# Tachyon Condensation and Black Hole Entropy

Atish Dabholkar
*Department of Theoretical Physics, Tata Institute of Fundamental Research, Mumbai 400005, India*
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String propagation on a cone with deficit angle $2\pi(1 - \frac{1}{N})$ is considered for the purpose of computing the entropy of a large mass black hole. The entropy computed using the recent results on condensation of twisted-sector tachyons in this theory is found to be in precise agreement with the Bekenstein-Hawking entropy.

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*Introduction.*—The statistical interpretation of Bekenstein-Hawking entropy of a black hole remains an outstanding problem in quantum gravity. For a black hole in $d$ spacetime dimensions, the entropy $S$ is given by a universal formula

$$S = \frac{A}{4G} \tag{1}$$

that depends only on the area $A$ of the event horizon and the $d$-dimensional Newton's constant $G$. Thermodynamically, this entropy behaves in every respect like ordinary entropy and unites the second law of thermodynamics with the area theorems of classical general relativity into an elegant generalized second law [1,2]. These beautiful results demand that, like any other entropy, the black hole entropy also must have a statistical interpretation in terms of underlying microstates.

Recent progress in string theory has shown that this is indeed true for a large class of supersymmetric black holes. The microstates of these special black holes are precisely countable and can completely account for their entropy [3,4]. These striking results can be extended to near-extremal black holes as well as to certain nonsupersymmetric charged black holes. At present, however, these methods cannot be applied to the more general case of a nonsupersymmetric neutral black hole.

We will address the problem of microstates of a Schwarzschild black hole from a very different perspective based on earlier ideas of 't Hooft [5], Susskind [6], and others [7–10]. 't Hooft [5] has advocated that the thermal entropy of the heat bath seen by a Schwarzschild observer should account for the Bekenstein-Hawking entropy. This would then offer the desired statistical interpretation of the entropy in terms of the near-horizon microstates of the heat bath. In field theory, the leading contribution to the thermal entropy is quadratically divergent in the ultraviolet [5] and is proportional to the area of the horizon. If the cutoff is of the order of the Planck length, then the thermal entropy is of the right order of magnitude to be identified with the Bekenstein-Hawking entropy.

In the context of field theory, there are several difficulties with this appealing idea. For example, the thermal entropy depends on an arbitrary cutoff and its precise identification with the Planck length is not clear. The thermal entropy depends on the species and the couplings of the various particles in the theory, whereas the black hole entropy is species independent. Finally, since the thermal entropy always starts at one loop, it is difficult to see how it can possibly account for the tree level black hole entropy which is inversely proportional to the coupling constant. 't Hooft has argued that it is necessary to understand the ultraviolet structure of the theory to address these questions, and therefore these difficulties will be resolved in the correct short-distance theory of quantum gravity.

In string theory, ultraviolet divergences are expected to be appropriately controlled, and Susskind, in particular, has argued that string theory offers a suitable framework for realizing this proposal [6]. We will pursue these ideas further by considering string theory on the near horizon geometry in Euclidean formalism following earlier work in [11,12].

*Strings on a cone.*—Consider a Schwarzschild black hole in four dimensions with a large mass $M$. The metric is given by

$$ds^2 = -\left(1 - \frac{2GM}{r}\right)dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega^2. \tag{2}$$

We are interested in the thermal entropy of string modes in this background near the horizon as seen by a Schwarzschild observer. To focus on the near-horizon region we choose coordinates $\rho = \sqrt{8GM(r - 2GM)}$ and $\eta = \frac{lt}{2GM}$ in which the metric becomes

$$ds^2 = -\left(\frac{\rho}{l}\right)^2 d\eta^2 + d\rho^2 + (2GM)^2 d\Omega^2. \tag{3}$$

Here $l$ is an arbitrary parameter that sets the length scale which in our context will be taken to be the string length. We see from the form of the metric that $\rho$ measures the proper distance from the horizon at $\rho = 0$ and $\eta$ is the proper time measured by a Schwarzschild observer located at a proper distance $\rho = l$. We will refer to this location as the "stretched horizon." Energy at the stretched horizon is related to asymptotic energy by the redshift factor $l/4GM$.

In the limit $GM \gg l$, the two-sphere at the horizon can be approximated by a flat two-dimensional transverse space and then the space has flat Minkowski geometry. Thus, in this limit, the Schwarzschild observers are exactly like Rindler observers in uniform acceleration in Minkowski space. More generally, for a $d$-dimensional black hole, the near horizon geometry is given by the $d$-dimensional Rindler metric with topology $\mathbf{R}^2 \times \mathbf{R}^{d-2}$,

$$ds^2 = -\rho^2 d\eta^2 + d\rho^2 + \sum_{i=1}^{d-2} (dx^i)^2, \qquad (4)$$

where $x^i$ are the coordinates of the $(d-2)$-dimensional transverse space and we have chosen units to set $l = 1$.

The observer at the stretched horizon sees a thermal bath at Rindler temperature $T = \frac{1}{2\pi}$ which, as usual, is inversely proportional to the periodicity of Euclidean time. Under Euclidean continuation, $\eta = -i\theta$, the metric in the $\mathbf{R}^2$ factor is given by the flat metric in polar coordinates,

$$ds^2 = \rho^2 d\tau^2 + d\rho^2. \qquad (5)$$

The Euclidean time $\tau$ is thus an angular variable and, when its periodicity is $2\pi$ corresponding to the Rindler temperature, the metric is smooth at the origin. The Rindler temperature at the stretched horizon is redshifted to the Hawking temperature $T_H = \frac{1}{8\pi GM}$ seen by the asymptotic observer.

The partition function $Z$ of this thermal bath in string theory would be given by a functional integral of all string modes on the Euclidean Rindler space. The thermal entropy of this heat bath $S$ would be given, as usual, by $S = -\frac{\partial F}{\partial T}$ as a derivative of the free energy, $F \equiv -T \ln Z$. Therefore, to calculate the entropy, we need to find the variation in the free energy of a string gas to order $\delta$ as we vary the temperature seen by the observer at the stretched horizon to $\frac{1}{2\pi} + \delta$. Changing the temperature changes the periodicity of Euclidean time in (5) to $2\pi - \delta$ to leading order. The Euclidean geometry is now conical with deficit angle $\delta$ and with a curvature singularity at the tip of the cone.

One of the difficulties in evaluating the partition function for strings on a cone with arbitrary deficit angle is that the cone is not a solution of the string equations of motion. The Einstein equation is not satisfied unless there is an explicit source at the tip to account for the curvature. One would thus require an off-shell formulation of string theory to evaluate this partition function.

We will instead proceed differently. We will consider string theory on a cone obtained as an orbifold $\mathbf{C}/\mathbf{Z}_N$ in [11–13]. This corresponds to the temperature $T = \frac{N}{2\pi}$ at the stretched horizon and the deficit angle is $\delta = 2\pi \times (1 - \frac{1}{N})$. Because the orbifold is a conformal field theory, for these special values of the deficit angle labeled by an integer the tree-level string equations of motion are indeed satisfied. In addition to the bulk modes, there are states from $N - 1$ twisted sectors labeled by $k = 1, \ldots, N - 1$. We will take both $k$ and $N$ to be odd to simplify the subsequent discussion. The ground state in each sector is tachyonic and its mass is given by $\alpha' m^2 = -2(1 - k/N)$. In the next section we evaluate the free energy $F(N)$ as a function of $N$ by properly taking into account infrared divergences due to the tachyons and then compute the entropy analytically continuing in $N$.

*Tachyon condensation and black hole entropy.*—The one-loop partition function for strings on the orbifold $\mathbf{C}/\mathbf{Z}_N$ can be easily written down. It is modular invariant and therefore ultraviolet finite as expected for a string partition function. This is already an improvement over the field theory calculation which was UV divergent. However, the partition function has severe infrared divergences because of the tachyons.

Infrared divergences, unlike ultraviolet divergences, are not a matter of renormalization but signify important physics. In this context, the existence of tachyons in the spectrum suggests that the thermal ensemble is unstable. It was speculated in [11,12] that these tachyons can condense, causing a phase transition, and the latent heat of this transition could account for the tree level black hole entropy. However, in the absence of a good candidate for the end point of tachyon condensation, it was not possible to pursue this further.

The recent work of Adams, Polchinski, and Silverstein [14] has provided new insights into the condensation of these tachyons. They have argued that tachyon condensation relaxes the cone to flat space. The most convincing evidence for this claim comes from the geometry seen by a D-brane probe in the substringy regime. In the probe theory, one can identify operators with the right quantum numbers under the quantum $\mathbf{Z}_N$ symmetry of the orbifold that correspond to turning on tachyonic vevs. By selectively turning on specific tachyons, the quiver theory of the probe can be "annealed" to successively go from the $\mathbf{Z}_N$ orbifold to lower $\mathbf{Z}_{N-2}$ orbifold all the way to flat space. The deficit angle seen by the probe in this case changes appropriately from $2\pi(1 - \frac{1}{N})$ to $2\pi(1 - \frac{1}{N-2})$.

These results are consistent with the assumption that in the field space of tachyons there is a potential $V(T)$ where we collectively denote all tachyons by $T$. The $\mathbf{Z}_N$ orbifold sits at the top of this potential, the various $\mathbf{Z}_M$ orbifolds with $M < N$ are the other extrema of this tachyonic potential, and flat space is at the bottom of this potential. Such a potential can also explain why a conformal field theory exists only for special values of deficit angles. We will be concerned here with the static properties such as the end point of tachyon condensation and the effective height of the tachyon potential and not so much with dynamical details of the process of condensation.

Let us return now to the computation of the black hole entropy. We would like to evaluate the free energy of a $\mathbf{Z}_N$ orbifold as a function of $N$. Now, the existence of tachyons in the orbifold implies that we are expanding the string field theory functional integral around a maximum. The asymptotic expansion provided by string perturbation theory around this point is as a result infrared (IR) divergent and essentially useless. To correctly evaluate the partition function, we must expand around the stable saddle point at the minimum of the potential. The leading semiclassical contribution to the partition function will be given by $Z \sim \exp -S_E$ where $S_E$ is the classical Euclidean action after condensation. To elaborate this point let us consider a toy model in field theory of a single scalar field $\phi$ with double well potential $-m^2\phi^2 + g^2\phi^4$. The perturbative expansion for the partition function around $\phi = 0$ is IR divergent, which signifies that we have expanded around the wrong saddle point. The stable saddle point is at $\phi^2 = \frac{m^2}{2g^2}$ and the leading semiclassical contribution to the partition function will be given by $Z \sim \exp -S_E$ where $S_E = \frac{-m^4}{4g^2}$ is the change in the classical Euclidean action after condensation.

It may seem difficult to evaluate the change in the classical action between the $\mathbf{Z}_N$ orbifold and flat space but we are helped by the fact that, for the orbifold conformal field theory, the equations of motion for the dilaton and the graviton are satisfied exactly. To extract this information, let us consider the Lorentzian string effective action, for concreteness, first to leading order in $\alpha'$,

$$S = \frac{1}{16\pi G}\int_M \sqrt{-g}\, e^{-2\phi}[R + 4(\nabla\phi)^2 - \delta^2(x)V(T)] + \frac{1}{8\pi G}\int_{\partial M}\sqrt{-g}\, e^{-2\phi}K\,, \tag{6}$$

where $K$ is the extrinsic curvature and $\delta^2(x)V(T)$ denotes the tachyon potential localized at the tip of the cone. The extrinsic curvature term is as usual necessary to ensure that the effective action reproduces the string equations of motion for variations $\delta\phi$ and $\delta g$ that vanish at the boundary.

The action is very similar to the one for a cosmic string in four dimensions. The tachyon potential supplies an 8-brane source term for gravity. Einstein equations imply $R = \delta^2(x)V(T)$ and therefore a conical curvature singularity at $x = 0$. Because of this equality, there is no source term for the dilaton and as a result the dilaton equations are satisfied with a constant dilaton. We see that the bulk contribution to the action is zero for the solution. The boundary has topology $\mathbf{R}^8 \times \mathbf{S}^1$. For a cone, the circle $\mathbf{S}^1$ has radius $r$ but the angular variable will go from 0 to $\frac{2\pi}{N}$. The extrinsic curvature for the circle equals $1/r$ and thus the contribution to the action from the boundary term equals $\frac{A}{4GN}$. We have to remember a factor of $-i$ in Euclidean continuation of $\sqrt{-g}$.

In the conformal field theory, we should worry about the higher order $\alpha'$ corrections to the effective action. These corrections are dependent on field redefinitions or equivalently on the renormalization scheme of the world-sheet sigma model. However, the total contribution of these corrections to bulk action must, nevertheless, vanish for the orbifold because we know that the equations of motion of the dilaton are satisfied with a constant dilaton which implies no source terms for the dilaton in the bulk. Thus, the entire contribution to the action comes from the boundary term even when the $\alpha'$ corrections are taken into account and we can reliably calculate it in a scheme independent way using the conical geometry of the exact solution at the boundary.

From the change in the classical action between the $\mathbf{Z}_N$ orbifold and flat space, we can calculate the leading semiclassical contribution to the free energy up to an $N$-independent additive constant:

$$F(N) = -\frac{A}{4G}\frac{(N-1)}{2\pi}. \tag{7}$$

The resulting entropy is

$$S = -2\pi\frac{\partial F(N)}{\partial N} = \frac{A}{4G}, \tag{8}$$

which is in precise agreement with the Bekenstein-Hawking entropy. Note that the entropy is independent of $N$ even though the latent heat scales with $N$.

Unlike the change in the classical action, the height of the tachyon potential is dependent on the renormalization scheme. It seems likely, however, that there exists a particular renormalization scheme of the $N = 2$ supersymmetric sigma model of the cone in which the beta function equations are reproduced exactly by (6) to all orders in $\alpha'$ and not just to leading order. This would be analogous to Calabi-Yau compactifications where one can argue that there always exists a scheme in which the metric is Ricci flat [15]. In such a renormalization scheme, the dilaton would be constant; the metric would be precisely conical as for the exact solution of the orbifold. In this scheme, the height of the tachyon potential $\mu_N$ for the $\mathbf{Z}_N$ would then be directly related to the deficit angle $\delta$ of the solution using Einstein equations from (6) and would be given by

$$\mu_N = 2\delta = 4\pi\left(1 - \frac{1}{N}\right). \tag{9}$$

It would be interesting to verify this prediction in some form of closed string field theory if the scheme dependence can somehow be taken into account.

*Conclusions and discussion.*—We end with a few comments and open problems. The calculation here may seem similar to the calculation of Gibbons and Hawking [16] or Teitelboim [17] where the entropy comes from the classical gravitational action. However, conceptually, it is fundamentally different. The Gibbons and Hawking calculation in canonical gravity is in an effective low energy theory and is insensitive to the high energy modes. In canonical gravity, a cone is not a saddle point. One can introduce energy density $\mu$ as a chemical potential for holding the area of the horizon fixed and then the cone of arbitrary deficit angle can be a saddle point [6,17]. But this chemical potential is not related to the potential of any dynamical field. There are no IR divergences in these calculations and the UV divergences simply renormalize Newton's constant in the effective theory.

Somehow, string theory, as a more complete theory of gravity, automatically supplies additional tachyonic fields at the Euclidean horizon. The string partition function is a functional integral over the bulk fields such as the metric as well as the twisted fields including the tachyons and thus is clearly different from the Gibbons and Hawking path integral. The cone is now a saddle point of the full string equations of motion only for integer $N$ but without having to introduce any chemical potential by hand. The UV finite partition function is now IR divergent, which implies a dynamical instability. One of the main motivations of this paper is to take seriously the dynamics of the new fields supplied by string theory. This dynamics at the tip seems to capture the physics of the heat bath close to the horizon. The entropy computed this way is universal in that it does not depend on what bulk theory we started with but is determined entirely by the potential of the tachyonic degrees of freedom near the horizon.

The UV divergence of entropy in field theory is intimately related to the puzzle of loss of information in black hole evaporation. If the entropy does have a statistical interpretation in terms of counting of states, then its divergence would suggest an infinite number of states associated with a finite mass black hole. As long as the black hole has an event horizon, it can apparently store an arbitrary amount of information in terms of correlations between the outgoing radiation and the high energy modes near the horizon. When the horizon eventually disappears, the information in these correlations is irretrievably lost. Finiteness of the entropy as we have found both in UV and IR, on the other hand, implies unitary evolution without information loss.

The tachyons at the tip of the cone are very similar to the thermal tachyons in string theory at finite temperature that come from the winding modes around the Euclidean time direction [18,19]. They cannot be interpreted as states in the spectrum but are rather new order-parameter fields. In the Lorentzian continuation, the tachyon condensate near the tip of the cone would seem to imply a Hagedorn-like phase near the Lorentzian horizon reminiscent of the "membrane paradigm" [20]. Apparently, the entropy of this phase accounts for the black hole entropy. A useful analogy is the deconfinement transition in large $N$ QCD. It is as though we are able to identify the order parameter for the deconfinement transition in the form of tachyons and compute the latent heat of the transition precisely from the potential to find a tree level contribution of order $N^2$ that is suggestive of gluon degrees of freedom. It would be desirable to figure out in a Hamiltonian formalism, the analog of the gluon degrees of freedom. Since the thermal entropy accounts for the entire tree-level entropy, it suggests that the Einstein action is wholly induced by the interactions of the UV degrees of freedom. Matrix theory [21] or a gauge theory dual [22] may be the right framework to address this question. It is interesting to explore the tachyon landscape irrespective of the problem of black hole entropy and to verify the form of the potential in a closed string field theory. It would also be interesting to pursue the implications of these results for the Hagedorn transition in string theory [23] as well as for holography [22,24,25].

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[1] J. D. Bekenstein, Phys. Rev. D **7**, 2333 (1973); **9**, 3292 (1974).
[2] S. W. Hawking, Phys. Rev. D **14**, 2460 (1976); Commun. Math. Phys. **43**, 199 (1975); Commun. Math. Phys. **87**, 395 (1982).
[3] A. Sen, Mod. Phys. Lett. A **10**, 2081 (1995).
[4] A. Strominger and C. Vafa, Phys. Lett. B **379**, 99 (1996).
[5] G. 't Hooft, Nucl. Phys. **B256**, 727 (1985).
[6] L. Susskind, Phys. Rev. D **49**, 6606 (1994); Phys. Rev. Lett. **71**, 2367 (1993); hep-th/9309145.
[7] W. H. Zurek and K. S. Thorne, Phys. Rev. Lett. **54**, 2171 (1985).
[8] L. Bombelli, R. Koul, J. Lee, and R. Sorkin, Phys. Rev. D **34**, 373 (1986).
[9] M. Srednicki, Phys. Rev. Lett. **71**, 666 (1993).
[10] C. Callan and F. Wilczek, Phys. Lett. B **333**, 55 (1994).
[11] A. Dabholkar, Nucl. Phys. **B439**, 650 (1995).
[12] A. Dabholkar, Phys. Lett. B **347**, 222 (1995).
[13] D. Lowe and A. Strominger, Phys. Rev. D **51**, 1793 (1995).
[14] A. Adams, J. Polchinski, and E. Silverstein, J. High Energy Phys. **110**, 29 (2001).
[15] D. Nemeschansky and A. Sen, Phys. Lett. B **178**, 365 (1986).
[16] G. W. Gibbons and S. W. Hawking, Phys. Rev. D **15**, 2752 (1977); S. W. Hawking, Phys. Rev. D **18**, 1747 (1978).
[17] M. Bañados, C. Teitelboim, and J. Zanelli, Phys. Rev. Lett. **72**, 957 (1994); S. Carlip and C. Teitelboim, Classical Quantum Gravity **12**, 1699 (1995).
[18] B. Sathiapalan, Phys. Rev. D **35**, 3277 (1987).
[19] Ya. I. Kogan, JETP Lett. **45**, 709 (1987).
[20] *The Membrane Paradigm,* edited by K. Thorne, R. Price, and D. Macdonald (Yale University Press, New Haven, CT, 1986).
[21] T. Banks, W. Fischler, S. Shenker, and L. Susskind, Phys. Rev. D **55**, 5112 (1997).
[22] O. Aharony, S. Gubser, J. Maldacena, and H. Ooguri, Phys. Rep. **323**, 183 (2000).
[23] J. Atick and E. Witten, Nucl. Phys. **B310**, 291 (1988).
[24] G. 't Hooft, gr-qc/9310026.
[25] L. Susskind, J. Math. Phys. (N.Y.) **36**, 6377 (1995).