Creation of a Monopole in a Spinor Condensate

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We propose a method to create a monopole structure in a multicomponent condensate by applying the basic methods used to create vortices and solitons experimentally in single-component condensates. We also show that by using a two-component structure for a monopole, we can avoid many problems related to the previously suggested three-component monopole. We discuss the observation and dynamics of such a monopole structure, and note that the dynamics of the two-component monopole differs from the dynamics of the three-component monopole.

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Introduction.—The experimental realization of spinor Bose-Einstein condensates [1,2] makes it feasible to extend the study of topological quantum objects into an entirely new field of physics. Ordinary single-component condensates have many topologically interesting properties such as the existence of vortices [3]. But in spinor condensates one can also study phenomena that cannot exist in the single-component systems. One example is a monopole structure in a spinor condensate with antiferromagnetic interactions as proposed by Stoof *et al.* recently [4]. Other novel possibilities also exist such as Skyrmions [5,6]. With Bose-Einstein condensates a crucial aspect is not only the existence and stability of topological structures, but also the methods for their creation and observation, as well as their dynamics. In this Letter we address all these aspects for a monopole structure in an experimentally relevant case of a multicomponent Bose-Einstein condensate.

A monopole is a topological defect in a vector field. It is characterized by a unit vector that is radial in respect to some unique central point (i.e., the "hedgehog" defect). In spinor condensates the vector quantity could be the local spin of the condensed atoms [7], but other choices are also possible [4,8]. Monopoles have been studied theoretically in two-dimensional condensates [7,8]. Recently some results for the three-dimensional case (such as density distribution, energy, and dynamics of such a defect) have been studied by Stoof *et al.* [4]. They described a monopole created in an antiferromagnetic spin-1 condensate such as ^{23}Na. The monopole was characterized by a spinor

$$\zeta = \sqrt{\frac{n}{2}} \begin{pmatrix} -m_x + im_y \\ \sqrt{2}\, m_z \\ m_x + im_y \end{pmatrix}, \tag{1}$$

where n is the condensate density and the vector $\mathbf{m} = \pm\mathbf{r}/r$ is a radial unit vector and has the spherically symmetric hedgehog structure. Stoof *et al.* demonstrated that this particular spin texture is a unique consequence of the unit winding number and minimization of the gradient energy. The monopole can also be displaced from the center of the trap without changing their argument. The spinor in Eq. (1) is nonmagnetized and can be achieved from the single-component mean-field ground state, $\zeta_0^T = \sqrt{n}\,(010)$, with local spin rotations. Consequently, at each position it resembles the ground state, and thus the absence of dynamical instabilities which lead to domain formation [9,10] is ensured.

Two-component monopole.—One is not, however, limited to the antiferromagnetic texture given in Eq. (1) when considering monopoles. We can alternatively map the vector $\mathbf{m}$ into an effective two-component system:

$$\zeta = \sqrt{n} \begin{pmatrix} -m_x + im_y \\ m_z \\ 0 \end{pmatrix}. \tag{2}$$

To ensure the stability of this texture against phase separation [11], the spin-1 condensate must have ferromagnetic interactions, which makes the ^{87}Rb spinor condensate a potential candidate. In other words, the preparation of a monopole is not limited to the antiferromagnetic ^{23}Na system as expected before [4], if we accept spinors that do not have the order-parameter space of the ground state. Also, the texture in Eq. (2) is not the texture of a spin monopole. The mapping of a three-dimensional vector into two wave functions is not unique, but all mappings are related by rotations and/or inversions. Therefore alternative mappings are possible by representing the radial vector $\mathbf{m}$ in a different basis, i.e., $\mathbf{m}' = \mathbf{R}\mathbf{m}$, where $\mathbf{R}$ is an operator that consists of rotations and inversions only. The angular momentum will point toward the north pole of the used coordinate system.

The static properties of the two-component monopole are similar to those of a three-component spinor (there are differences in dynamics, as we shall discuss later). Because it should also be easier to create experimentally, we focus on the two-component case. We note that as the spinor in Eq. (2) is not the mean-field ground state locally, some relaxation towards the true ground state is to be expected. But this relaxation can easily be made very slow and can thus be ignored at time scales of interest [9].

We describe the condensate with a multicomponent wave function and label the components as ψ_m with the spin projection quantum number m ($m = 0, \pm 1$). The relevant mean-field Gross-Pitaevskii (GP) equations are [12]

$$i\hbar \frac{\partial \psi_{-1}}{\partial t} = \mathcal{L}\, \psi_{-1} + \lambda_a(|\psi_{-1}|^2 + |\psi_0|^2)\psi_{-1}\,, \qquad i\hbar \frac{\partial \psi_0}{\partial t} = \mathcal{L}\, \psi_0 + \lambda_a|\psi_{-1}|^2\psi_0\,, \tag{3}$$

where $\mathcal{L} = -\frac{\hbar^2}{2m}\nabla^2 + V_{\mathrm{trap}} + \lambda_s(|\psi_{-1}|^2 + |\psi_0|^2)$, $\lambda_s = \frac{4\pi\hbar^2}{m}(a_0 + 2a_2)$, $\lambda_a = \frac{4\pi\hbar^2}{m}(a_2 - a_0)$, and a_F is the s-wave scattering length in the total hyperfine two-atom F channel. For a cylindrically symmetric trapping potential we use $V_{\mathrm{trap}} = m\omega_r^2(x^2 + y^2)/2 + m\omega_z^2 z^2/2$.

Monopole structure and stability.—To understand and test our approach for monopole creation we study the monopole structure by solving Eq. (3) numerically. In order to create the monopole as an initial state of our numerical study we force the texture given in Eq. (2) [or Eq. (1)] into the order parameter (this should not be confused with the actual proposal for experimental creation of the monopole, to come later). The imprinted spinor is then propagated in imaginary time until sufficient convergence is reached. If the monopole is at the center of the condensate the imprint has to be done only once at the beginning of the iteration, otherwise the imprint must be repeated in the course of the iteration to prevent the monopole from drifting away from the intended location, to a location with lower energy. In Fig. 1 we show the typical density distribution of the spin-1 monopole located at the center of a trap.

By looking at the individual condensate components we gain relevant insight into the structure of the monopole. Here ψ_{-1} has a vortex at $z = 0$ with a core size that is a function of z. On the other hand, ψ_0 goes through a π phase shift as we move from positive to negative z values; consequently, this component relates to a soliton in a single-component condensate. The $m = 0$ component atoms fill the vortex line everywhere else except at the origin, where the density of the $m = 0$ component also vanishes. Therefore the intersection of the vortex line with the soliton plane gives rise to a monopole core. Thus, if we can experimentally create a vortex and a soliton in a two-component system, we can obtain a monopole.

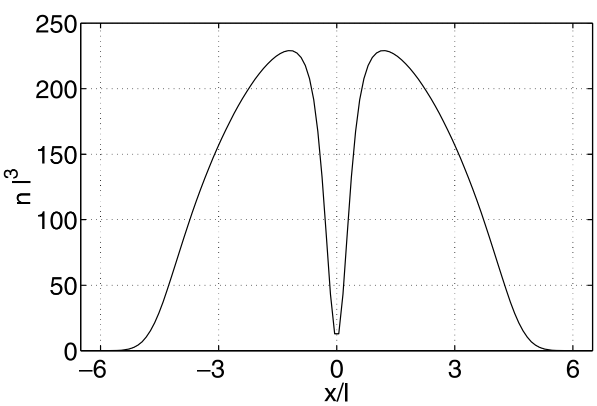

FIG. 1. The total density ($l = \sqrt{\hbar/m\omega}$) of the spin-1 condensate with 5×10^4 ^{87}Rb atoms when $y = z = 0.05$ (origin is not present in our discretization scheme). The monopole is at the center of a spherically symmetric trap with a trap frequency $\omega = \omega_r = \omega_z = (2\pi)50$ Hz.

Creation of monopoles.—The separate look into each spin component of the monopole structure suggests a possible way to create it. For example, we can prepare a spinor condensate with $2/3$ population at the $m = 1$ state and the rest at $m = 0$ state, e.g., with an rf pulse [13]. A "blueprint" of the monopole is achieved by creating a vortex into the $m = 1$ component and a soliton (with π phase discontinuity) into the $m = 0$ component, both at the trap center. The vortex line should lie along the phase discontinuity in the $m = 0$ wave function. In Fig. 2 we demonstrate the time evolution of such a mixture in a cigar-shaped trap in real time, when the soliton and vortex were created using the phase-imprint method [14]. With other excitations abounding, it is clear that we nevertheless have a monopole inside the condensate.

In our numerical studies we have used a certain amount of smoothing to reduce the amount of noise that would be created if a phase imprint is too abrupt. Smoothing for a vortex was done by assuming that not only do we have a phase mask, but also a narrow (on the order of the coherence length) beam that bores a hole through the $m = 1$ component along the vortex line. For a soliton the π phase jump was done at the distance of the order of the coherence length. Without such smoothing our numerical approach becomes unstable. It is not clear how much

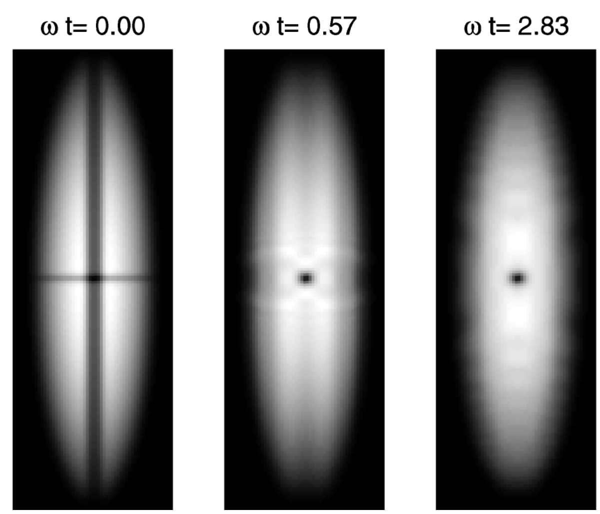

FIG. 2. The time evolution of the total density of a condensate with 5×10^4 ^{87}Rb atoms in a cylindrically symmetric trap with frequencies $\omega = \omega_r = (2\pi)250$ Hz and $\omega_z = (2\pi)50$ Hz. Initially there is a vortex at the $m = -1$ component (perpendicular line) and a soliton at the $m = 0$ component (horizontal line). The vortex and the soliton are imprinted at $t = 0$ and the figures show the cut of the total density in the $y = 0.09$ plane. The x axis is along the horizontal direction.

this smoothing is required in actual experiments, although some amount would always be present. To obtain this combined phase imprint experimentally is probably complicated, but at least the two main ingredients, namely experimental creation of vortices [15] and solitons [16] have already been achieved.

Observation of monopoles.—The monopole core has roughly the size of the healing length, and it is inside the condensate. Thus its direct observation is difficult. But the same methods used to observe vortices [15,17] and vortex rings [18] can be applied to observe monopoles as well. One should first let the condensate expand and then image the 3D structure of the different m states using two orthogonal probe beams. As for separating the different m states, one can use an appropriate Stern-Gerlach apparatus [9].

We have studied the behavior of a freely expanding monopole using the time-dependent generalization of the monopole GP equation in Ref. [4]. This approximation assumes equal scattering lengths, but at the time scales of interest the role of differing scattering lengths is, in fact, negligible. In the limit that $a_2 = a_0$ all the components feel the same spherically symmetric potential (external potential and the mean-field terms) and are not sensitive to the phase of the other components. Therefore a single-component GP equation is sufficient for modeling the expanding monopole.

In Fig. 3 we show an example of the time evolution of the ratio of the monopole core size to the system radius once the trapping potential is turned off. Expansion is qualitatively similar to vortex expansion in a scalar condensate. At short times the monopole core size, ξ, adjusts (almost) instantaneously to the local density [19] and one expects the size of the core to scale as the healing length, $\xi_0 = 1/\sqrt{4\pi a n}$. If we model the wave function as

$$\Psi(r) = A \exp\left(-\frac{r^2}{2R^2}\right)\exp\left(\frac{i\beta r^2}{2}\right), \qquad (4)$$

where A is the normalization factor, the monopole grows faster than the expanding condensate, or more precisely

$$\frac{\xi_0}{R} = \frac{\pi^{1/4}R^{1/2}}{\sqrt{4\pi a N}}. \qquad (5)$$

This happens as long as the characteristic time for adjustment of the core size, $\tau_{ad} \sim \hbar/n\lambda_s$, is much less than the expansion time $\tau_{ex} \sim R/c_s$, where c_s is the sound velocity. The parameters in Fig. 3 imply that in this regime the condensate can expand by an order of magnitude, and in the end of this regime the monopole size compared to the condensate size has greatly increased. At later times the atoms in the cloud will evolve as free particles and ξ_0/R will settle to some constant value. We also compared the results obtained with a single component GP equation (at small times) against the solution of the multicomponent GP equations (3) and found that the two approaches give essentially the same results.

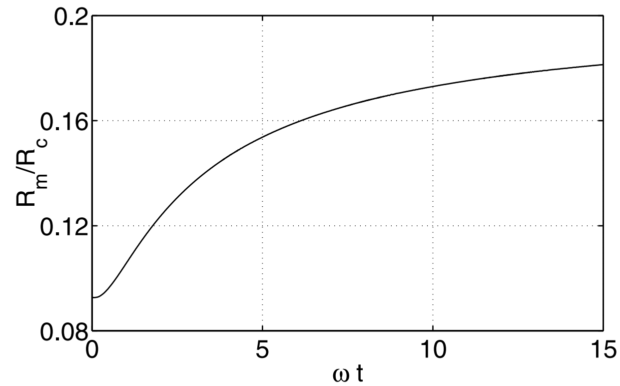

FIG. 3. Time evolution of the ratio of the monopole core size R_m to the condensate size R_c. These sizes were determined from locations where the density was one-half of the maximum density. Initially we assumed 5×10^4 condensed rubidium atoms in a spherically symmetric trap with a trap frequency $\omega = (2\pi)50$ Hz.

Monopole dynamics.—The dynamics of a monopole is quite interesting. As expected, the monopole at the origin is stable and stationary. A displaced monopole, on the other hand, behaves differently [20]. The monopole precesses around the trap center inside the condensate, just like a displaced vortex line does. It returns to its initial location after $T = 2\pi/\Omega_P$. For a vortex close to the center the precession frequency Ω_P coincides with the frequency of the anomalous mode ω_a. For a disk-shaped trap an analytic result is available [21] and is given by

$$\omega_a = -\frac{3}{2}\frac{\hbar}{mR^2}\ln\left(\frac{R}{\xi}\right), \qquad (6)$$

where R is the radius of the system and ξ is the healing length. Even though the trap geometry in our example is nowhere near the disk shape, we expect that Eq. (6) gives a reasonable order of magnitude estimate. Especially so since the $m = -1$ atoms are "squeezed" between lobes of $m = 0$ atoms, thus making the disk-shaped approximation rather justified.

As a test case we take a spherically symmetric trap with trap frequency $\omega = (2\pi)50$ Hz and a ^{87}Rb condensate with 5×10^4 atoms. Setting R to the Thomas-Fermi radius of the system and calculating ξ from the Thomas-Fermi result at the trap center we get an estimate $\omega T \approx 35$. This value is fairly close to the value actually seen in our 3D simulation of the monopole dynamics. If the monopole was displaced by one-fifth of the Thomas-Fermi radius our numerical result is $\omega T \approx 38$. The above estimate is surprisingly accurate, in particular, because the precessing vortex should feel the mean field of the other component.

The dynamical behavior of the three-component monopole of Eq. (1) is different from above. In this case one has a vortex at the $m = -1$ state and an antivortex at the $m = 1$ state. As a displaced vortex and an antivortex precess in opposite directions, the monopole core will vanish only to reappear at the opposite side as soon as the vortices have precessed that far. This recurrence is almost perfect [20]. Partial revival of a monopole has also been predicted in case of a 2D monopole [8]. As the vortices precess in opposite directions, the order parameter becomes magnetized and can no longer be represented in the form given by Eq. (1). Therefore, the order parameter is no longer in the order parameter space of the ground state. Obviously, our proposed approach to create monopoles experimentally applies to the three-component case as well. But then one needs to create a vortex/antivortex in the $m = \pm 1$ state, respectively, in addition to the soliton in the $m = 0$ state.

In Ref. [4] the dynamics of the monopole were due to the two different scattering lengths. In our inhomogeneous spinor condensate the dynamics are not intimately connected with differing scattering lengths. Setting $a_2 = a_0$ does not change our results qualitatively and even the quantitative changes are small. Therefore it seems clear that in an inhomogeneous spinor condensate the dynamical behavior of the monopole goes beyond the model suggested by Stoof *et al.* in Ref. [4].

If we displace the monopole from the center of the spherically symmetric trap, the energy of system decreases as a function of the displacement (in qualitative agreement with the results in Ref. [4]). A soliton in a dissipative environment turns into a gray soliton and accelerates to the edge of the condensate [22]. The vortex, on the other hand, is expected to spiral away from the condensate [23]. By inserting a small imaginary part to the time step we saw that both of these expectations are fulfilled. Estimates for the time scales of these processes go beyond the model used in this paper. But in single-component condensates the vortex lifetimes can be several seconds [17], and this time scale is also, presumably, the upper limit of the monopole lifetime. As the cores of the soliton and the vortex are filled with the other component, one can suspect that the thermal component plays a less significant role than it does in a one-component system. Also problems caused by the dynamical instability of the soliton can be avoided in the two-component system [24].

To summarize, we have proposed and demonstrated numerically a method to create monopoles in three-dimensional Bose-Einstein condensates, and shown that monopole creation is not limited to the antiferromagnetic spinor condensates. In addition, we have studied the detection by expansion of such monopoles, and also the dynamics of displaced monopoles in a trap. For the creation of two-dimensional monopoles, an off-resonant Raman beam has been suggested [7], but so far there has not been any suggestions for creating a three-dimensional monopole in a realistic experiment. Also, replacing a single vortex at one component with a lattice of vortices in our approach could lead to creation of multiple monopoles, and allow one to study interactions between monopoles.

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[1] D. M. Stamper-Kurn *et al.,* Phys. Rev. Lett. **80**, 2027 (1998).
[2] M. D. Barrett *et al.,* Phys. Rev. Lett. **87**, 010404 (2001).
[3] F. Dalfovo *et al.,* Rev. Mod. Phys. **71**, 463 (1999), and references therein.
[4] H. T. C. Stoof *et al.,* Phys. Rev. Lett. **87**, 120407 (2001).
[5] H. T. C. Stoof, cond-mat/0002375.
[6] U. Al Khawaja and H. T. C. Stoof, Nature (London) **411**, 918 (2001).
[7] J. J. Garcia-Ripoll *et al.,* Phys. Rev. A **61**, 053609 (2000).
[8] T. Busch and J. R. Anglin, Phys. Rev. A **60**, R2669 (1999).
[9] H.-J. Miesner *et al.,* Phys. Rev. Lett. **82**, 2228 (1999).
[10] J. Stenger *et al.,* Nature (London) **396**, 345 (1998).
[11] E. Timmermans, Phys. Rev. Lett. **81**, 5718 (1998).
[12] H. Pu *et al.,* Phys. Rev. A **60**, 1463 (1999).
[13] M.-O. Mewes *et al.,* Phys. Rev. Lett. **78**, 582 (1997).
[14] L. Dobrek *et al.,* Phys. Rev. A **60**, R3381 (1999).
[15] M. R. Matthews *et al.,* Phys. Rev. Lett. **83**, 2498 (1999).
[16] S. Burger *et al.,* Phys. Rev. Lett. **83**, 5198 (1999); J. Denschlag *et al.,* Science **287**, 97 (2000).
[17] K. W. Madison *et al.,* Phys. Rev. Lett. **84**, 806 (2000); J. Mod. Opt. **47**, 2715 (2000); Phys. Rev. Lett. **86**, 4443 (2001).
[18] B. P. Anderson *et al.,* Phys. Rev. Lett. **86**, 2926 (2001).
[19] E. Lundh *et al.,* Phys. Rev. A **58**, 4816 (1998).
[20] For the movies of the time evolution, visit the web page http://www.physics.helsinki.fi/~jpjmarti/Monopole/Info.html.
[21] A. A. Svidzinsky and A. L. Fetter, Phys. Rev. Lett. **84**, 5919 (2000).
[22] P. O. Fedichev *et al.,* Phys. Rev. A **60**, 3220 (1999).
[23] P. O. Fedichev and G. V. Shlyapnikov, Phys. Rev. A **60**, R1779 (1999).
[24] B. P. Anderson *et al.,* Phys. Rev. Lett. **86**, 2926 (2001).