

Heat-Flux Measurement in High-Prandtl-Number Turbulent Rayleigh-Bénard Convection

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We report Nusselt number measurements from high Prandtl number turbulent thermal convection experiments. The experiments are conducted in four fluids with the Prandtl number Pr varying from 4 to 1350 and the Rayleigh number Ra from 2×10^7 to 3×10^{10} , all in a single convection cell of unity aspect ratio. We find that the measured Nusselt number decreased about $\sim 20\%$ over the range of Pr spanned in the experiment. The measure data are also found in good agreement with the prediction of a recent theory over the extended range of Pr covered in the experiment.

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In the study of turbulent Rayleigh-Bénard convection, an important issue [1,2] is to determine how the Nusselt number depends on the three parameters of the system, i.e., the Rayleigh number Ra , the Prandtl number Pr , and the aspect ratio Γ . In the present work Γ is held at constant, so we write $Nu = Nu(Ra, Pr)$ and focus on its Pr dependence. Comparing to that between Nu and Ra , the $Nu(Pr)$ relationship is considerably less established and has attracted increasing attention recently [3,4]. More specifically, there are different predictions and claims regarding the regime $Pr \geq 1$ as to whether Nu is a decreasing function of Pr or is simply independent of it [5–8]. Because Pr is dictated by the properties of the convecting fluids, its variable range is rather limited for a given fluid. To determine the relation between Nu and Pr over a wide range, one needs to compare data obtained from different experiments conducted with different convection cells (sometimes of different geometries). This inevitably introduces uncertainties to the interpretation of the data, since different experiments may be subject to different systematic errors. Recently, Ahlers and Xu [9] made a high precision experiment on the Pr dependence of Nu over the range of Pr from 4 to 34 and their results appear to support the theory of Grossmann and Lohse [8] that Nu decreases with increasing Pr for $Pr > 1$. However, the values of several prefactors in the final result of the theory were obtained by fitting the experimental data from [9] and the same data were later used to show agreement between theory and experiment. Also, the theory is supposed to be valid for very large values of Pr , but the fitting data covered only a relatively narrow range of it. And it is over an extended range of Pr that the fact Nu is a decreasing function of Pr is established with certainty. It is therefore essential that another experimental test be made and over the range of Pr beyond that covered by the data used for the determination of the prefactors.

In this Letter, we report Nusselt number measurement over a wide varying range of Pr and Ra . Three organic liquids 1-pentanol, triethylene glycol, and dipropylene glycol [10], plus water are used as convecting fluids. With these fluids, we achieve a combined range of Pr spanning from 4 to 1353 and that of Ra from 2×10^7 to 3×10^{10} , all in a single convection cell of unity aspect ratio. The Rayleigh

and Prandtl numbers are calculated based on recently published properties of the fluids [11]. The convection cell is a vertical cylinder with diameter and height L both equal to 19 cm. The surfaces of the upper and lower copper plates are gold plated and the sidewall is made of Pyrex glass. The temperature of the upper plate is regulated by passing cold water through a chamber fitted on its top. To ensure temperature uniformity of the top plate, the cooling chamber is fitted with two inlets and two outlets and with stirring blades driven by the incoming water jets. The lower plate is heated at a constant rate with an imbedded film heater. The temperature difference ΔT between the two plates is monitored by four thermistors imbedded inside the plates. The measured relative temperature difference between the two thermistors in the same plate is less than 1% for both plates at all Ra . The Rayleigh number is defined as $Ra = \alpha g L^3 \Delta T / \nu \kappa$, with g being the gravitational acceleration, and α , ν , and κ , respectively the volume expansion coefficient, the kinematic viscosity, and the thermal diffusivity of the convecting fluid. The Prandtl number is defined as $Pr = \nu / \kappa$. For each data set (same Pr but different Ra), it is essential that Pr remains constant while Ra is varied. This is achieved by adjusting the heating power supplied to the bottom plate and the temperature of the cooling water flowing over the top plate in such a way that the mean fluid temperature in the center of the cell remains unchanged while the desired ΔT is obtained. To maintain the constancy of the fluid's mean temperature in the cell so that it is not subject to room temperature fluctuations and also to reduce the heat leakage, the convection cell (with thermal insulation) is placed inside a thermostat box with its temperature matched to that of the fluid in the cell center. The thermostat has a temperature stability of $\pm 0.2^\circ\text{C}$. With this setup, we are able to maintain the near constancy of the Pr for each data set. The local temperature at cell center is monitored with a small thermistor. Because of the relatively high viscosity and low thermal conductivity of these fluids, we typically waited overnight after each adjustment of Ra/Pr before starting the next measurement.

In most thermal convection studies, the Oberbeck-Boussinesq (OB) approximation [12] is assumed, where

the temperature dependencies of all fluid properties other than density are ignored and the density's temperature dependence is reflected only through the buoyancy term in the Boussinesq equation. Among the fluids used in this study, the viscosities of triethylene glycol (*F3*) and dipropylene glycol (*F4*) have quite strong dependence on temperature, whereas the viscosity of 1-pentanol (*F2*) has a rather weak temperature dependence, similar to that of water (*F1*). The other fluid properties (thermal conductivity, heat capacity, volume expansion coefficient) of the three fluids have weak temperature dependence, similar to or even weaker than those of water. Thus we expect *F3* and *F4* to exhibit the so-called non-OB effect, while the non-OB effect for *F2* and water should be weak. One measure of the non-OB effect is the ratio of the temperature drop across the top boundary layer over that across the bottom boundary layer, i.e., $x = \Delta_t/\Delta_b$ [13] with $x = 1$ corresponds to the OB limit. We found that for *F2*, its x value remains less than 1.1 for most values of ΔT (or Ra) and increased to 1.2 only for the highest ΔT in the experiment. For *F3* and *F4*, however, x increases to as high as 1.6. The strong non-OB effect for the higher Pr fluids of *F3* and *F4* also manifests as asymmetric mean temperature profile across the height of the convection cell, i.e., the mean temperature in the central bulk region T_c is warmer than the average temperature of the top and bottom plates $(T_t + T_b)/2$, in agreement with a previous study of non-OB convection [14]. Nevertheless, it appears that the non-OB effect shows up in the measured Nu only for the highest Ra and Pr where departures from a simple power law are observed (see below), which agrees with an earlier study of the non-OB effect [13]. Because of the asymmetric temperature profile, all fluid properties are evaluated based on the temperature T_c measured at the center of the convection cell [15].

To account for heat leakage through the sidewall, supporting rods, and the bottom plate, we carried out the following correction procedure. First, the cell is pumped to vacuum [16] and the effective thermal conductivity of the empty cell is measured. This is done for the various combinations of the thermostat temperature (or fluid's temperature at cell center) and the top-bottom temperature difference ΔT corresponding to those used in the actual experiment. With the known thermal conductivity of the sidewall material, we then obtain the effective thermal conductivity of the empty cell other than that of the sidewall for each of the above temperature combinations. Second, to obtain the heat current passing through the sidewall when the latter is coupled to the fluid, we adopted the method proposed by Ahlers [17]. Again, this is done for each data point, which involved numerically determining the two-dimensional temperature distribution for each case. We used a viscous layer δ_v of fixed thickness in our corrections for the sidewall leakage (model 2 in Ref. [17]), but taking $\delta_v = 2$ mm instead of 1 mm since our cell is larger than the one used in [17]. We also made corrections

using model 1 of Ref. [17] and the obtained result leads to similar conclusions.

Figure 1(a) shows a log-log plot of the measured Nu versus Ra for nine values of Pr, with the corresponding fluids indicated on the graph. The errors for Pr indicate its range of variation for the corresponding data set. First of all, the figure shows that Nu decreases monotonically and consistently with increasing Pr. It is also seen that data points near the high-Ra end in the two highest Pr data sets appear to bend downward, thus deviating from a simple power law. As shown by Wu and Libchaber [13], this is a manifestation of the non-OB effect; and so these data are not used in the power law fits $Nu \sim Ra^\beta$ (straight lines) and all subsequent analyses. For the high-Pr fluids, we find that after taking into account all the heat leakage the exponent β in general is increased by 0.04 as compared to that for the uncorrected data. Of this, roughly 0.025 to 0.02 can be attributed to corrections for leakage through the bottom plate, with the rest due to sidewall conduction. For water, the overall correction to β is less than 0.01. Note

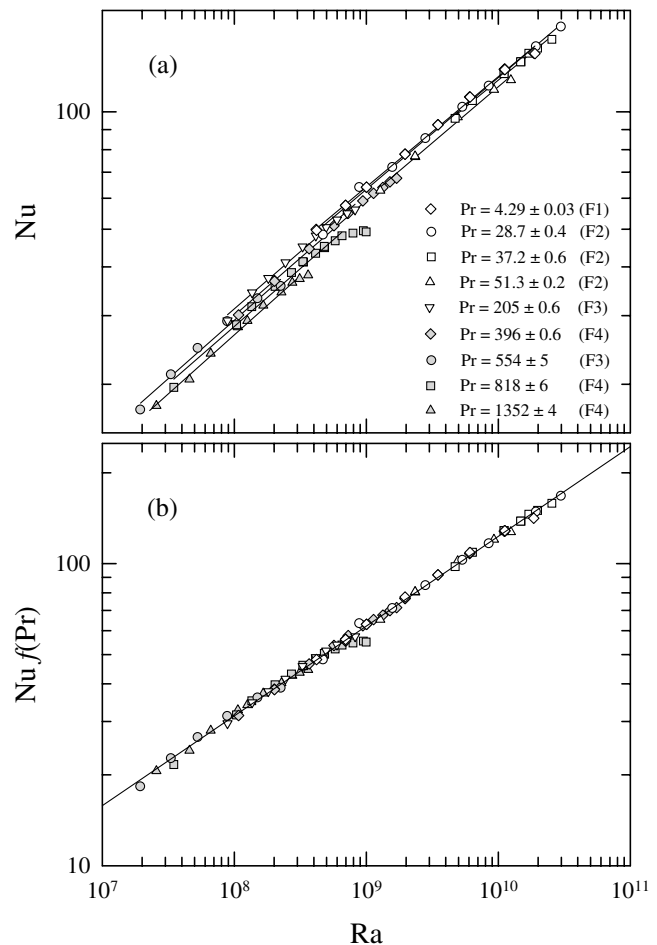


FIG. 1. (a) Measured Nu vs Ra for nine values of Pr. The solid lines are power law fits to the individual data sets (see text); (b) A log-log plot of $Nu/(Pr)$ vs Ra. The codes in the parentheses represent the fluids used, as defined in the text. The solid line is a fit to the entire data set: $Nu \sim Ra^{-0.297}$.

that the recent experiment by Xu, Bajaj, and Ahlers has put in doubt the existence of a simple power-law scaling of Nu with Ra [18]. Here we shall use the power-law fit simply to extract the Pr dependence of Nu, very much in the same spirit as in Ref. [9]. The exponent β for the power-law fits shown in Fig. 1(a) varies from 0.281 to 0.307 from the lowest to highest Pr. To extend the scaling range in Ra and thus to better extract the Ra dependence of Nu, we multiply a factor $f(\text{Pr})$ to the measured Nu such that all data points would collapse on a single line [note that $f(\text{Pr})$ is a function of Pr only and thus is a constant for each data set]. The results are shown in Fig. 1(b) where it is seen that over the range of Ra from 2×10^7 to 3×10^{10} , all data points can be well represented by a simple power law (solid line) $\text{Nu}f(\text{Pr}) \sim \text{Ra}^\beta$ with $\beta = 0.297$ (the “best exponent” for the whole set). Again, the few points that show clear non-OB effect are displayed in the figure but not included in the analysis.

With the scaling behavior of Nu established over a wide range of Ra, we fit each data set with the power-law $\text{Nu} = A(\text{Pr})\text{Ra}^\beta$ using the “best exponent” for the whole set [19]. The so obtained $A(\text{Pr}) = \text{NuRa}^{-0.297}$ are shown in a log-log plot in Fig. 2. It is seen that over the range of Pr from 4.3 to 1353, Nu shows clear decrease with increasing Pr ($\sim 20\%$). If a power law fit is attempted, we would obtain $\text{NuRa}^{-0.297} = 0.14 \text{Pr}^{-0.03}$ which is shown as the solid line in the figure. Recently, Grossmann and Lohse [8] (GL) extended their earlier model [2] to include very large Pr regimes. In essence, the theory predicts that Nu is a decreasing function of Pr for $\text{Pr} \geq 1$ and will eventually saturate at very large values of Pr. But as we already point out, experimental data had to be used to fix several prefactors needed in the final result of the theory and later the same data were used to compare with the theoretical result [8,9]. Thus it is crucial that the theory be tested with experimental data that were not used in the fitting and especially over the range of Pr beyond that of the fitting data.

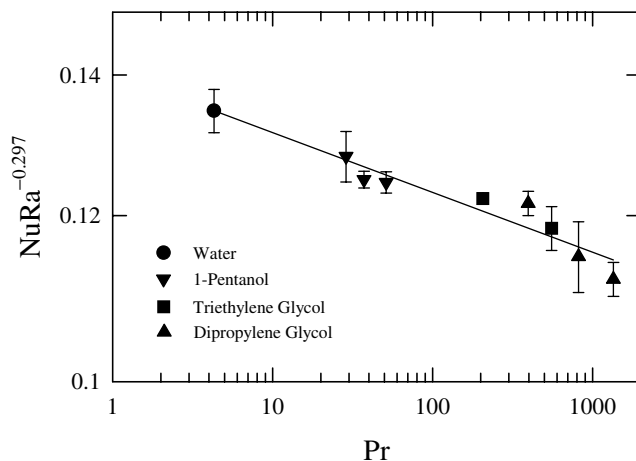


FIG. 2. Log-log plot of the compensated $\text{NuRa}^{-0.297}$ vs Pr. The solid line represents a power law fit $\text{Nu} = 0.14 \text{Pr}^{-0.03} \text{Ra}^{0.297}$ to the data points.

Figure 3 shows two theoretical curves of $\text{NuRa}^{-1/4}$ vs Pr in a log-log plot for two values of Ra, 1.78×10^9 (upper curve) and 1.78×10^7 (lower curve) [20]. Shown on the same figure are our data for the corresponding values of Ra (solid squares) and the data of Ahlers and Xu [9] (open circles) which were present in the original figure. For $\text{Ra} = 1.78 \times 10^7$ our data and the theoretical curve are seen to be in very good agreement, considering the very wide range of Pr and the fact that this is not a fitting for our data. For $\text{Ra} = 1.78 \times 10^9$, data and theory appear to follow the same trend and may be regarded as in qualitative agreement with each other, though not quantitatively. As pointed out in Ref. [17], the method we used to account for the sidewall heat leakage is probably an over correction and the effect is larger for larger values of Ra (hence larger Nu) since the relative importance of sidewall conduction decreases with increasing Nu. This could be one reason for the quantitative difference between theory and the experimental data for the higher value of Ra, as seen in Fig. 3.

In addition to that due to the cell’s heat leakage, another possible source of systematic errors for the measured Nu is the fluids’ parameters. Among them, the thermal conductivity χ most directly affects Nu ($= QL/\chi\Delta T$, where Q is the actual heat flux across the cell). Although we did not make our own fluid properties measurement, we compared the values of χ we used [11] with those from other sources [21,22]. We find that the relative difference in χ between these sources are generally in the order of 0.1% or less, depending on temperature. The exception is dipropylene glycol at $T = 50^\circ\text{C}$, which is 1.06%. Based on this, and also considering the differences between the measured and the literature values of χ in Ref. [9] (the maximum of which is 3%), we put the error in our Nu due to the uncertainties in fluid properties at $\leq 5\%$. In Ref. [9], it was

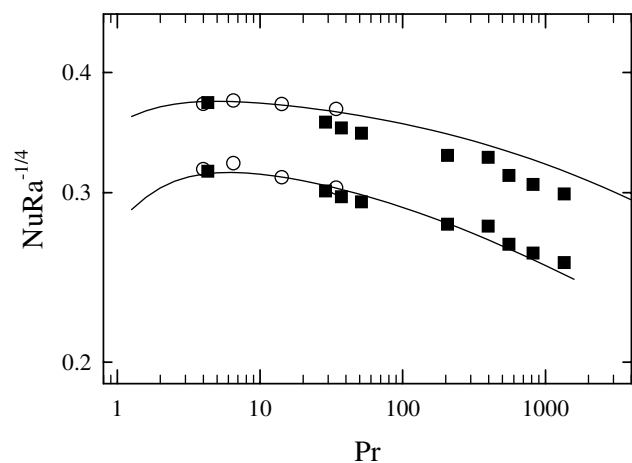


FIG. 3. Comparison between our data (solid squares) and the theoretical prediction of Grossmann and Lohse (Ref. [8]), for two values of Ra (1.78×10^9 and 1.78×10^7 for the upper and lower curves, respectively). The open circles are experimental data of Ahlers and Xu (Ref. [9]).

found that Nu decreased for about $\sim 2\%$ – 3% when Pr was increased from 4 to 34, but the authors pointed out that systematic errors could not be ruled out for the small drop. As can be seen from Fig. 3, the trend that Nu decreases with increasing Pr is established unambiguously only over an extended range of Pr . Since our Nu decayed about 20% over the varying range of Pr of the experiment, we could conclude that Nu does decrease with increasing Pr for the present regime. This conclusion is incompatible with the long standing prediction made by Kraichnan [5] that Nu is independent of Pr in the high Pr (> 0.1) and moderate Ra regime. Our data suggest that if this regime exists, the transitional Pr should be at least in the order of few thousands not 0.1. Indeed the GL theory suggests that above a certain value of Pr , Nu may become independent of Pr , and their numerical results appear to suggest that this Ra -dependent transitional Pr is around 10^4 for the range of Ra covered in our experiment.

In summary, our heat flux measurement in turbulent thermal convection over the range of Pr from 4 to 1353 and Ra from 2×10^7 to 3×10^{10} show that the Nusselt number Nu is a decreasing function of Pr and may be described by a power law of Pr with an exponent ~ -0.03 . A good agreement is also found between our data and the recent theoretical result of Grossmann and Lohse over the extended range of Pr covered in the experiment.

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