Growth of Bose-Einstein Condensates from Thermal Vapor

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We report on a quantitative study of the growth process of ^{87}Rb Bose-Einstein condensates. By continuous evaporative cooling we directly control the thermal cloud from which the condensate grows. We compare the experimental data with the results of a theoretical model based on quantum kinetic theory. We find quantitative agreement with theory for the situation of strong cooling, whereas in the weak cooling regime a distinctly different behavior is found in the experiment.

PACS numbers: 03.75.Fi, 05.30.Jp

The nonequilibrium path to the formation of a Bose-Einstein condensate [1] is one of the most intriguing physical processes in ultracold quantum gases. When the critical temperature of this phase transition is crossed, the atoms accumulate in the ground state of the system, which leads to the characteristic dense core in the atomic distribution and to the long-range phase coherence. It is an experimental challenge to monitor this process in time, aiming for a precise understanding of the formation of Bose-Einstein condensates. Experimental studies of the growth of Bose-Einstein condensates have so far focused either on verifying the effect of bosonic stimulation [2] or on verifying the case of attractive interactions between the atoms, which gives rise to special dynamics [3]. However, the key role of the thermal cloud, from which the condensate grows, has not yet been addressed in experiments.

In a pioneering theoretical study Kagan *et al.* [4] proposed the appearance of a quasicondensate with strong phase fluctuations prior to the formation of long-range phase coherence. Stoof [5] presented the idea of nucleation followed by a growth according to a kinetic equation. A simple quantitative theory has been developed by Gardiner *et al.* [6,7] on the basis of quantum kinetic theory. The key equation describing the time dependence of the number N_0 of atoms in the condensate reads

$$\dot{N}_0 = 2W^+(N_0)\{(1 - e^{[\mu_C(N_0)-\mu]/k_B T})N_0 + 1\}, \quad (1)$$

where $\mu_C(N_0)$ is the chemical potential of the condensate, μ the chemical potential of the thermal vapor, and $W^+(N_0)$ an analytically known rate factor [7]. Starting from $N_0 = 0$ the condensate starts to grow at a finite rate, which turns quickly into exponential growth, before it reaches its asymptotic equilibrium value when the condensate chemical potential approaches that of the thermal cloud. This equation describes only the growth of the population in the ground state of the system, and it neglects certain scattering processes among atoms in higher lying states. Especially in the initial period the redistri-

bution of population between higher lying states becomes important and modifies the growth [7]. When the scattering terms are included one finds reasonable agreement with the experiment described in [2], but qualitative and quantitative discrepancies remain [7,8]. Moreover, Eq. (1) neglects the dynamics of the thermal component by assuming its chemical potential to be stationary. The influence of the dynamics of the thermal cloud was included in the recent models [8,9]. A purely numerical approach by Monte Carlo simulations is presented in [10].

We have identified and studied different stages of formation when the Bose-Einstein condensate grows from a thermal gas. In the experiment a cloud of thermal atoms is prepared above the critical temperature and then cooled into the quantum degenerate regime by continuous evaporation for a duration of up to 6 s. This strategy allows us to observe the formation process in slow motion. A distinct growth characteristic is revealed by directly controlling the properties of the thermal vapor. In comparison, previous experimental and most theoretical work addressed the situation in which a short radio-frequency pulse is applied to remove hot atoms, then turned off leaving the system to evolve toward thermal equilibrium without any further forced evaporation.

For a quantitative understanding it is crucial to prepare a well-defined initial state of the thermal cloud. As described previously [11], we load about 10^9 atoms in the $|F = 1, m_F = -1\rangle$ hyperfine ground state into a magnetic quadrupole-Ioffe configuration trap (QUIC trap). The trapping frequencies are $\omega_\perp = 2\pi \times 110$ Hz in the radial and $\omega_z = 2\pi \times 14$ Hz in the axial direction. In the magnetic trap we perform forced evaporative cooling for about 25 s by continuously lowering a radio frequency to a final value of $\nu_{rf,0} = 2120$ kHz. This prepares a purely thermal cloud of $N_i = (4.2 \pm 0.2) \times 10^6$ atoms at a temperature of $T_i = (640 \pm 30)$ nK. The minimum of the trap is determined by atom laser output coupling [12] and corresponds to a radio frequency of 1955 kHz. The stability of

the magnetic trap enables us to approach the formation of the condensate very slowly. The measured stability [13] and the good reproducibility of the experiment show that the formation of the condensate is not triggered by fluctuations of the magnetic field. When operating the trap at full current, we have measured a slow upwards drift of the bottom of the trapping potential of 5 kHz/s, which is due to thermally induced motion of the trap coils.

After this preparation of a purely thermal cloud, the atoms are cooled through the Bose-Einstein condensation phase transition. The radio-frequency field jumps abruptly to a lower frequency $\nu_{rf,1}$ and remains at this value for a period of up to 6 s. This limits the depth of the magnetic trap to ϵ_{cut}, which is expressed in units of T_i by the evaporation parameter $\eta = \epsilon_{cut}/k_B T_i$. Atoms with an energy larger than ϵ_{cut} are lost from the trap due to spin flips into a nontrapped state. Equilibration of the cloud by elastic collisions leads to a decrease of temperature and the Bose-Einstein condensate forms during this process.

The time evolution of the condensation is probed by releasing the atoms from the magnetic trap after different evolution times, for otherwise identical repetitions of the experiment. The atom cloud is imaged by absorption imaging after 17 ms of ballistic expansion. Atom numbers are extracted from the absorption images by two-dimensional fits to the atomic density distribution. The thermal cloud is fitted by a Bose-Einstein distribution under the assumption of zero chemical potential, and the condensed fraction is fitted by a Thomas-Fermi distribution. The error in the determination of the atom number is estimated to be below 10%. Here the "condensate number" refers to the integral under the Thomas-Fermi profile. Temperatures are determined from a Gaussian fit to the far-out wings of the thermal atom cloud. The statistical error on the temperature determination is 5%, whereas the systematic error is 10%–15%.

The initially prepared atomic sample corresponds to $\eta = 6$, where the gravitational sag of the atom cloud must be taken into account. Gravity pulls the atomic cloud away from the center of the magnetic trap and thereby reduces the dimensionality and the efficiency of evaporative cooling. We have performed experiments in the range $0.75 < \eta < 4.8$, corresponding to radio frequencies between 1990 and 2095 kHz. For $\eta > 4.6$ we could not obtain Bose-Einstein condensates within the 6 s period of the measurement due to the low efficiency of the cooling. For $\eta = 0.75$ we did not observe the formation of a condensate since the atom number of the initial thermal cloud was reduced too much for evaporative cooling to proceed.

Figure 1 shows the measured data for an evaporation parameter $\eta = 1.4$. Every data point is an average over three measurements, and the error bars show the statistical error. The final temperature of the cloud is (220 ± 20) nK and the final condensate fraction is $(55 \pm 7)\%$. The solid line is a numerical calculation of the condensate growth according to a model based on quantum kinetic theory, where
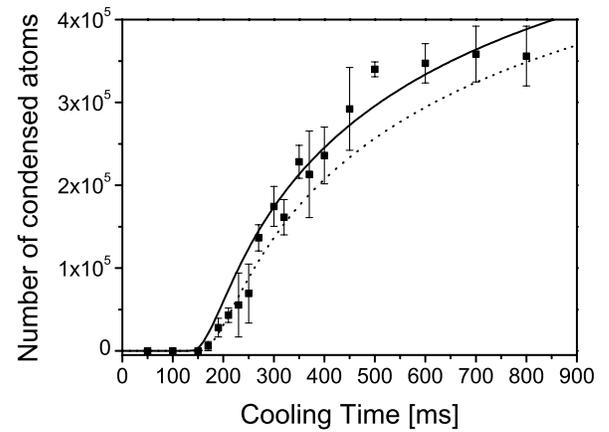
FIG. 1. Growth curve of a Bose-Einstein condensate for an evaporation parameter $\eta = 1.4$. The lines are the results of the numerical simulation of the growth for the starting conditions $N_i = 4.4 \times 10^6$ and $T_i = 610$ nK (solid) and $N_i = 4.2 \times 10^6$ and $T_i = 610$ nK (dotted). Every data point is averaged over three identical repetitions of the experiment with statistical errors shown by the bars.

the influence of the thermal atoms is taken into account [8]. The model was improved to include the continuous evaporation of atoms, the gravitational sag of the atomic cloud, the experimentally observed magnetic field drift, and three-body loss of atoms from the condensate [14]. We find quantitative agreement between the model and the measured data without any free parameters entering into the calculation, which suggests that the model describes this regime of the formation of Bose-Einstein condensates sufficiently. The best agreement between data and theory has been obtained for the calculation with the starting conditions $T_i = 610$ nK and N_i in the range $(4.2-4.4) \times 10^6$. These parameters lie within the range of the experimental uncertainties.

We have compared the time of the onset of Bose-Einstein condensation after the beginning of the final radio-frequency cooling with the calculated curves and find excellent agreement for the situation of strong cooling (see Fig. 2). To quantify the initiation time t_i for the measured data we have fitted the function

$$N_0(t) = N_{0,f}(1 - e^{-\Gamma(t-t_i)}), \qquad (2)$$

which describes the final stage of the condensate growth towards equilibrium (see Fig. 3). Γ describes a relaxation rate towards equilibrium. This functional form was suggested in Ref. [15] and subsequently found to be a reasonable approximation to more sophisticated calculations [7,9]. To account for the range of validity of the equation we have restricted the fitting range to condensate numbers larger than 30% of $N_{0,f}$, where we have obtained good agreement with the data. For low values of the evaporation parameter, i.e., rapid cooling, the initiation time approaches a minimum of 200 ms. For the example shown in Fig. 1 we obtain an initiation time of $t_i = (215 \pm 10)$ ms. This time corresponds to $30\tau_0$, where $\tau_0 = (n\sigma v)^{-1}$ is the
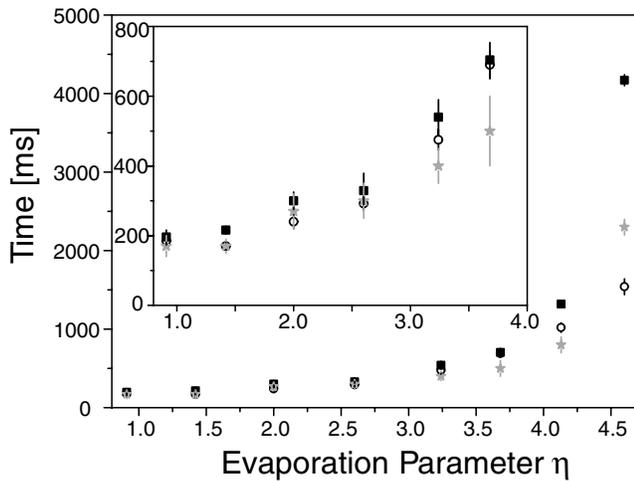
FIG. 2. Initiation times for the growth of Bose-Einstein condensates. The squares show the measured initiation times according to Eq. (2), the open circles show the times when a condensate is first detected, and the gray stars show the time the condensate population exceeds 10^4 as obtained from theoretical simulations. The inset shows an enlargement of the results for small η.

classical collision time in a thermal gas (n: mean atomic density, σ: scattering cross section, v: average thermal velocity). For the homogeneous Bose gas the first stage of formation was predicted to be on the order of τ_0 [4]. Early numerical calculations have confirmed that this should also be valid for the case of a harmonic confining potential, provided that all scattering processes are included [7,9]. Most theoretical treatments require sufficient ergodicity, meaning that ergodic mixing occurs on the order of a few col-
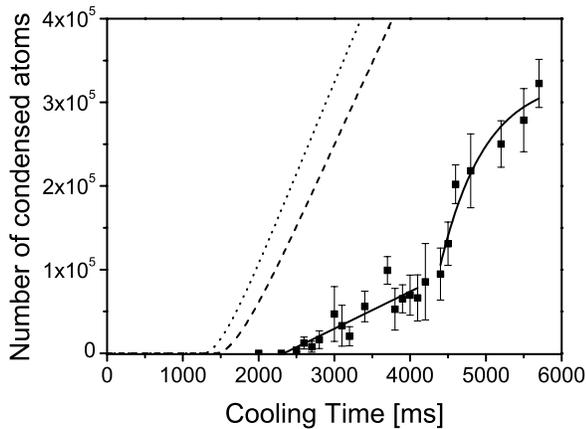
FIG. 3. Condensate growth curve for $\eta = 4.6$. The kink in the growth curve is clearly seen. It occurs at a condensate fraction of $N_0/N = 3.5\%$. The linear growth rate is inferred from a linear fit (straight line). The relaxational part of the growth curve is fitted by Eq. (2) (curved line). The initial rate in this regime is 8 times larger than the linear growth rate. Every data point is averaged over four identical repetitions of the experiment with statistical errors shown by the bars. The results of the numerical calculation are shown for the same initial parameters as in Fig. 1: $N_i = 4.4 \times 10^6$ and $T_i = 610$ nK (dashed) and $N_i = 4.2 \times 10^6$ and $T_i = 610$ nK (dotted).

lision times. At temperatures below a few μK this does not describe the experimental situation. Evaporation in the magnetic trap becomes one-dimensional due to the gravitational sag of the atom cloud. This leads to a significantly lower evaporation rate [16], and the ergodic mixing times can be longer than 10 times the collision time [10].

The measured initiation times show a pronounced divergence at $\eta \simeq 4.6$ and become much longer than the ergodic mixing time of approximately ten τ_0. Very slow cooling through the phase transition does not affect only the initiation time but it also changes the characteristics of the growth curve. We detected a slow, approximately linear growth of a condensate prior to the initiation time, determined from Eq. (2). The open circles in Fig. 2 indicate when a condensate is observed for the first time and the gray stars show the time the condensate population exceeds 10^4 as obtained from theoretical simulations of the growth experiments [14].

In Fig. 3 the two distinct stages of condensate formation can be seen for the case of slow cooling ($\eta = 4.6$): first, there is an approximately linear growth at a constant rate of $\dot{N}_0 = (44 \pm 8) \times 10^3$ s^{-1}, followed by a very rapid growth with an initial rate 8 times larger. The slow linear growth of the condensate indicates a less pronounced bosonic stimulation during this stage of the condensate formation. The numerical calculation does not reproduce the measured condensate growth data accurately in this regime. The calculation rather shows a growth rate of the same order of magnitude as the second stage of the observed growth, suggesting that the model works well if the system is sufficiently far below the transition temperature. For the thermal cloud the calculated temperatures agree with the experimentally determined values. We deduce that a possible heating rate of the atoms in the trap is below our sensitivity of 5 nK/s and thus probably negligible. The measured decrease in the total atom number for long time scales seems to be slightly larger than expected from the simulations. Whereas three-body losses in the condensate are included in the calculations, we neglect three-body recombination involving thermal atoms. The factor of 6 enhancement for these processes is negated by the much lower density of the thermal cloud: for the given trap parameters and a temperature of 300 nK one obtains a lifetime of more than 40 s [17,18]. Similarly, the background limited lifetime of the magnetic trap is at least 40 s.

We attribute the discrepancy between the theoretical model and the measured condensate numbers for slow cooling to approximations in the theoretical model which, for example, do not take into account coherence effects in the scattering process between the atoms. Possibly, the two-stage growth can be explained by the formation of quasicondensates which have been predicted as an early stage in the formation process of a Bose-Einstein condensate [4]. Quasicondensates exhibit strong phase fluctuations that die out as the system grows. These phase

fluctuations are due to a population of low-lying excited states in the trap. Density fluctuations are suppressed due to their comparatively large excitation energy, and therefore the density exhibits a Thomas-Fermi distribution. The growth rate could then be significantly smaller than expected from the assumption of Bose stimulation. Indeed, we find the growth rate to be almost 1 order of magnitude smaller than calculated from our model based on quantum kinetic theory. The time scale for both stages of the evolution is longer than the ergodic mixing time. The system is closer to equilibrium compared to the case of rapid cooling, i.e., small values of the evaporation parameter η. However, the kink in the growth curves clearly shows that the condensate is not formed adiabatically. In this case the atom number should increase according to $N_0 = N[1 - (T/T_c)^3]$ when the temperature is lowered smoothly.

It was recently found that 3D Bose-Einstein condensates can exhibit strong phase fluctuations in elongated traps [19]. The phase fluctuations are characterized by the parameter $\delta_L^2 = (T/T_c)(N/N_0)^{3/5}\delta_c^2$, where δ_c^2 is a factor that mainly depends on the trap geometry and atom number. Its value is about 0.15 for our experiment. The kink in the growth curves is observed at temperatures higher than $0.9T_c$, thus large phase fluctuations $\delta_L^2 > 1$ can be expected in the regime $N_0/N < 4\%$. We find the transition from linear growth to relaxational growth for slow cooling always in the range $1.5\% < N_0/N < 3.5\%$, which is compatible with the theoretical expectation. When the phase fluctuations have died out, i.e., $\delta_L^2 \ll 1$, a true Bose-Einstein condensate forms. This stage of the evolution then shows the same growth characteristics as observed for rapid cooling. So far calculations of the kinetics of the growth process have not taken into account the large aspect ratio of cigar-shaped traps. Therefore axial phase fluctuations may not be included in these models. Evidence for axial phase fluctuations in a steady state of a very elongated condensate is reported in [20].

In conclusion, we have investigated the influence of the thermal atom cloud on the growth of a Bose-Einstein condensate. The measured data have been compared to a numerical model based on quantum kinetic theory. We find the minimum initiation time for condensate growth to be 30 times the classical collision time, in quantitative agreement with our calculations. This comparatively long time scale is presumably due to slow ergodic mixing of the trapped atomic cloud. Very slow cooling of the cloud enables us to observe a two stage process during the formation of a Bose-Einstein condensate, which is possibly beyond the approximations made in the model.

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