Comment on "Bicritical and Tetracritical Phenomena and Scaling Properties of the SO(5) Theory"

Recently, Hu [1] used Monte Carlo (MC) simulations on an SO(5) rotator model and concluded that the multicritical point which characterizes the simultaneous ordering of the SO(3) "antiferromagnetic" 3-component and of the U(1) "superconducting" 2-component order parameters, $\vec{s}$ and $\vec{t}$, has the critical behavior of the *isotropic* 5-component rotator model. This contradicts the renormalization group (RG) in $d = 4 - \epsilon$ dimensions, which states that (a) to a high order in ϵ, the isotropic SO(n) fixed point (IFP) is *unstable* for $n > n_c$, with $n_c < 4$ [2] and (b) to order ϵ, this multicritical point is described by the anisotropic biconical fixed point [3,4]. Measurements of isotropic 5-component critical exponents at this multicritical point were proposed as "measuring the number 5," confirming the SO(5) theory for high-T_c superconductivity [5].

Here I show that, in fact, at $d = 3$ the multicritical point *must* be tetracritical, being characterized by the *decoupled* fixed point (DFP): asymptotically the free energy breaks into a sum of the two free energies, $\vec{s}$ and $\vec{t}$ exhibit the Heisenberg ($n = 3$) and XY ($n = 2$) critical exponents, and the two critical lines cross each other at finite angles, with the crossover exponent $\phi = 1$.

The stability of the DFP follows from an exact argument, which was already presented in 1976 [2,6]: at this point, the coupling term $w|\vec{s}|^2|\vec{t}|^2$ scales like the product of two energylike operators, having the dimensions $(1 - \alpha_n)/\nu_n$, where α_n and ν_n are the specific heat and correlation length exponents. Thus, the combined operator has the dimension $d - \lambda_D$, where

$$\lambda_D = \frac{1}{2}\left(\frac{\alpha_2}{\nu_2} + \frac{\alpha_3}{\nu_3}\right) \qquad (1)$$

is the scaling exponent which determines the RG flow of the coefficient of this term near the DFP. The known negative values of α_2 and α_3 at $d = 3$ [7] then yield $\lambda \cong -0.087 < 0$, and the DFP is stable, in contrast to the order-ϵ extrapolation to $\epsilon = 1$ [3,4].

Reference [1] used a discrete spin model, with $|\vec{s}|^2 + |\vec{t}|^2 = 1$. This is believed to be in the same universality class as a Ginzburg-Landau-Wilson theory, with the quartic term $u(|\vec{s}|^2 + |\vec{t}|^2)^2$ (where initially $u \to \infty$) [8]. Reference [1] then added a coupling $w|\vec{s}|^2|\vec{t}|^2$. Quantum fluctuations [9] and RG iterations [2] then also generate a term $v(|\vec{s}|^4 - |\vec{t}|^4)$. There exist six fixed points in the $u - v - w$ parameter space, of which *only one* is stable [10]. For a continuous transition, the above argument implies an RG flow away from the vicinity of the unstable IFP, at $v = w = 0$, to the DFP, where $2u + w = 0$. This flow may be slow, since the related exponents λ_I^v and λ_I^w are small: the asymptotic DFP behavior can be observed only if $wX^{\lambda_I^w}$ becomes comparable to u, which is large. Here, $X = \max(L, \xi)$, with L the sample size and $\xi \sim (T - T_c)^{-\nu}$ the correlation length (T_c is the temperature at the multicritical point). The simulations of Ref. [1], which begin close to the ITP ($u \gg v, w$) and use relatively small L, apparently stay in the *transient* regime which exhibits the isotropic exponents. To observe the true asymptotic decoupled behavior, one should start with a more general model, allowing different interactions for $\vec{s}$ and for $\vec{t}$, relax the strong constraint $|\vec{s}|^2 + |\vec{t}|^2 = 1$, and use much larger X. The latter is also needed due to the small value of λ_D. These requirements may be impossible for realistic MC simulations.

All the above statements assume that the initial Hamiltonian is the region of attraction of the DFP. Alternatively, one should expect a *first order transition.* The possibility that *both* the IFP and the DFP are stable is highly unlikely, given the wide evidence that $n_c < 4$ and the proof of Ref. [10]. The apparent experimental observation of IFP exponents [5] may still indicate that the initial Hamiltonian is close to the ITP.

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