Feedback Control of Atomic Motion in an Optical Lattice

N. V. Morrow, S. K. Dutta, and G. Raithel
FOCUS Center, Physics Department, University of Michigan, Ann Arbor, Michigan 48109-1120
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We demonstrate a real-time feedback scheme to manipulate wave-packet oscillations of atoms in an optical lattice. The average position of the atoms in the lattice wells is measured continuously and nondestructively. A feedback loop processes the position signal and translates the lattice potential. Depending on the feedback loop characteristics, we find amplification, damping, or an entire alteration of the wave-packet oscillations. Our results are well supported by simulations.

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The development of stochastic cooling of particle beams [1] has demonstrated that ensembles of particles can be manipulated using feedback. Recent advances in the manipulation of small quantum-mechanical systems using ultrafast laser pulses [2,3], and in the manipulation of laser-cooled atoms using light-shift potentials [4], have renewed the interest in feedback control applied to such systems. Real-time feedback presents the possibility of continuous control of the state of a system, e.g., the motion of an atomic ensemble. The motion of atoms can be monitored through nondestructive position or momentum measurements. Proposed methods of position measurement include the observation of output light from a cavity, in which atoms interact with the standing-wave light field of a cavity mode [5], and the observation of the position-dependent fluorescence of an atom on a strong transition [6]. Nondestructive measurements of the velocity distribution of atomic clouds can be performed using recoil-induced resonances [7]. Feedback can be applied to the atoms by the means of a time-dependent optical potential [4]. In this Letter, we study a real-time feedback scheme acting on the wave-packet motion of atoms in a 1D optical lattice. Results of nondestructive position measurements are entered into a feedback circuit that translates the lattice potential under the atoms, such as to alter the course of atomic oscillations.

The presented real-time approach to feedback on quantum systems differs from the recently developed method of learning control. There, measurement results of one or more observables obtained during test realizations of a quantum process are used to progressively alter the Hamiltonian of the process [2,3,8] until a Hamiltonian is found that delivers a desired state evolution or final state of the system. In any given realization, a freshly prepared initial state of the system is used, and the Hamiltonian is predetermined. The learning-controlled evolution occurs in steps between the realizations, and destructive quantum measurement techniques may be used. In contrast, in the real-time feedback scheme discussed in this paper the Hamiltonian is continuously modified using input from a continuous and nondestructive measurement.

In an optical lattice [9], atoms are cooled and localized at the bottom of periodic light-shift potential wells that are formed due to the interference of multiple laser beams. The average deviation of the atoms from the lattice sites, $\langle \Delta x(t) \rangle$, can be measured using the optical lattice itself, without the use of additional laser beams. Following Ehrenfest's theorem, an atom displaced from its equilibrium position experiences a restoring electric-dipole force $F = (d/dt)\langle p \rangle = \langle -\nabla U \rangle$, where U is the lattice potential. The semiclassical electric-dipole force F, which is derived assuming classical lattice light fields, corresponds to an imbalance in the quantum-mechanical rates of stimulated Raman transition processes: the displaced atom preferentially absorbs photons from some lattice beams and reemits them into others. In a 1D lattice formed by two counterpropagating laser beams with wavelength λ, the average photon exchange rate between the two beams due to the presence of an atom, $\langle \dot{n} \rangle$, and F is related by

$$F = 2\hbar k_L \langle \dot{n} \rangle, \quad (1)$$

where $2\hbar k_L = 2h/\lambda$ is the net change in momentum due to one redistributed photon. Averaged over an ensemble of N atoms in the lattice, the force F translates into a power difference between the beams

$$\Delta P = Nc\langle F \rangle. \quad (2)$$

Usually, the lattice wells are approximately harmonic and the average displacement of the atoms, $\langle \Delta x(t) \rangle$, is much smaller than the spacing of the lattice wells. In this case, $\langle F(t) \rangle \propto \langle \Delta x(t) \rangle \propto \Delta P(t)$. Thus, a measurement of $\Delta P(t)$ directly reveals $\langle \Delta x(t) \rangle$. Such measurements have previously been used to study wave-packet oscillations in optical lattices [10,11]. Here, the power exchange $\Delta P(t)$ is measured and entered into an analog circuit, which translates the lattice by an amount proportional to $\Delta P(t)$. Thereby, a unique feedback control circuit is implemented, which uses macroscopic electronic components to act on a mesoscopic sample of atoms trapped in identical, microscopic quantum wells.

We perform our experiment in a vapor-cell magneto-optic trap (MOT) of ^{87}Rb atoms. Atoms periodically collected and precooled in the MOT are further cooled in a

3D optical molasses and then transferred into a vertical 1D lin $\perp$ lin optical lattice [9]. The lattice is formed by two counterpropagating laser beams with orthogonal linear polarizations, single-beam intensities of 1.2 $\mathrm{mW/cm^2}$, and a detuning of -27 MHz with respect to the $5S_{1/2}$, $F = 2 \rightarrow 5P_{3/2}$, $F = 3$ transition ($\lambda = 780$ nm, $\Gamma/2\pi = 6$ MHz). Because of polarization-gradient laser cooling [12], within a few hundred μs the atoms collect in the lowest few bound states of the lattice potential. The feedback experiments are performed on atomic samples that have reached steady-state conditions in the lattice.

As seen in Fig. 1a, the lattice laser beam is spatially filtered in an optical fiber and divided into two lattice beams using a polarizing beam splitter (PBS). In the chamber, the beams intersect at the location of the atomic cloud at an angle near 180°. A pair of photodiodes, D1 and D2, with circular sensitive areas of 1 $\mathrm{mm^2}$, are positioned into the outgoing lattice beams such that they selectively detect the light that has interacted with the atomic cloud; the cloud also has a cross section $\sim$1 $\mathrm{mm^2}$. The two photodiodes are connected with opposite polarity to a transimpedance amplifier with 1 MHz bandwidth, yielding the power difference $\Delta P(t)$. Two phase modulators, PM1 and PM2, are located in the arms of the lattice. PM1 is used to apply a sudden shift to the lattice, while PM2 is used to apply the feedback, as shown in the circuit diagram (Fig. 1b).

To illustrate our concept of wave-packet manipulation, let us consider an atom that, at time $t = 0$, is at rest near the minimum of the potential. If we apply a small sudden shift to the lattice (e.g., such as to simulate a mirror displacement $\lesssim 0.1\lambda$), the atom will gain extra potential energy and will start to oscillate (Fig. 2a). After half the oscillation period, $t = T/2$, the atom will be localized near its classical turning point on the opposite side of the potential. If at that time the lattice is again shifted in the same direction as in the first shift, the atom will lose the extra potential energy (Fig. 2b) and return to the minimum of the potential. Figure 2c shows a result of this double-shift experiment. The experiment confirms that an initial disturbance of the atoms can be compensated by a suitable time-delayed reaction.

In the double-shift experiment, the time-dependent Hamiltonian of the system is still predetermined. Since our position measurement method is continuous and nondestructive, the logical extension of control is to apply the reaction shift continuously and in direct response to the measured average displacement $\langle \Delta x(t) \rangle$ of the atoms. For that purpose, the measured power difference $\Delta P(t)$ is sent to a feedback circuit, which adjusts the position of the lattice by applying a reaction voltage $Y(t)$ to the PM2. By changing the gain of the feedback circuit, we can control the magnitude of the response shift applied to the lattice. Further, by inverting the output, we can apply the response shift such that $\langle \Delta x(t) \rangle$ is decreased (negative gain) or increased (positive gain).

In Fig. 3, the experimentally observed behavior of the atoms for different signs of the feedback gain is shown. In the case of negative feedback, the oscillations are damped efficiently, and are mostly absent after one period. The displayed case corresponds to optimal damping of the atomic motion; by analogy with classical control theory, this case is given the designation of "gain -1." All other gain settings are calibrated in reference to it. In the case of positive feedback (gain $+1$), the oscillations are amplified, as expected, and their coherence is preserved longer than in the case of no feedback. Thus, negative gain can be used to suppress common-mode oscillations of the atoms, while positive gain can be used to extend the coherence time of the wave-packet motion.

We have varied the feedback gain from -3.5 to $+3.5$. The results in Fig. 4 show that moderate positive feedback slows the decay of the wave-packet oscillation, while at our largest positive gains the wave-packet amplitude appears stationary or even growing for times $t \gtrsim 50$ μs. As before, for gain -1 the wave-packet oscillations are quickly

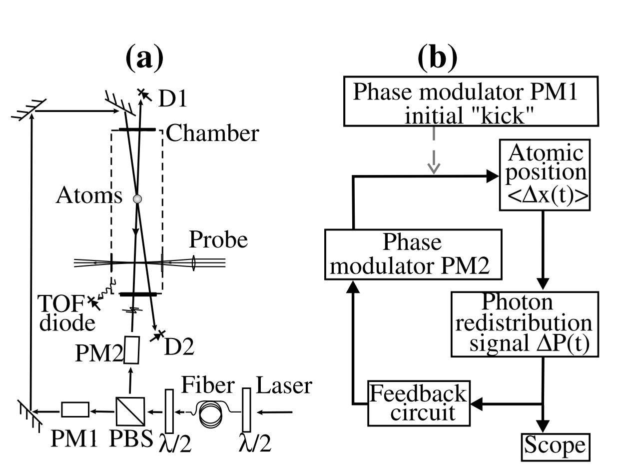

FIG. 1. (a) Schematic of the optical lattice setup and the setup used for time-of-flight measurements. (b) Block diagram of the feedback loop used to control wave-packet oscillations in the lattice.

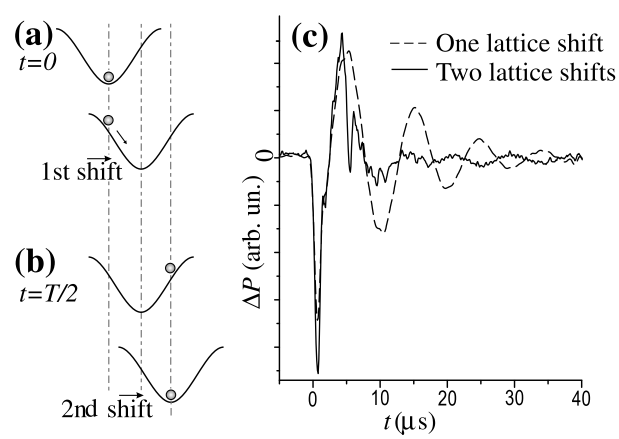

FIG. 2. Double-shift experiment. (a) At $t = 0$, an initial shift is applied to atoms located in the lattice wells. (b) After half the oscillation period, a second shift of equal size can be applied. (c) Resultant wave-packet oscillations, measured experimentally with and without the application of the second shift.

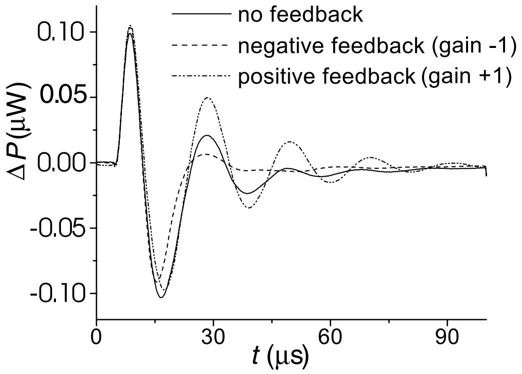

FIG. 3. Wave-packet oscillations for the case of no feedback, positive feedback (gain $+1$), and negative feedback (gain -1). The initial common-mode displacement of the atoms generated by PM1 amounts to 0.075λ and is applied in the form of an approximately linear ramp of ≈ 1 μs duration.

damped out. Stronger negative feedback does not further improve the damping of the motion; in contrast, at large negative gain (< -2) the motion is altered into a long-lasting oscillation with half the natural oscillation period of the atoms in the lattice.

The dramatic reduction of the wave-packet oscillations in the case of gain -1 raises the following concern. Feedback-induced heating could cause a loss of atoms from the lattice potential wells, and—due to the N dependence in Eq. (2)—a reduction of $\Delta P(t)$. This reduction could be misinterpreted as a damping of the wave-packet oscillation. To eliminate this possibility, we have conducted temperature measurements of the atoms using the conventional time-of-flight (TOF) technique [13]. At a variable delay time t after the initiation of the wave-packet oscillation, the lattice light is suddenly turned off. The fluorescence of the atoms, as they fall through a thin sheet of resonant probe light, is collected by a large-area photodiode (see Fig. 1a and [14]). The TOF signal yields the temperature T_0 of the atoms at the time of their release [15]. For given lattice parameters and feedback gain, we measure T_0 as a function of the time delay t. The general increase of T_0, observed for $t \lesssim 30$ μs in Fig. 5, is caused by the dephasing of the wave-packet oscillation. At later times, the temperature increase is balanced and eventually undone by the effect of polarization-gradient cooling. Further, Fig. 5 shows that, generally, negative feedback does not increase the temperature. These observations corroborate our interpretation of the photon exchange signals and mollify the above raised concern.

To quantitatively model the experiment, we have analyzed the temporal response of the feedback circuit. The circuit generates a PM2 voltage $Y(t) \propto \Delta P(t)$, whereby an electronic lag of 2 to 3 μs occurs. The photodiode amplifier adds 1 to 2 μs delay to the overall feedback response time. Using an interferometric setup, we have further verified that over the relevant frequency range the lattice shift $S(t)$ generated by PM2 due to the applied voltage $Y(t)$ does not exhibit any additional time or phase shifts. The total lag of the response therefore is ~ 4 μs, which is about a quarter of the natural oscillation period of the atoms in the lattice.

We have simulated the feedback experiment using quantum Monte Carlo wave function simulations (QMCWF). To implement feedback, the value of $(d/dt)\langle k\rangle(t)$ of an ensemble of 10 000 trapped atoms is tracked, where k is the momentum in units of $2\pi/\lambda$. Making the harmonic approximation for the lattice potential near the minima, $U(x) = U_0 k_L^2 x^2$ [9], the average displacement is obtained from $\langle \Delta x(t)\rangle = \frac{\hbar}{2k_L U_0}(d/dt)\langle k\rangle(t)$. The lattice depth U_0 amounts to $h \times 250$ kHz for our lattice. The lattice position $S(t)$ generated in the simulation then is obtained from

$$S(t) = G \int_0^\infty \langle \Delta x(t - \tau)\rangle g(\tau)\, d\tau\,, \qquad (3)$$

where G is a gain parameter and $g(\tau)$ is a normalized response function of the feedback. Based on our analysis of the setup, we approximate $g(\tau)$ by a linearly rising and falling function with total rise and fall times of 2 μs and a 1-μs-wide flat top. The model function is centered at $\tau = 5$ μs. Figure 6 shows simulation results for different

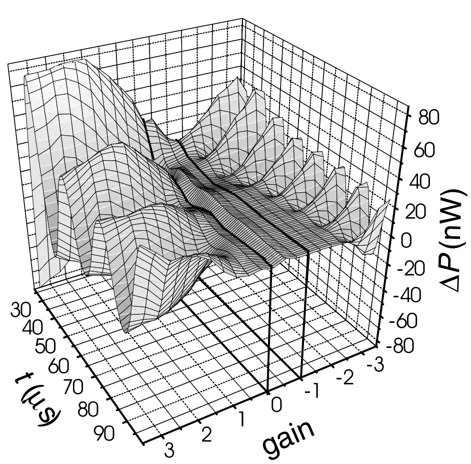

FIG. 4. Wave-packet oscillations as a function of the gain of the feedback circuit. First peak omitted for clarity.

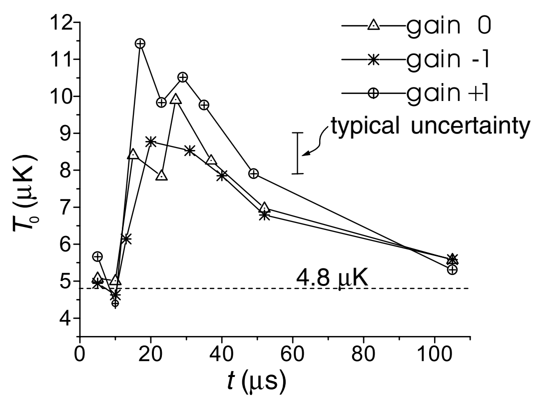

FIG. 5. Temperatures $T_0(t)$ derived from time-of-flight measurements for the indicated gain values. The horizontal dashed line shows the steady-state value of T_0.

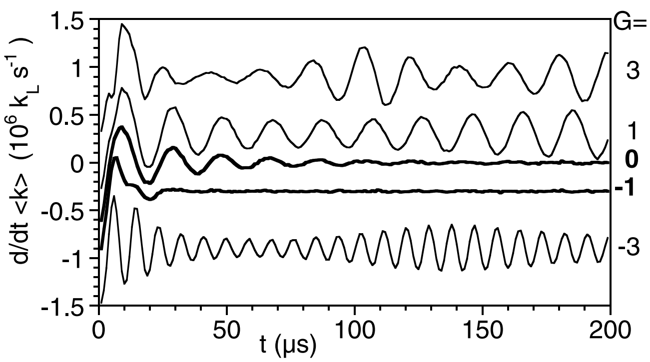

FIG. 6. QMCWF simulations of the feedback experiment for the indicated values of the theoretical gain parameter G defined in the text. The initial shift is 0.075λ. The curves are offset from zero for clarity.

values of G. Comparing Figs. 6 and 4, we observe good agreement. It is a perhaps coincidental fact that in the simulations the most efficient damping of the wave-packet oscillation occurs exactly at the value $G = -1$. In simulations not shown, we have varied $\tau_{\mathrm{av}} = \int g(\tau)\tau\, d\tau$ from 3 to 7 μs and found that it only weakly affects the behavior at $G = -1$. For $G = +1$, the variation of τ_{av} causes variations of the frequency and the amplitude of the feedback-sustained oscillations of order 20%. At large negative gain, we have generally observed a much stronger dependence of the frequency and amplitude of the oscillations on the detailed properties of $g(\tau)$.

The lattice shift $S(t)$ generated by PM2 is related to $Y(t)$ by $S(t) = \alpha Y(t)$ with a constant, frequency-independent factor $\alpha = 0.92\lambda/\mathrm{kV}$. This factor follows from the specifications of the PM2 and has been independently verified through interferometric calibration. Using this calibration, we have measured the shift $S(t)$ for an initial sudden displacement of 0.075λ and gain -1. The result and a corresponding theoretical result for $G = -1$ are displayed in Fig. 7. We find that the neutralization of the wave-packet oscillation largely occurs within a single period of the feedback-free oscillation. We observe reasonable agreement between theory and experiment; we attribute quantitative differences to experimental uncertainties in the lattice parameters and in the calibration factors of both phase modulators, and to unaccounted details of the response function $g(\tau)$.

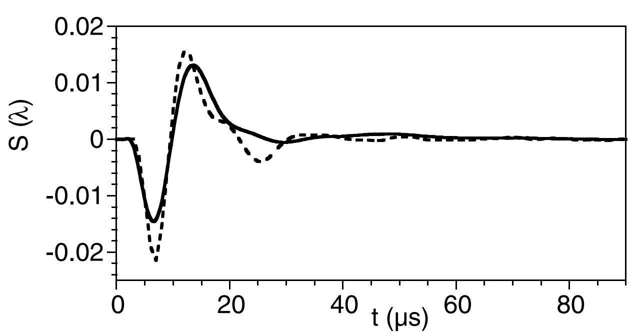

FIG. 7. Experimental (solid line) and theoretical (dashed line) results for the feedback-generated lattice shift $S(t)$ observed for gain -1.

We have shown that we can enhance, dampen, or alter atomic oscillations in an optical lattice through real-time continuous feedback. Strong feedback can lead to sustained wave-packet oscillations, while moderate negative feedback can efficiently remove common-mode motion of the atoms. We have taken advantage of the fact that any common-mode motion leads to a power exchange $\Delta P(t)$ that scales as the number of atoms N and therefore is very significant. As an extension of our work, we plan to use feedback to control the temperature-increasing effect of random common-mode excitations of the atoms due to mirror vibrations etc. A further application of our technique could be stochastic cooling of the atoms [4]. It will, however, be difficult to detect the stochastic power exchange signal, which will scale as $\sqrt{N}$ and will therefore be very weak [4]. Stochastic cooling requires phase space mixing, which could be enhanced by the use of optical potentials with large anharmonicity. Coherence preservation due to feedback may be another focus of future studies. For gains of order one, we have observed undamped oscillations over hundreds of μs. Under such conditions, an equilibrium between the heating caused by the ongoing excitation of the wave-packet oscillation and polarization-gradient laser cooling exists. It will be interesting to study the nature of this equilibrium and the phase diffusion rate of the self-sustained oscillation. Overall, the initial success of our technique shows an intriguing extension of classical control theory into the quantum world.

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