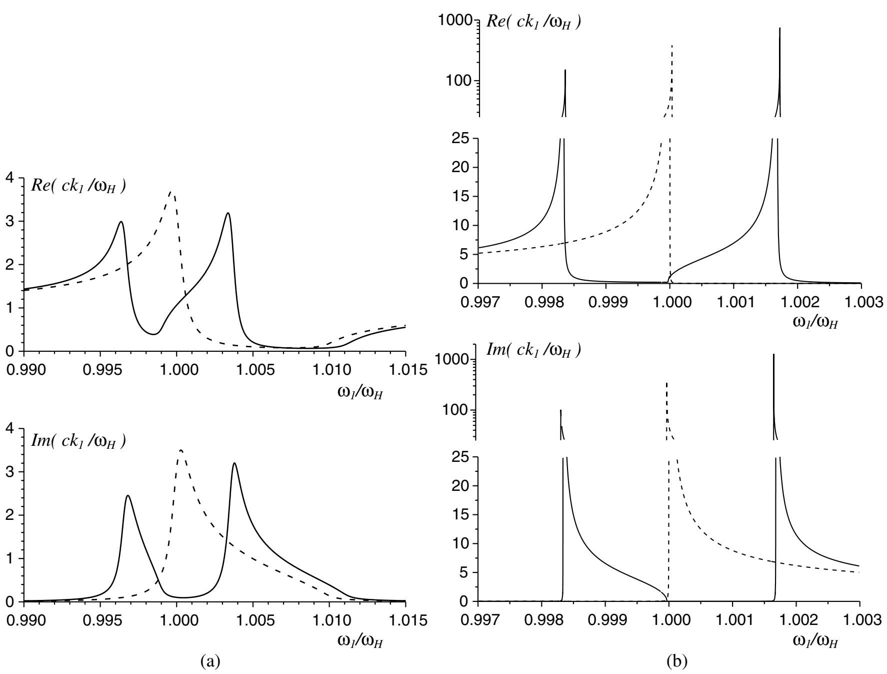

FIG. 3. Formation of the transparency window. Energy flux of driving field is 250 kW/cm^2, $\omega_1/2\pi = 100$ GHz. (a) $v = 10^{-2}$, $\omega_2/\omega_H = 0.9$, $s = 5 \times 10^{-4}$, $\xi_{\mathrm{EC}} = 0$ (dashed line), $\xi_{\mathrm{EC}} = 2.5 \times 10^{-4}$ (solid line). (b) $v = 0.078$, $\omega_2/\omega_H = 0.72$, $s = 3.5 \times 10^{-7}$, $\xi_{\mathrm{EC}} = 0$ (dashed line), $\xi_{\mathrm{EC}} = 2 \times 10^{-5}$ (solid line).

The possibility to transport EC waves through the region of cyclotron absorption seems to be rather interesting for applications at magnetic fusion devices for EC plasma heating and diagnostic. Generation of EIT windows gives a tool to control the region of electromagnetic wave absorption (or radiation) in the case of extended zone of EC resonance in a homogeneous magnetic field. Another possible research line for studying the EIT effect in classical systems is associated with the progress in the theory of "inversion-free" lasing. In quantum systems the EIT regime is, essentially, the excitation threshold for one of the inversion-free lasing schemes (for the so-called P scheme, see, e.g., [13] and [14]). Hence, creation of the classical EIT version makes it possible to hope for development of the classical variant of the quantum P scheme.

However, specific proposals will require generalization of the theory developed here to the case of collisionless resonance interaction of waves and particles.

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