Stochastic Heating and Acceleration of Electrons in Colliding Laser Fields in Plasma

Z.-M. Sheng,[1,2] K. Mima,[1] Y. Sentoku,[1] M. S. Jovanović,[1] T. Taguchi,[3] J. Zhang,[2] and J. Meyer-ter-Vehn[4]

[1]*Institute of Laser Engineering, Osaka University, 2-6 Yamada-oka, Suita, Osaka 565-0871, Japan*

[2]*Laboratory of Optical Physics, Institute of Physics, Chinese Academy of Science, Beijing 100080, China*

[3]*Department of Electrical Engineering, Setsunan University, Neyagawa, Osaka, Japan*

[4]*Max-Planck-Institut für Quantenoptik, D-85748 Garching, Germany*

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We propose a mechanism that leads to efficient acceleration of electrons in plasma by two counterpropagating laser pulses. It is triggered by stochastic motion of electrons when the laser fields exceed some threshold amplitudes, as found in single-electron dynamics. It is further confirmed in particle-in-cell simulations. In vacuum or tenuous plasma, electron acceleration in the case with two colliding laser pulses can be much more efficient than with one laser pulse only. In plasma at moderate densities, such as a few percent of the critical density, the amplitude of the Raman-backscattered wave is high enough to serve as the second counterpropagating pulse to trigger the electron stochastic motion. As a result, even with one intense laser pulse only, electrons can be heated up to a temperature much higher than the corresponding laser ponderomotive potential.

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Electron acceleration by intense laser fields in plasmas has been the subject attracting a great deal of attention recently due to the advent of high power laser pulses and their potential applications. Various acceleration mechanisms have been proposed, including the plasma wave acceleration [1,2], the direct laser acceleration [3–5], and the mixed acceleration from both the transverse and the longitudinal fields [6,7].

In this Letter, we discuss a new kind of direct laser acceleration of electrons in two counterpropagating laser fields. Recently, there is a proposal, so called the superradiant amplification of ultrashort laser pulses by use of a counterpropagating laser pulse in plasma [8]; generation of periodic accelerating structures in a similar configuration has also been suggested [9]. Both of these phenomena are associated with the *coherent motion of electrons* driving by the two colliding laser pulses slightly detuned by the electron plasma frequency. Here, in contrast, our mechanism of electron acceleration results from the *stochastic motion of electrons,* which occurs when the amplitudes of two laser pulses exceed some thresholds, now easily accessible with chirped pulse amplification lasers. Moreover, the present mechanism is insensitive to their frequency differences; also it can work without a self-focusing channel, essential for the betatron resonance mechanism [5]. In plasma at a few percent of the critical density, the Raman backscattering wave of a driving pulse can serve as the second counterpropagating laser pulse, which could be intense enough to trigger stochastic acceleration if the first pulse has an intensity over $I\lambda^2 \sim 10^{18}\ \mathrm{W\,cm^{-2}\,\mu m^2}$. Therefore the present mechanism could be dominant in certain cases in laser interaction with underdense plasma. It may help to explain how the maximum electron energy can exceed the dephasing limit for particle acceleration from wave breaking observed in some particle-in-cell (PIC) simulations [10].

There have been considerable studies on electron motion in multiwave systems [11–14]. It is well-known that the Hamiltonian in multiwaves is usually not integrable. In this case, electron motion becomes stochastic when certain thresholds of the wave amplitudes are exceeded. It has been proposed by Mendonca that the formation of suprathermal electron tails observed in laser plasma interaction may be explained by the occurrence of stochastic motion of electrons in two electromagnetic waves [13,15]. However, this and other earlier studies on electron motion in a standing longitudinal plasma wave or two electromagnetic wave have been limited to the problem of the stochastic instability near the separatrices, and mostly in nonrelativistic electron motion. The potential of this stochastic instability with particle acceleration in plasma with powerful lasers focused up to relativistic intensities has not been fully explored. Here, with single particle calculations and particle-in-cell simulations, we demonstrate how electrons can be accelerated much more efficiently with the presence of a second counterpropagating pulse even at a very small amplitude than without it.

We start by considering the electron motion in two colliding planar laser fields in vacuum. The laser pulses can be described by their vector potential $\mathbf{A}_i = a_i(\xi_i)\cos(\xi_i + \psi_i)\hat{\mathbf{y}} \equiv A_i\hat{\mathbf{y}}$, where $i = 1, 2$, $\xi_1 = x - t$, and $\xi_2 = k_2(x + t)$; the frequencies of the two laser pulses are ω_1 and ω_2, respectively; x and t are normalized to c/ω_1 and ω_1^{-1}, respectively; k_2 and $\omega_2(= k_2 c)$ are normalized to ω_1/c and ω_1, respectively; and ψ_i are constants. The first pulse propagates in the positive x direction and the second one propagates in the negative x direction. The Hamiltonian for electrons is given by $H = [1 + (\mathbf{P} + \mathbf{A})^2]^{1/2}$, where the canonical momentum $\mathbf{P} = \mathbf{p} - \mathbf{A}$ is normalized by mc and vector potential $\mathbf{A}$ by mc^2/e. Since $\mathbf{A}$ is independent of y, one finds that $P_y = \mathrm{const} = p_{y0}$. For simplicity, we assume $p_z = 0$ in the following. Therefore the longitudinal motion can be described by Hamiltonian $H = [1 + p_x^2 + (p_{y0} + A_1 + A_2)^2]^{1/2}$. This Hamiltonian is still more complicated than that for electron motion in counterpropagating plasma waves [11]. Even in the nonrelativistic limit, making the canonical transform with $\eta = x - t$, $F_2 = \eta p_\eta$, and $p_\eta = p_x$, assuming $p_{y0} = 0$ and $\epsilon = a_2/a_1 \ll 1$, the resulting Hamiltonian contains two perturbation terms oscillating at different frequencies.

The instability regime for stochastic motion can be examined conveniently in geometry by the use of the surface of section plots at $\phi_2 = \xi_2 + \psi_2 \,\mathrm{mod} 2\pi = \mathrm{const}$. Figure 1 shows the surface of section plots for two lasers at the same frequency. When the amplitudes of the two pulses are the same, the electron trajectories in longitudinal momentum space are symmetric about zero as shown in Fig. 1(a). Electron trapping is found around $(p_x, \eta) = [0, -(2N + 1)\pi/2]$ by the ponderomotive potential resulting from the beating of the two laser pulses. Stochastic motion first appears around the separatices. According to the Kolmogorov-Arnol'd-Moser (KAM) theorem [12], there exist many KAM tori around a separatix. Local stochastic motion sets in when nearby KAM tori overlap. Thresholds for local stochastic motion have been estimated to be about $a_1 a_2 = 1/16$ by Mendonca [13]. When the amplitude of the forward-moving pulse a_1 is larger than a_2 for the backward-moving pulse, the electron trajectories becomes nonsymmetric about zero; stochastic motion of electrons spreads widely in positive momentum space. The larger the amplitude a_1, the wider the region for stochastic motion in positive momentum space, as shown in Fig. 1(b). For a given a_2, the width for stochastic motion scales roughly proportional to a_1^2. One notes that there remains regular motion for electrons trapped around $(p_x, \eta) = (0, 0)$, where acceleration cannot occur. However, with the increase of a_1 or a_2 further, this trapping island is gradually suppressed. Before it is fully suppressed, bifurcation occurs at certain amplitudes when the trapping island splits into two parts as shown in Fig. 1(c). This shows a transition of the stochastic motion from a local to a global one, where, in the later case, electrons initially at rest or small energy will be driven into stochastic motion and gain energy from laser fields. Until now, we have taken the frequencies of the two pulses to be the same. If the frequency of the second pulse is changed, the basic features are qualitatively similar, indicating that the stochastic motion is not sensitive to the frequency difference of the two pulses. One notes that there is no stochastic motion if the two pulses copropagate.

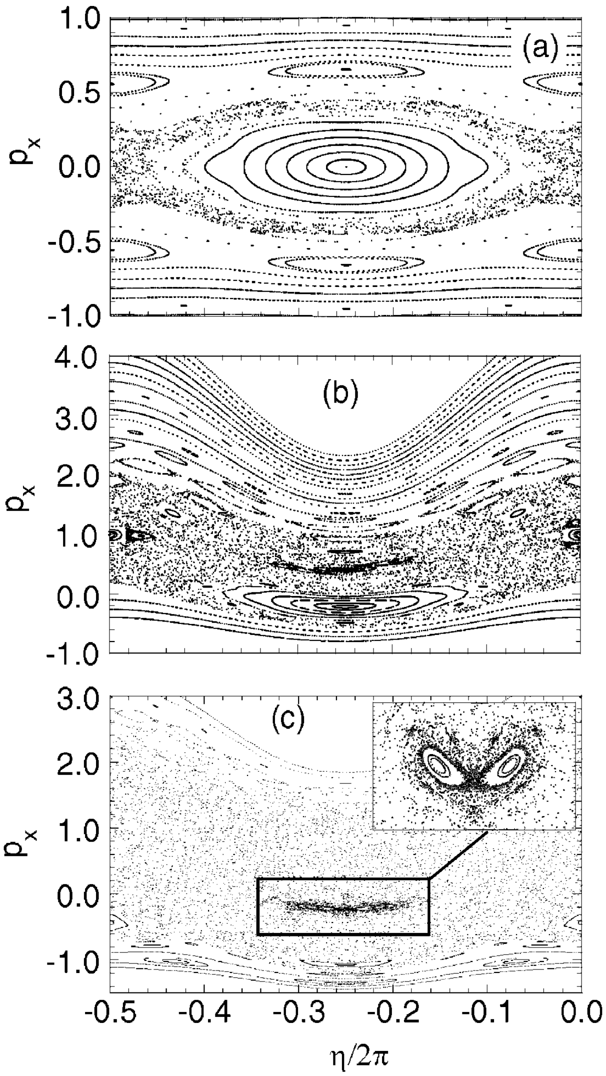

FIG. 1. Surface of section plots at $\xi_2 + \psi_2 = 2N\pi$ for electron motion in counterpropagating laser fields. (a) $a_1 = a_2 = 0.3$; (b) $a_1 = 1.0$ and $a_2 = 0.1$; (c) $a_1 = 1.0$ and $a_2 = 0.42$. Here $\eta = (\xi_1 + \psi_1) \,\mathrm{mod} 2\pi$ and $\omega_{01} = \omega_{02}$. Note that η is in the period of π when the two pulse frequencies are the same.

The Liapunov exponents can provide a quantitative measure of the degree of stochasticity for a given Hamiltonian system [12]. Let the test electron initially at rest and the amplitudes of the infinite planar pulse increase from zero in a function $\tan(t/t_L)$ with $t_L = 50$ laser cycles. The Liapunov exponents are calculated when the laser fields reach the maximum amplitudes or $t > 2t_L$. As either the amplitude a_1 or a_2 increases, the Liapunov exponents increase suddenly as the laser amplitudes exceed some threshold amplitudes. Since we start with electrons at rest initially, the obtained threshold corresponds to that for the global stochasticity around the fundamental trapping island in the surface of the section plot. This is shown in Fig. 2 by the solid line marked with $v_{x0} = 0$. It is approximately $a_1 a_2 \sim 1/2$, which is larger than that estimated by Mendonca for local stochastic motion [13], but close to that for the bifurcation of the fundamental trapping island, as expected. It is worthwhile to point out that the so-called global stochastic motion is relative and limited only to the region between some upper and lower boundaries in longitudinal momentum, beyond which, the electron motion becomes regular again. Therefore one can understand that it also depends on its initial velocity whether the trajectory of a test electron is stochastic or regular. If electrons

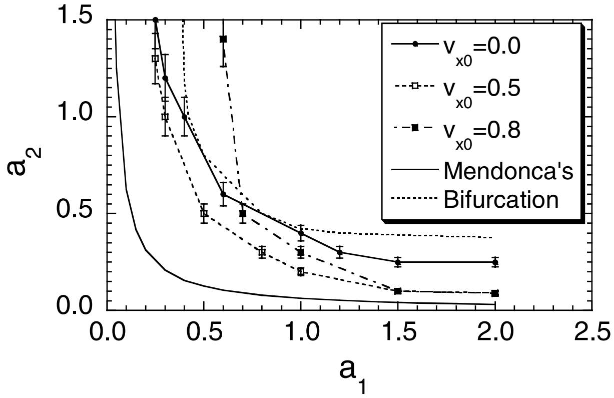

FIG. 2. Threshold amplitudes for stochastic motion for different initial electron velocities obtained numerically. Also shown are the thresholds for local stochastic motion by Mendonca and for the occurrence of bifurcation for trajectories trapped in the fundamental island around $(p_x, \eta) = (0, 0)$.

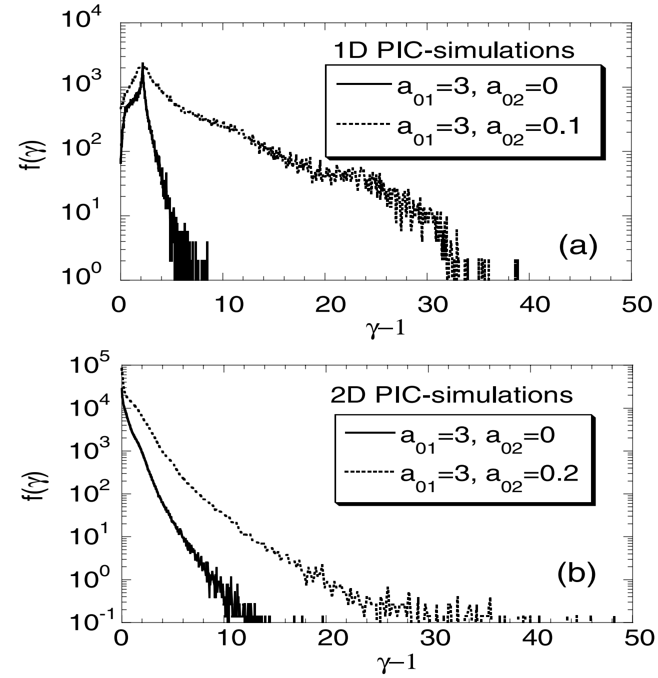

FIG. 3. Snapshots of electron energy distributions from PIC simulations of laser interaction with plasma slab $L = 50\lambda$. (a) With 1D PIC code in plasma at $n/n_c = 0.01$ at $t = 350$ laser cycles; (b) with 2D PIC codes in plasma at $n/n_c = 0.01$ and the laser beam diameter is 12 laser wavelengths at $t = 300$. The laser pulses are semi-infinite.

have some initial longitudinal velocity, such as that driven by the ponderomotive force at the front of the forward-propagating pulse, the threshold amplitudes can either reduce or increase as shown in Fig. 2. For $v_{x0} \geq 0.5$, the threshold amplitude of the counterpropagating a_2 reduces to only about 0.1 when $a_1 > 1.5$.

To confirm this acceleration mechanism, we have performed numerical simulations with PIC codes. We first try to simulate it with a 1D PIC code since it is essentially a one-dimensional effect. In simulations, the plasma is homogeneous, which occupies a region of $L = 50 \sim 100\lambda$, where λ is the incident laser wavelength. The laser pulses, which are semi-infinite and at the same frequency, increase to the maximum amplitudes in ten laser cycles. The electron energy distributions shown in Fig. 3(a) have been obtained in plasma with $n = 0.01n_c$ (n_c is the critical density). Both the electron temperatures and the maximum electron energy are much higher with a counterpropagating pulse than without it. The temperature increases up to about 3 MeV at 350 laser cycles after interaction with the counterpropagating pulse, which is 5 times larger than without it, even though its amplitude is only at $a_2 = 0.1$. We find that the temperature tends to be saturated after interaction for a certain time. This can be explained by the surface of the section plot, which shows that stochastic motion is found only in limited phase space around the separatices. For very energetic electrons, their trajectories remain regular, and therefore net energy gain from the laser fields does not occur. Notice that the peaks near $\gamma = 3$ in the distributions are due to ponderomotive push with semi-infinite pulses, which would be absent for pulses with finite duration. This simple example demonstrates obviously that the presence of the second counterpropagating pulse is very important to trigger the stochastic motion, which can lead to effective energy transfer from laser fields to electrons. Figure 3(b) has been obtained with a 2D PIC code, where the s-polarized laser pulses focused at a diameter of 12 laser wavelengths are incident into the underdense plasma slab with the same parameters as for 1D simulation. The s polarization rules out possible electron acceleration through the Betatron resonance mechanism [5]. In this simulation, when with the second counterpropagating pulse $a_2 = 0.2$, the hot electron temperature is about twice that without it. Meanwhile, the quasistatic current and the corresponding quasistatic magnetic field with $a_2 = 0.2$ are found to be more than doubled than without it. However, the acceleration in the 2D simulation appears to be not as efficient as in the 1D case. This comes because high energy particles, usually having larger transverse momenta, tend to escape from the pulse center in the 2D geometry and therefore experience shorter acceleration time than in the 1D case. In passing, we mention that, if we change the initial phases of the incident laser pulses, the electron energy distributions can be changed. This is particularly evident for the relatively low energy part in the distribution function. However, the high energy tail appears not very sensitive to the initial phase difference of the laser pulses.

To verify that electrons gain energy mainly from the transverse laser fields rather than from the longitudinal fields, one can make use of the relation $\gamma = 1 + \Gamma_\parallel + \Gamma_\perp$ following the equation of motion for electrons [5], where $\Gamma_\parallel = -\int_0^t dt' E_x v_x$ and $\Gamma_\perp = -\int_0^t dt' E_\perp v_\perp$; E_x and $E_\perp$ are the normalized longitudinal and transverse electric fields, respectively. Here $\Gamma_\parallel$ stands for the energy gain due to the longitudinal electric field, while $\Gamma_\perp$

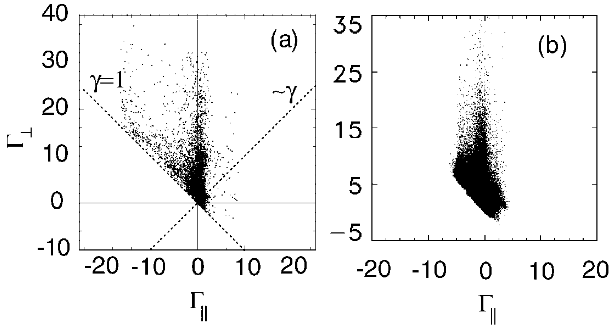

FIG. 4. Electron energy gain from longitudinal fields versus that from transverse laser fields in plasma with density $n = 0.01$. (a) $a_1 = 3.0$ and $a_2 = 0.1$ in 1D simulation at $t = 350$ laser cycles; (b) $a_1 = 3.0$ and $a_2 = 0.2$ in 2D simulation at $t = 300$.

represents the direct laser acceleration by the transverse field. The energy gain from the laser field is eventually directed in the longitudinal direction through the Lorentz force. Figure 4 shows electrons distributed in $\Gamma_\parallel \sim \Gamma_\perp$ space found in 1D and 2D simulations. It demonstrates that electrons are accelerated mainly by the transverse laser fields in both cases.

When increasing the plasma density up to $n = 0.04n_c$ or beyond, we find that electrons can be accelerated to a similar level with or without the injection of the second pulse at $a_2 = 0.1$ in 1D simulations. This can be explained by the presence of Raman-backscattered waves, which have an amplitude even higher than 0.1 as seen in the simulation, large enough to trigger the stochastic motion of electrons. In the case with $n = 0.01n_c$ and $L = 50\lambda$, however, the Raman-backscattered wave has an amplitude smaller than 0.1. Therefore a second counterpropagating wave is necessary to trigger the stochastic motion of electrons.

To see the dependence of electron temperatures on the pulse amplitudes, we take plasma at low density $n = 0.01n_c$ to avoid the high Raman-backscattered wave and change either a_1 or a_2. For the interested forward acceleration rather than the isotropic heating, we usually take $a_1 \gg a_2$. As found in 1D PIC simulations, the hot electron temperature scales similar to $T \sim a_1^{\delta_1} a_2^{\delta_2} t^{\delta_3}$, where t is the interaction duration. For the case with semi-infinite laser pulses, we find $\delta_1 \sim 2$ and $\delta_2 \sim 0.5$; while for pulses with finite pulse duration, we find $\delta_1 \sim 1$ and $\delta_2 \sim 0.5$ after the laser pulses pass through the plasma region. These different scaling laws with the intensity of the first pulse are related to the different ponderomotive push of the two cases. Scaling to the interaction duration is normally similar to $\delta_3 \approx 0.5 \sim 1.0$ before the high electron temperature becomes saturated. One notes that, although the final energy is relevant to the pulse shapes and duration, the present acceleration mechanism itself is irrelevant to these pulse parameters.

In summary, we propose a scheme that can efficiently accelerate electrons by the use of two counterpropagating laser pulses. The acceleration is triggered by stochastic motion of electrons. The threshold amplitudes for stochastic motion have been found numerically. Particle-in-cell simulations show that this mechanism can be dominant in laser interaction with underdense plasma, where the counterpropagating wave can either be the Raman-backscattered wave or the reflected wave of an incident pulse from overdense plasma. Energetic electrons generated through this scheme move predominantly in the direction of the pulse with higher intensity.

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[1] M. N. Rosenbluth and C. S. Liu, Phys. Rev. Lett. **29**, 701 (1972); T. Tajima and J. M. Dawson, Phys. Rev. Lett. **43**, 267 (1979); for a review, see E. Esarey *et al.,* IEEE Trans. Plasma Sci. **24**, 252 (1996).
[2] D. Umstadter *et al.,* Phys. Rev. Lett. **76**, 2073 (1996); E. Esarey *et al.,* Phys. Rev. Lett. **79**, 2682 (1997).
[3] G. Schmidt and T. Wilcox, Phys. Rev. Lett. **31**, 1380 (1973); C. I. Moore *et al.,* Phys. Rev. Lett. **74**, 2439 (1995); F. V. Hartmann *et al.,* Phys. Rev. E **51**, 4833 (1995); B. Rau *et al.,* Phys. Rev. Lett. **78**, 3310 (1997); E. Esarey *et al.,* Phys. Rev. E **52**, 5443 (1995); C. J. McKinstrie and E. A. Startsev, Phys. Rev. E **56**, 2130 (1997); B. Quesnel and P. Mora, Phys. Rev. E **58**, 3719 (1998).
[4] G. R. Smith and A. N. Kaufman, Phys. Rev. Lett. **34**, 1613 (1975); Phys. Fluids **21**, 2230 (1978); C. R. Menyuk *et al.,* Phys. Rev. Lett. **58**, 2071 (1987); S. Kawata, *ibid.* **66**, 2072 (1991); M. S. Hussein and M. P. Pato, *ibid.* **68**, 1136 (1992); J. Meyer-ter-Vehn and Z. M. Sheng, Phys. Plasmas **6**, 641 (1999).
[5] A. Pukhov *et al.,* Phys. Plasmas **6**, 2847 (1999); C. Gahn *et al.,* Phys. Rev. Lett. **83**, 4772 (1999).
[6] F. Brunel, Phys. Rev. Lett. **59**, 52 (1987).
[7] W. L. Kruer and K. Estabrook, Phys. Fluids **28**, 430 (1985).
[8] G. Shvets *et al.,* Phys. Rev. Lett. **81**, 4879 (1998).
[9] G. Shvets *et al.,* Phys. Rev. E **60**, 2218 (1999).
[10] K.-C. Tzeng, W. B. Mori, and T. Katsouleas, Phys. Rev. Lett. **79**, 5258 (1997).
[11] G. M. Zaslavskii and N. N. Filonenko, Sov. Phys. JETP **25**, 851 (1968); G. Schmidt, Comments Plasma Phys. Control. Fusion **7**, 87 (1982).
[12] A. J. Lichtenberg and M. A. Lieberman, *Regular and Stochastic Motion* (Springer-Verlag, New York, 1981), and references therein.
[13] J. T. Mendonca and F. Doveil, J. Plasma Phys. **28**, 485 (1982); Phys. Rev. A **28**, 3592 (1983).
[14] Z.-M. Sheng *et al.,* Phys. Rev. Lett. **85**, 5340 (2000); Z.-M. Sheng and J. Meyer-ter-Vehn, in *Superstrong Fields in Plasmas,* edited by M. Lontano *et al.,* AIP Conf. Proc. No. 426 (AIP, Woodbury, NY, 1998), p. 153.
[15] D. W. Forslund *et al.,* Phys. Rev. Lett. **54**, 558 (1985).