Unification and Hierarchy from 5D Anti–de Sitter Space

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We show that perturbative high scale unification and a solution to the hierarchy problem are possible with extra dimensions in the context of the warped geometry of 5D anti–de Sitter space (AdS_5). This is possible because the couplings for bulk gauge bosons run logarithmically below the AdS_5 curvature scale. The calculation is done in five dimensions, rather than in the effective theory, which is strongly coupled above the TeV scale.

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The hierarchy problem is the fact that there is a large discrepancy between the Planck scale and the electroweak scale of roughly 16 orders of magnitude. This is hard to understand in the standard model of particle physics, even with a very small parameter in some fundamental Lagrangian, because quantum corrections would produce a large scale for the Higgs and hence the whole standard model. Solutions to the hierarchy problem include theories with supersymmetry or technicolor, or, more recently, extra dimensions. There are two apparent weaknesses of theories that use extra dimensions to produce the hierarchy. The first is that dangerous operators, such as those that violate baryon number, can occur with a much larger coefficient than in 4D theories, as the natural scale is TeV rather than $M_{\rm Pl}$, and has been addressed in several places [1–3]. The second weakness is that it appears that we must abandon an intriguing feature of supersymmetric grand-unified-theory (GUT) models, which the gauge couplings appear to unify near the Planck scale. There have been suggestions for addressing this problem; for example, it was discussed in the context of power law running for large extra dimensions [4] and taking advantage of the large dimension [5]. Even if these mechanisms were to yield unification, it would never be at a scale higher than a TeV, because none exists in the large extra dimensional models.

The particular model that we explore here and in [6] involves a single warped extra dimension [7]. It provides a qualitatively and conceptually new way of addressing the hierarchy problem that has opened up an entirely new branch of "beyond the standard-model" physics. The enormous ratio of the Planck scale to the TeV scale arises naturally, as it is generated as an exponential of distance in a single extra (fifth) dimension. This is possible because the warp factor essentially appears as a conformal factor at any fixed location in the fifth dimension; the natural scale varies from the Planck scale on the "Planck brane" to the TeV scale on the "TeV brane."

Warped dimensions are very interesting because of the position dependence of the natural energy scale. Although locally a TeV brane observer sees the TeV scale as the scale at which gravity becomes strongly interacting, this is not true for the global theory. Scales as high as the Planck scale appear, but in a different place in the fifth dimension. Gauge bosons that live in the bulk exist for energies about a TeV and can be weakly coupled over the entire range of energies. Although this might seem surprising for a TeV brane observer, it is not at all perplexing from the perspective of an observer on the Planck brane.

The warped (RS1) scenario therefore contrasts sharply to the large-extra-dimension scenarios, where the TeV scale is truly the cutoff for the theory. As the couplings are clearly not unified at observed scales, they must very quickly unify before the low cutoff scale. In [4], it was argued that with *power law* unification, this might happen. However, such unification is not reliable, as the theory also very quickly becomes strongly coupled. In four dimensions, unification occurs due to a slow logarithmic evolution of couplings that is under much better control.

This loss of the phenomenologically striking prediction of gauge coupling unification at a very high GUT energy scale was also thought to be true in the warped models. But it was mistakenly assumed that there is no way to have weakly coupled physics at a high scale, and that there is power law running of the couplings because there are five dimensions. We demonstrate that the strong coupling is an artifact of an effective theory calculation, and one can in fact perform perturbation theory in the full five-dimensional theory. Couplings will run logarithmically. We regard it as a major advance that one can study physics above the TeV scale in a controlled fashion. With unification proceeding in this way, one is much closer to the goal of unifying all couplings including gravity.

We show that the couplings run with β functions that are essentially multiples of the standard-model β functions. This, of course, ensures unification only with U(1) normalization of an SU(5) GUT [8]. We do not address this issue here in any detail. The details of the precision with which unification occurs is model dependent, but generically, the couplings unify at the level of the standard model, and higher precision is possible with supersymmetry or additional scalars. The calculation we do is sufficiently accurate only to establish approximate unification;

with corrections included, the unification might be better (or worse).

The scale at which unification occurs can depend on the cutoff scale where the theory becomes strongly interacting, which is again a model-dependent parameter, though ultimately one would hope to understand the microscopic physics sufficiently well to pin it down. However, for a cutoff close to the anti–de Sitter (AdS) curvature scale, we would find high scale unification as in the standard model. What is clear is that even if we view unification as a clue to physics underlying the standard model, there are many possible solutions. The physics of the warped models that address the hierarchy is entirely different from the physics of supersymmetric models. For example, the particle content at intermediate scales is much richer than the supersymmetry desert.

Reference [9] also considered unified theories with bulk gauge bosons. However, there the standard-model fields were put on the Planck brane, not the TeV brane. We abandon this assumption in favor of theories that directly address the hierarchy. Furthermore, we use a different regularization and calculational scheme. Rather than use a Pauli-Villars regulator, we use a cutoff depending on position, in a way more fully described in Ref. [6]. Furthermore, we assume a cutoff for the five-dimensional theory above the AdS curvature scale.

We begin by postulating the presence of a fifth dimension, and an anti–de Sitter space metric:

$$ds_5^2 = \frac{1}{k^2 z^2}(dt^2 - dx^2 - dz^2). \qquad (1)$$

The fifth dimension is bounded by two four-dimensional subspaces: the Planck brane at $z = 1/k$ and the TeV brane at $z = 1/T$. T is related to the size of the extra dimension R by $T = k\exp(-kR)$ and defines the energy scale on the TeV brane. If we take $T \approx$ TeV, we can naturally explain the weak scale in the standard model if the standard-model fermions and Higgs are confined to the TeV brane. Since the fifth dimension is finite, it can be integrated out to get an effective four-dimensional theory valid at energies below T.

We now put gauge bosons in the bulk [9–14]. To best perform a trustworthy calculation, we choose to do perturbation theory in the five-dimensional theory up to high scales. In five dimensions, where we know that physics is nonrenormalizable, the theory is ill-defined at high energy. We will see this corresponds to the fact that there is cutoff dependence and correspondingly regulator dependence in our result. But because the background is strongly curved, there is a large logarithmic running which is completely calculable. In principle, one can do a four-dimensional holographic calculation [15–21]; however, not only would the theory be strongly coupled, but one would need a model for the TeV brane.

To study the 5D theory, we work in position space for the fifth dimension, but momentum space for the other four. The propagator for a gauge boson in the Feynman–'t Hooft gauge is

$$\langle A^\mu A^\nu \rangle = -iG_p^{1,0}(z,z')\eta^{\mu\nu}, \qquad (2)$$

where the Green's function $G_p^{1,0}$ satisfies

$$\left[\partial_z^2 - \frac{1}{z}\partial_z + p^2\right]G_p^{1,0}(z,z') = zk\delta(z - z'). \qquad (3)$$

Applying Dirichlet boundary conditions, corresponding to positive parity under the orbifold transformation, this Green's function can be computed explicitly [6]. The propagators for fields of other spin and mass involve similar Green's functions.

In our analysis, we assume the cutoff, Λ, is greater than k as is expected and necessary for consistency. In the calculation, the scale Λ always appears multiplied by the warp factor at a given position in the fifth dimension, so that effectively there is a position-dependent cutoff. The obvious way to implement this cutoff is to integrate up to momentum $q = \Lambda/kz$ at a point z in the bulk. This is almost correct. As we argue more fully in [6], the correct procedure is to impose boundary conditions at the scale $\Lambda/(kq)$ on the Green's functions that appear in the Feynman graphs. That is, we work with a brane at the Planck scale, and a second brane at $z = \Lambda/(kq)$. With the second brane at an energy-dependent position, we integrate out the high-energy modes to derive a 5D Wilsonian effective action valid at energy q. Including the warp factor in the cutoff, as is necessary from general covariance, guarantees logarithmic running of the coupling as in four dimensions. Moving the brane in (or equivalently, imposing q-dependent boundary conditions) can be viewed as a choice of regulator. It is important to recognize that the answer is indeed cutoff and regulator dependent, since the cutoff is where the theory goes nonperturbative. In our calculation, we demonstrate the potential existence of Λ-dependent logarithms.

Our choice of regulator is in part motivated by the AdS conformal field theory (AdS/CFT) correspondence [15–19]. The idea is that string theories in certain AdS backgrounds are probably dual to conformal field theories in flat backgrounds. For example, the global symmetry group in AdS_5, SO(4, 2), is isomorphic to the conformal group in four dimensions. In particular, translations in z in AdS_5 correspond to scale transformations in the CFT. So we might suspect that integrating out a range of scales in the field theory, that is, performing a renormalization-group flow would be equivalent to integrating out a length of the fifth dimension in the 5D theory [22,23]. Since renormalization-group flows transform the whole action, including the implicit boundary conditions, it makes sense that the normalization of the Green's functions should depend on scale. Furthermore, as we discuss more fully in [6], this regulator is necessary to get the correct high-energy contribution from light Kaluza-Klein (KK) modes.

When we use this regulator to evaluate the boson self-energy graphs we find that the contribution of bulk fields is enhanced by a factor

$$I^{\sigma,m}(\Lambda,q) = 2q^4 \int_{1/k}^{\Lambda/(kq)} \frac{du}{ku} \int_{u}^{\Lambda/(kq)} \frac{dv}{kv} [G_q^{\sigma,m}(u,v)]^2. \quad (4)$$

Here σ and m refer to spin and mass. The functions $I^{\sigma,m}(\Lambda,q)$ are only weakly q dependent and can be expanded as $I^{\sigma,m}(\Lambda,q) = I_0^{\sigma,m}(\Lambda) + I_1^{\sigma,m}\frac{q}{k} + \ldots$. Numerical results confirm that it is a good approximation to include only the zeroth order term. The functions $I_0(\Lambda)^{\sigma,m}$ can be found numerically. Their general qualitative features and some sample values for various bulk fields are discussed in [6]. Roughly, $I_0 \approx 1 + (\Lambda - k)/\pi k$. The contribution to the β function from gauge bosons is essentially multiplied by the number of KK modes with mass beneath the strong coupling scale. In [9], the curvature scale was above the cutoff, so this was not observed. If one ignores the fact that there are ghost states and calculates with Pauli-Villars with a higher cutoff, one would find a similar effect. Notice that although the calculation has this nice four-dimensional interpretation, it was performed in the full five-dimensional theory.

The one-loop β function is

$$\beta(g) = -\frac{g^3}{4\pi^2}\left[C_2(G)\left(\frac{11}{3} I_0^{1,0} - \frac{1}{6} I_0^{1,i}\right) - \frac{2}{3} C(r_f) I_0^{1/2,0} n_f - \frac{1}{3} C(r_s) n_s I_0^{2,0}\right]. \quad (5)$$

The $I_0^{1,i}$ terms come from the contribution of A_5. We have included n_f massless bulk Majorana fermions and n_s massless bulk complex scalars. $C_2(G)$ is the quadratic group Casimir and $C(r)$ is the Dynkin index for the appropriate representations. Particles which are localized on the TeV brane, such as the matter of the standard model, contribute only to running below energy T. However, it is important to keep in mind that any model with charged fields on the TeV brane requires bulk charged matter in the bulk. From the holographic viewpoint, this corresponds to the fact that TeV brane matter is the bound states of the near CFT theory at higher energy scales.

Notice also that the coefficient of the logarithm scales linearly with Λ. Moreover, there is the large logarithm of conventional unification which is not present for standard power law scaling. The additional power law corrections that we have omitted are not logarithmically enhanced but reflect the nonrenormalizability of the five-dimensional gauge theory.

There are many possible theories one might consider which include standard-model TeV brane matter and bulk gauge bosons. One essential feature of any of these models is that baryon number violation be suppressed. The X and Y gauge bosons of an SU(5) model would necessarily mediate baryon-number violation with a scale suppression of only a TeV. We can suppress baryon number violation or eliminate the X and Y. Two possibilities for eliminating them are either that they do not exist or that their coupling to standard-model matter is forbidden [24–26]. In either case, there might be a unified group in a higher dimension or some other reason to expect a single coupling at high energy. We simply ask the question, given the low-energy measured couplings, do they unify at a high-energy scale with the assumption of the U(1) normalization of a GUT model? Of course, if there is no unified group, for the unification to be meaningful requires additional physics to occur at the unification scale. Otherwise, the lines cross and then diverge at higher energies. If the X and Y gauge bosons are present, unification occurs at the standard scale.

It should be borne in mind that there is a good deal of uncertainty in the models. In addition to the question of whether or not there is a contribution from X and Y gauge bosons, there is the question of what fermion and scalar fields exist in the bulk. We expect there to exist charged fermions and scalars to explain the fields confined to the TeV brane. We therefore consider an arbitrary number of scalars and fermions. The scaling depends relatively weakly on this parameter; depending on the value, one can obtain very exact unification or unification roughly at the level of the standard model. It should also be kept in mind that the threshold corrections to this calculation can be large since we focused only on the large log term. There are additional power law corrections between k and Λ/k, for example, that can modify our results and should be included in future work.

For illustration, we pick a fairly specific but very simple model. We take $\Lambda = k$ and put four Majorana fermion doublets in the bulk. These represent the preonic states in the CFT which condense to form the standard model (SM) Higgs doublet at low energy. We should also include bulk fields for the SM fermions. But since particles which transform in complete SU(5) multiplets do not affect unification or the unification scale (although they do affect the value of the couplings at this scale), we simply represent these fields with $n_g = 3$ in the following. This lets us compare to the SM most easily. The results for the couplings at a scale M_G are

$$\alpha_j^{-1}(M_G) = \alpha_j^{-1}(M_Z) - \frac{2}{\pi}\log\left(\frac{M_G}{M_Z}\right)\left[-b_j \frac{11}{12} I_0^{1,0}(\Lambda) + b_j \frac{1}{24} I_0^{1,i}(\Lambda) + c_j \frac{n_f}{12} I_0^{1/2,0}(\Lambda) + \frac{n_g}{3}\right]. \quad (6)$$

Here, $b_j = (0,2,3)$, $c_j = (\frac{3}{5},1,0)$, and relevant numerical values for I_0 are $I_0^{1,0}(1) = 1.024$, $I_0^{1,i}(1) = 0.147$, and $I_0^{1/2,0}(1) = 1.009$. There are additional terms in the above equations proportional to $I_1(\Lambda)\frac{M_G}{k}$, which we assume to be small. The couplings are shown for this case in Fig. 1, starting with the observed values [27] of $\alpha_3(M_Z)$, $\alpha_e^{-1}(M_Z)$, and θ_w. The standard model is shown for comparison.

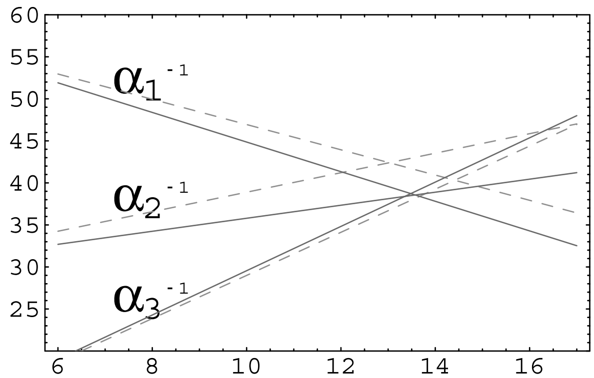

FIG. 1. α^{-1} as a function of $\log_{10}(M_G/M_Z)$. Unification of couplings for $\Lambda = k$ (solid lines). The standard model is shown for comparison (dashed lines).

Recall that the Λ dependence of the β function is roughly $[1 + (\Lambda - k)/\pi k]$. The logarithm of (M_G/TeV) is divided by this quantity. For example, if we take $\Lambda = 5$, M_G drops from 10^{14} to 10^8, but unification still occurs. Accelerated unification was considered in a different scenario in [28]. Their models also have the feature that the unification scale is connected to observable quantities.

Unlike in flat extra dimensions, in warped extra dimensions parameters run logarithmically. This is because one sees only a few KK modes; one does not get the sum of all KK contributions that adds up to very rapid power law running [4]. In theories with a cutoff that is high compared to the curvature scale, this running can be faster than the standard model; if the cutoff is low, unification is very similar to a four-dimensional model. Even with a high cutoff, a theory that has complete SU(5) representations with states of mass less than k would retain a high unification scale. In this Letter, we have presented a procedure for running couplings in AdS_5, and relating high-energy parameters to their value at the infrared scale (e.g., $\approx$TeV). This involved a regularization scheme motivated by AdS/CFT duality and the Wilsonian effective action.

One might argue that supersymmetry looks better from the point of view of unification. However, additional threshold corrections are required even in that case for unification, so the net result is also model dependent, especially when one accounts for the absence of a definite model due to the doublet-triplet splitting problem. It seems fair to say that both scenarios are possibilities at this point and that it is premature to deduce knowledge of physics up to very high-energy scales based on unification.

It is clear that there are many possibilities in terms of models and parameters, and the detailed predictions for the high-energy couplings from the low-energy ones will vary. One can, for example, consider the supersymmetric version of this theory or alternative GUT groups. Furthermore, we include only the logarithmically enhanced contribution. There are further threshold corrections arising from power law running between k and Λ, as well as higher order terms in the expansion of $I(\Lambda, q)$. These are of course in addition to the standard subleading corrections. We therefore view this work (and that of [6]) as a first step towards a more detailed and more general analysis.

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