## Periodically Dressed Bose-Einstein Condensate: A Superfluid with an Anisotropic and Variable Critical Velocity

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We consider a two-component atomic gas illumined by two intersecting laser beams which induce Raman coupling between the components. This spatially periodic coupling modifies the dispersion relation of the gas. Properties of a Bose-Einstein condensate of such a gas are strongly affected by this modification. Using the quasiparticle excitation spectrum derived from a Bogoliubov transformation, the Landau critical velocity is found to be anisotropic and can be widely tuned by varying properties of the dressing laser beams.

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In numerous physical systems, particles confined to a medium can be treated as free particles whose properties are modified by the medium. For example, electrons in a periodic solid can be treated as particles with properties which can be engineered by modifying the periodic structure. In this Letter, we consider similarly the task of engineering novel macroscopic quantum behavior in an atomic gas by placing the gas in a properly constructed periodic medium.

A periodic potential for an atomic gas can be produced by intersecting two or more laser beams. A polarizable atomic gas illuminated by two intersecting off-resonant laser beams with identical polarization and frequency experiences a spatially periodic potential proportional to the light intensity; i.e., the atoms reside in a crystalline medium made of light. Atoms in such media have been studied in the nondegenerate [1] and quantum degenerate [2] regimes. A small frequency difference  $\omega = \omega_1 - \omega_2$  between the laser beams can induce Bragg transitions between identical internal but different momentum states. Such Bragg transitions have been used to probe Bose-Einstein condensates [3].

In this Letter, we present a different scheme for engineering properties of an ultracold gas. Rather than considering a spatially periodic potential, we consider a spatially periodic coupling between two internal states of an atomic

gas; i.e., we consider using laser beams which effect Raman rather than Bragg transitions. An atomic gas in this medium is characterized by a tunable dressed-state dispersion relation. In creating a Bose-Einstein condensate of such a gas, one may explore how the nature of free single-particle excitations can modify macroscopic properties of a quantum fluid. In particular, we show that this novel quantum fluid has a variable and anisotropic superfluid critical velocity.

Let us consider a uniform dilute gas composed of atoms of mass m with two internal ground states,  $|a\rangle$  and  $|b\rangle$ , separated by an energy  $\hbar\omega_0$ , and an excited state  $|e\rangle$  (Fig. 1). This gas is exposed to two laser beams (labeled 1 and 2), with wave vectors  $\mathbf{k}_1$  and  $\mathbf{k}_2$  and frequencies  $\omega_1$  and  $\omega_2$ , respectively. The beams are polarized so that beam 1 connects states  $|a\rangle$  and  $|e\rangle$  and beam 2 connects states  $|b\rangle$  and  $|e\rangle$ , while both beams have a large detuning  $\Delta$  from these transitions. These laser beams can induce Raman transitions between the two internal ground states. A transition from state  $|a\rangle$  to  $|b\rangle$  results in a momentum kick of  $\hbar\mathbf{k} = \hbar(\mathbf{k}_1 - \mathbf{k}_2)$ . Such coupling can be represented in position space as a spatially periodic coupling between the two internal states, proportional to  $e^{i\mathbf{k}\cdot\mathbf{r}}|b\rangle\langle a|+e^{-i\mathbf{k}\cdot\mathbf{r}}|a\rangle\langle b|$ .

A gas exposed to these beams can be described by the second-quantized dressed-state Hamiltonian [4]

$$\mathcal{H} = \sum_{\mathbf{q}} \left[ \left( \frac{\hbar^2 q^2}{2m} - \frac{\hbar \omega_0}{2} \right) a_{\mathbf{q}}^{\dagger} a_{\mathbf{q}} + \left( \frac{\hbar^2 q^2}{2m} + \frac{\hbar \omega_0}{2} \right) b_{\mathbf{q}}^{\dagger} b_{\mathbf{q}} \right] + \hbar \omega_1 \left( c_1^{\dagger} c_1 - N_1 - \frac{1}{2} \right) + \hbar \omega_2 \left( c_2^{\dagger} c_2 - N_2 - \frac{1}{2} \right) + \sum_{\mathbf{q}} \left( \epsilon c_2^{\dagger} b_{\mathbf{q}+\mathbf{k}/2}^{\dagger} c_1 a_{\mathbf{q}-\mathbf{k}/2} + \epsilon^* c_1^{\dagger} a_{\mathbf{q}-\mathbf{k}/2}^{\dagger} c_2 b_{\mathbf{q}+\mathbf{k}/2} \right).$$

$$(1)$$

Here  $a_{\bf q}$  and  $b_{\bf q}$  ( $a_{\bf q}^{\dagger}$  and  $b_{\bf q}^{\dagger}$ ) are the annihilation (creation) operators for atoms with wave vector  ${\bf q}$  in internal states  $|a\rangle$  or  $|b\rangle$ , respectively. The operators  $c_1$  and  $c_2$  ( $c_1^{\dagger}$  and  $c_2^{\dagger}$ ) are photon annihilation (creation) operators for photons in beams 1 and 2.

Now define states  $|a_{\mathbf{q}}\rangle \equiv |N_1 + 1; N_2; a, \mathbf{q} - \mathbf{k}/2\rangle$  and  $|b_{\mathbf{q}}\rangle \equiv |N_1; N_2 + 1; b, \mathbf{q} + \mathbf{k}/2\rangle$  which are connected by

a Raman transition. The notation indicates that for state  $|a_{\bf q}\rangle$  there are  $N_1+1$  photons in beam 1,  $N_2$  photons in beam 2, and one atom is in state  $|a\rangle$  with momentum  $\hbar {\bf q} - \hbar {\bf k}/2$ . These two states have the same total momentum [5], which we define as  $\hbar {\bf q}$  by a choice of reference frame.

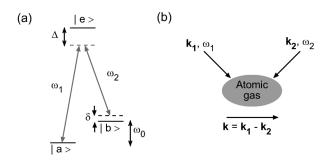


FIG. 1. Engineering properties of a periodically dressed atomic gas. (a) Laser beams of frequency  $\omega_1$  and  $\omega_2$  induce Raman transitions between internal states  $|a\rangle$  and  $|b\rangle$ . (b) Such a Raman transition imparts a momentum transfer of  $\hbar \mathbf{k} = \hbar(\mathbf{k}_1 - \mathbf{k}_2)$ , where  $\mathbf{k}_1$  and  $\mathbf{k}_2$  are the wave vectors of the Raman coupling lasers.

Diagonalizing the Hamiltonian in the subspace of states  $|a_{\bf q}\rangle$  and  $|b_{\bf q}\rangle$  yields the dressed-state atomic eigenstates  $|\pm_{\bf q}\rangle$  with energies

$$\hbar \omega_{\mathbf{q}}^{\pm} = \frac{\hbar^2}{2m} \left( q^2 + \frac{k^2}{4} \right) \pm \frac{\hbar}{2} \sqrt{\left( \delta - \frac{\hbar \mathbf{q} \cdot \mathbf{k}}{m} \right)^2 + \Omega^2}, \tag{2}$$

where  $\delta = (\omega_1 - \omega_2) - \omega_0$  is the detuning from the Raman resonance. We define  $\Omega = 2\epsilon\sqrt{(N_1+1)(N_2+1)}/\hbar$  as the (real) two-photon Rabi frequency, while  $\epsilon$  is related to the product of dipole matrix elements which couple states  $|a\rangle$  and  $|b\rangle$  to the excited state  $|e\rangle$ . Creation operators  $\pi_{\bf q}$  (for state  $|+_{\bf q}\rangle$ ) and  $\mu_{\bf q}$  (for state  $|-_{\bf q}\rangle$ ) are defined as

$$\begin{pmatrix} a_{\mathbf{q}} \\ b_{\mathbf{q}} \end{pmatrix} = \mathcal{R} \left( \frac{\theta_{\mathbf{q}}}{2} \right) \begin{pmatrix} \pi_{\mathbf{q}} \\ \mu_{\mathbf{q}} \end{pmatrix}, \tag{3}$$

where  $\mathcal{R}(\theta_{\mathbf{q}}/2) = e^{-i\sigma_y\theta_{\mathbf{q}}/2}$  ( $\sigma_y$  is a Pauli matrix) represents a rotation by the angle  $\theta_{\mathbf{q}}/2$  where  $\tan\theta_{\mathbf{q}} = \Omega/(\delta + \hbar\mathbf{q} \cdot \mathbf{k}/m)$ .

The dressed-state dispersion relation [Eq. (2)] is shown in Fig. 2. The ground state occurs in the lower dressed state at a wave vector  $\mathbf{Q}$ , which is near either  $\pm \mathbf{k}/2$  depending on the sign of  $\delta$ .

Let us now consider the effects of Raman coupling on a two-component Bose-Einstein condensate, which is now formed of a macroscopic population of atoms in the lowest energy state  $|-_Q\rangle$ . This condensate has a two-branch excitation spectrum, reflecting the presence of two internal states (and, correspondingly, two dressed-state levels). As with a scalar Bose-Einstein condensate, weak repulsive interactions should yield phononlike excitations at low energies and a free-particle-like dispersion relation at high energies. However, we expect properties of a periodically dressed Bose-Einstein condensate to reflect the anisotropy of the dressed-state dispersion relation.

Let us now treat explicitly the effects of atomic collisions by writing the many-body Hamiltonian as

$$\mathcal{H} = \sum_{\mathbf{q}} (\hbar \omega_{\mathbf{q}}^{-} \mu_{\mathbf{q}}^{\dagger} \mu_{\mathbf{q}} + \hbar \omega_{\mathbf{q}}^{+} \pi_{\mathbf{q}}^{\dagger} \pi_{\mathbf{q}}) + \mathcal{H}_{\text{int}}. \quad (4)$$

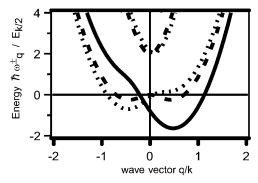


FIG. 2. Dressed-state dispersion relation: single particle energies  $\hbar \omega_{\bf q}^\pm$  vs (normalized) wave vector q/k parallel to the momentum transfer  $\hbar {\bf k}$ . Energies are scaled by  $E_k = \hbar^2 k^2/2m$ . For the traces shown,  $\hbar \delta/E_k = 3/2$  (solid lines), 0 (dashed lines), or -1/2 (dotted lines), while for all traces  $\hbar \Omega/E_k = 1$ . For  $\delta \neq 0$ , the dispersion relation is asymmetric. As  $\delta$  changes sign, the momentum of the lowest energy state changes discontinuously. For  $\delta = 0$ , the ground state is degenerate.

Considering only elastic binary collisions (which conserve the number of atoms in each of the internal states) characterized by identical s-wave scattering lengths a, we may write the interaction Hamiltonian  $\mathcal{H}_{\text{int}}$  as

$$\mathcal{H}_{\text{int}} = \frac{g}{2} \sum_{\mathbf{q}} (n_{\mathbf{q}} n_{-\mathbf{q}} - N). \tag{5}$$

Here  $g = (4\pi\hbar^2 a/mV)$ , V is the volume occupied by the gas, and N is the number of atoms in the gas. We express  $n_{\bf q} = \sum_{\bf k} (a^{\dagger}_{\bf k+q} a_{\bf k} + b^{\dagger}_{\bf k+q} b_{\bf k})$ , the spatial Fourier transform of the density operator, as

$$n_{\mathbf{q}} = \sum_{\mathbf{k}} (\pi_{\mathbf{k}+\mathbf{q}}^{\dagger} \ \mu_{\mathbf{k}+\mathbf{q}}^{\dagger}) \mathcal{R} \left( \frac{\theta_{\mathbf{k}} - \theta_{\mathbf{k}+\mathbf{q}}}{2} \right) \begin{pmatrix} \pi_{\mathbf{k}} \\ \mu_{\mathbf{k}} \end{pmatrix}. \quad (6)$$

Note that elastic collisions do not generally conserve the number of atoms in the upper and the lower dressed state, respectively.

We now make use of the Bogoliubov approximation [6] in which we assume a macroscopic population of  $N_0$  atoms in the lowest energy state of wave vector  $\mathbf{Q}$  and substitute  $\mu_{\mathbf{Q}} = \mu_{\mathbf{Q}}^{\dagger} = \sqrt{N_0}$ . To proceed, we consider the four-component vectors  $\mathbf{v} = (\mu_{\mathbf{Q}+\mathbf{q}}, i\mu_{\mathbf{Q}-\mathbf{q}}^{\dagger}, \pi_{\mathbf{Q}+\mathbf{q}}, i\pi_{\mathbf{Q}-\mathbf{q}}^{\dagger})$  and  $\mathbf{w} = (\mu_{\mathbf{Q}+\mathbf{q}}^{\dagger}, i\mu_{\mathbf{Q}-\mathbf{q}}, \pi_{\mathbf{Q}+\mathbf{q}}^{\dagger}, i\pi_{\mathbf{Q}-\mathbf{q}}^{\dagger})$ . The Bose commutation relations of the dressed-state annihilation and creation operators can be expressed as  $[v_i, w_j] = \delta_{ij}$ . Isolating terms of order  $N^2$  and N, we may now approximate the Hamiltonian as

$$\mathcal{H} \simeq \hbar \omega_{\mathbf{Q}}^{-} N + \frac{g}{2} (N^{2} - N)$$

$$+ \sum_{\mathbf{q} \neq 0} \frac{w_{i} H_{ij} v_{j}}{2} + \hbar \omega_{\mathbf{Q}}^{+} \pi_{\mathbf{Q}}^{\dagger} \pi_{\mathbf{Q}}, \qquad (7)$$

where H is a  $4 \times 4$  matrix of the form  $H_{ij} = \mathcal{E}_{ij} + \mu x_i x_j$ . The diagonal matrix  $\mathcal{E}_{ij}$  has entries  $\mathcal{E}_{\mathbf{q}}^-, -\mathcal{E}_{-\mathbf{q}}^-, \mathcal{E}_{\mathbf{q}}^+,$  and  $-\mathcal{E}_{-\mathbf{q}}^+,$  where  $\mathcal{E}_{\mathbf{q}}^{\pm} = \hbar(\omega_{\mathbf{Q}+\mathbf{q}}^{\pm} - \omega_{\mathbf{Q}}^{-})$ . The chemical potential is given by  $\mu = gN$ , and we define

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the four-component vector  $x = (\cos \Delta_{\mathbf{q}}, -i \cos \Delta_{-\mathbf{q}}, \sin \Delta_{\mathbf{q}}, -i \sin \Delta_{-\mathbf{q}})$ , where  $\Delta_{\mathbf{q}} = (\theta_{\mathbf{Q}+\mathbf{q}} - \theta_{\mathbf{Q}})/2$ .

The quasiparticle energies and their creation and annihilation operators are found by diagonalizing the matrix H [7]. The eigenvalues of H are found to be  $\hbar \tilde{\omega}_{\mathbf{Q}+\mathbf{q}}^{-}$ ,  $-\hbar \tilde{\omega}_{\mathbf{Q}-\mathbf{q}}^{+}$ ,  $\hbar \tilde{\omega}_{\mathbf{Q}+\mathbf{q}}^{+}$ , and  $-\hbar \tilde{\omega}_{\mathbf{Q}-\mathbf{q}}^{+}$ , by which we define the lower and upper quasiparticle excitation energies at wave vector  $\mathbf{Q} \pm \mathbf{q}$ .

Figure 3 shows the quasiparticle spectrum calculated for excitations parallel to the Raman transition momentum transfer. In choosing parameters for this calculation, we have in mind an experimentally convenient realization with a Bose-Einstein condensate of <sup>87</sup>Rb. One may choose the internal ground hyperfine states  $|a\rangle = |F| = 1, m_F =$  $-1\rangle$  and  $|b\rangle = |F| = 2, m_F = 1\rangle$  which can be connected by a two-photon Raman transition. Choosing these states has the benefit that the Raman transition frequency is insensitive to magnetic field fluctuations, both hyperfine states are magnetically trappable, and, specifically in <sup>87</sup>Rb, inelastic collisions are scarce. Furthermore, the scattering lengths for all elastic collisions are nearly the same, justifying the assumption made in Eq. (5). We consider the case of counterpropagating Raman laser beams which are nearly resonant with the  $D_2$  transition ( $k \simeq 4\pi/\lambda$ , where  $\lambda = 780$  nm). The chemical potential  $\mu = h \times 2.5$  kHz corresponds to a condensate density of  $3 \times 10^{14} \text{ cm}^{-3}$ .

As shown in Fig. 3, the quasiparticle energies are higher than the free dressed-state energies due to the repulsive interactions between atoms. Comparing the quasiparticle spectrum to that for the two-component condensate in the absence of Raman coupling  $(\Omega \to 0)$ , one sees that the effect of the dressing lasers is to introduce avoided crossings to the spectrum. The spectrum for quasiparticle excitations near the condensate momentum  $\mathbf{Q}$  (i.e., wave vectors  $\mathbf{Q} + \mathbf{q}$  for small  $\mathbf{q}$ ) is linear, describing phononlike

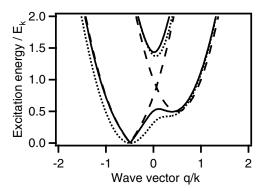


FIG. 3. Bogoliubov quasiparticle dispersion relation of a periodically dressed Bose-Einstein condensate. We consider a gas of  $^{87}$ Rb exposed to counterpropagating Raman beams, with  $\hbar\delta = -E_k/2$ ,  $\hbar\Omega = E_k$ , and  $\mu = 0.3E_k = h \times 2.5$  kHz. Excitations parallel to **k** are considered. Lower and upper quasiparticle excitation branches are shown (solid lines). The quasiparticle energy spectrum is higher than the free dressed-state dispersion relation (dotted lines). Compared to the dispersion relation in the case of no Raman coupling (dashed lines), the effect of Raman coupling is to induce a level anticrossing.

excitations. For excitations parallel to the direction of momentum transfer and making small q approximations to the Hamiltonian, we find the lower excitation spectrum to have the limiting value  $\hbar \tilde{\omega}_{\mathbf{Q}+\mathbf{q}}^- = c^* \hbar q$ , where  $c^* = \sqrt{\mu/m^*}$  is the Bogoliubov speed of sound corresponding to an effective mass  $m^*$  determined by the curvature of the dressed-state dispersion relation at its minimum. We note further that the complex matrix H which appears in the Hamiltonian has two positive and two negative real eigenvalues [8], as is required for the stability of the condensate.

Finally, we consider the implications of the dressed-state dispersion relation for the superfluidity of the periodically dressed Bose-Einstein condensate. An explanation for the dissipationless flow of a superfluid below a critical velocity  $v_L$  was provided by Landau, who used kinematic arguments to define  $v_L = \min E(\mathbf{q})/\hbar q$ , where  $E(\mathbf{q})$  is the quasiparticle excitation energy at wave vector  $\mathbf{q}$  [9]. For weakly interacting scalar Bose-Einstein condensates, the Landau critical velocity  $v_L$  is equal to the speed of sound  $c = \sqrt{\mu/m}$ . This contrasts with the behavior of superfluid <sup>4</sup>He, for which there exists a secondary minimum in the excitation spectrum corresponding to rotons [10]. The Landau critical velocity in that case is set by the roton minimum [11].

The dispersion relation of periodically dressed Bose-Einstein condensates suggests an analogy to superfluid  $^4$ He and a reduction of the superfluid velocity below the speed of sound. Figure 4 shows the Landau critical velocity calculated for flow parallel to the Raman momentum transfer  $\mathbf{k}$ , as the Raman laser detuning  $\delta$  is varied. For large values of  $|\delta|$ , the critical velocity equals the speed of sound  $\sqrt{\mu/m}$ . As  $|\delta|$  is lowered, the critical velocity

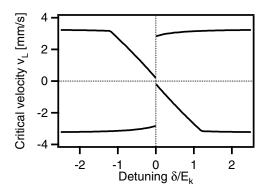


FIG. 4. Landau critical velocity in a periodically dressed Bose-Einstein condensate. Velocities aligned with (positive  $v_L$ ) or counter to (negative  $v_L$ ) the Raman momentum transfer  $\mathbf{k}$  are considered, and we take  $\hbar\Omega=E_k$ . At large detunings  $|\delta|$ ,  $v_L$  has the same magnitude for flow in both directions, with a value approaching the Bogoliubov speed of sound  $c=\sqrt{\mu/m}=3.3$  mm/s for a <sup>87</sup>Rb condensate at the density of  $3\times 10^{14}$  cm<sup>-3</sup>. At smaller detunings, an anisotropy in  $v_L$  develops. For  $\{\delta>0,v_L>0\}$  and  $\{\delta<0,v_L<0\}$ ,  $v_L$  is determined by the effective speed of sound  $c^*$ . In the regimes  $\{\delta>0,v_L<0\}$  and  $\{\delta<0,v_L>0\}$ , the magnitude of  $v_L$  is lowered due to the secondary minimum in the dispersion relation.

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becomes anisotropic. In one direction,  $v_L = c^* = \sqrt{\mu/m^*}$  as determined by phonon scattering. In the other direction,  $v_L$  is dramatically lowered due to scattering at the secondary minimum (an "artificial roton") in the dispersion relation. We may approximate the artificial roton minimum as occurring at energy  $\hbar |\delta|$  and quasiparticle momentum  $\hbar k$ , obtaining  $|v_L| \simeq |\delta/k|$  in this regime. Thus, the superfluid properties can be controlled by varying the detuning and by changing the relative angle between the Raman laser beams.

One may also vary the intensity of the Raman beams, thereby changing  $\Omega$ . An important impact of changing the Raman coupling strength is not only altering the Landau critical velocity, but also controlling the degree to which quasiparticles in the lower or upper excitation branches can be created by scattering off an obstacle. That is, the Landau criterion determines the onset of dissipation for a moving superfluid but does not describe the *strength* of such dissipation. A large Raman coupling would enhance scattering into the secondary minimum of the lower excitation branch, while in the limit  $\Omega \to 0$ , this scattering rate clearly vanishes. A detailed calculation of this dissipation rate (similar to Refs. [12–14]) will be given elsewhere.

In summary, we have described a means of engineering a novel quantum fluid composed of dressed-state atoms in a spatially periodic Raman coupling medium. This quantum fluid should be amenable to study using current techniques for probing ultracold atomic gases, such as methods for studying collective excitations at various length scales [15] and for probing aspects of superfluidity [13,16]. Further theoretical work should address a number of issues such as the effect of a trapping potential, the behavior near the Raman resonance where the ground state is degenerate, and the possible extension of our scheme to condensates with more than two internal states, such as spinor Bose-Einstein condensates [17].

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- [8] Taking the characteristic polynomial as  $P(\lambda) = \det(H \lambda I)$ , one finds (1)  $P(\lambda \to \pm \infty) \to \infty$ , (2) P(0) > 0, and (3) by evaluating  $P(\lambda)$  at plus or minus the dressed-state eigenenergies, one finds  $P(\lambda)$  crosses zero for some  $\lambda > 0$  and  $\lambda < 0$ . Thus,  $P(\lambda)$  has four zeros on the real axis.
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