Intrinsic and Extrinsic Nature of the Orbital Angular Momentum of a Light Beam

A. T. O'Neil, I. MacVicar, L. Allen, and M. J. Padgett

Department of Physics and Astronomy, University of Glasgow, Glasgow, G12 8QQ, Scotland

We explain that, unlike the spin angular momentum of a light beam which is always intrinsic, the orbital angular momentum may be either extrinsic or intrinsic. Numerical calculations of both spin and orbital angular momentum are confirmed by means of experiments with particles trapped off axis in optical tweezers, where the size of the particle means it interacts with only a fraction of the beam profile. Orbital angular momentum is intrinsic only when the interaction with matter is about an axis where there is no net transverse momentum.

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Introduction.—Some 65 years ago Beth [1] demonstrated that circularly polarized light could exert a torque upon a birefringent wave plate suspended in the beam by the transfer of angular momentum. The angular momentum associated with circular polarization arises from the spin of individual photons and is termed spin angular momentum.

More recently, Allen *et al.* [2] showed that for beams with helical phase fronts, characterized by an $\exp(il\phi)$ azimuthal phase dependence, the orbital angular momentum in the propagation direction has the discrete value of $l\hbar$ per photon. Such beams have a phase dislocation on the beam axis that in related literature is sometimes referred to as an optical vortex [3]. In general, any beam with inclined phase fronts carries orbital angular momentum about the beam axis which, when integrated over the beam, can be an integer or noninteger [4,5] multiple of $\hbar$.

In this paper, we experimentally examine the motion of particles trapped off axis in an optical tweezers and are able to associate specific aspects of the motion with the distinct contributions of spin and orbital angular momentum of the light beam. The interpretation of the experiments, when combined with a numerical calculation of the spin and orbital contributions derived from established theory, allows a distinction to be made between the intrinsic and extrinsic aspects of the angular momentum of light.

Angular momentum of a light beam.—The cycle-averaged linear momentum density, **p**, and the angular momentum density, **j**, of a light beam may be calculated from the electric, **E**, and magnetic, **B**, fields [6]:

$$\mathbf{p} = \varepsilon_0 \langle \mathbf{E} \times \mathbf{B} \rangle, \tag{1}$$

$$\mathbf{j} = \varepsilon_0 (\mathbf{r} \times \langle \mathbf{E} \times \mathbf{B} \rangle) = \mathbf{r} \times \mathbf{p}. \tag{2}$$

Equation (2) encompasses both the spin and orbital angular momentum density of a light beam.

Within the paraxial approximation, the local value of the linear momentum density of a light beam is given by [2]

$$\mathbf{p} = i\omega \frac{\varepsilon_0}{2} (u^* \nabla u - u \nabla u^*) + \omega k \varepsilon_0 |u|^2 \mathbf{z}$$
$$+ \omega \sigma \frac{\varepsilon_0}{2} \frac{\partial |u|^2}{\partial r} \Phi, \tag{3}$$

where $u \equiv u(r, \phi, z)$ is the complex scalar function describing the distribution of the field amplitude. Here σ describes the degree of polarization of the light; $\sigma = \pm 1$ for right- and left-hand circularly polarized light, respectively, and $\sigma = 0$ for linearly polarized.

The cross product of this momentum density with the radius vector $\mathbf{r} = (r, 0, z)$ yields an angular momentum density. The angular momentum density in the z direction depends upon the Φ component of **p**, such that

$$j_z = r p_\phi. \tag{4}$$

The final term in Eq. (3) depends upon the polarization but is independent of the azimuthal phase and, consequently, this term may be linked directly to the spin angular momentum. The first term in Eq. (3) depends upon the phase gradient and not the polarization, and so gives rise to the orbital angular momentum.

For many mode functions, u, such as for circularly polarized Laguerre-Gaussian modes, Eqs. (3) and (4) can be evaluated analytically such that the local angular momentum density in the direction of propagation is given by [2]

$$j_z = \varepsilon_0 \left\{ \omega l |u|^2 - \frac{1}{2} \omega \sigma r \frac{\partial |u|^2}{\partial r} \right\}. \tag{5}$$

The angular momentum integrated over the beam is readily shown to be equivalent to $\sigma\hbar$ per photon for the spin and $l\hbar$ per photon for the orbital angular momentum [2], that is

$$J_z = (l + \sigma)\hbar. \tag{6}$$

A theoretical discussion of the behavior of local momentum densities has been published elsewhere [7], and it should be noted that the local spin and orbital angular momentum do not have the same functional form.

As is well known, spin angular momentum does not depend upon the choice of axis and so is said to be *intrinsic*. The angular momentum which arises for any light beam from the product of the z component of linear momentum about a radius vector, may be said to be an *extrinsic* because its value depends upon the choice of calculation axis.

Berry showed [8] that the orbital angular momentum of a light beam does not depend upon the lateral position of the axis and can therefore also be said to be intrinsic, provided the direction of the axis is chosen so that the transverse momentum is zero. When integrated over the whole beam the angular momentum in the z direction is

$$\mathbf{J}_z = \varepsilon_0 \iint dx\, dy\, \mathbf{r} \times \langle \mathbf{E} \times \mathbf{B} \rangle. \qquad (7)$$

If the axis is laterally displaced by $\mathbf{r}_0 \equiv (r_{0x}, r_{0y})$ it is easy to show that the change in the z component of angular momentum is given by

$$\Delta J_z = (r_{0x} \times P_y) + (r_{0y} \times P_x)$$
$$= r_{0x} \varepsilon_0 \iint dx\, dy\, \langle \mathbf{E} \times \mathbf{B} \rangle_y \qquad (8)$$
$$+ r_{0y} \varepsilon_0 \iint dx\, dy\, \langle \mathbf{E} \times \mathbf{B} \rangle_x.$$

The angular momentum is intrinsic only if ΔJ_z equals zero for all values of r_{0x} and r_{0y}. This condition is satisfied only if z is stipulated as the direction for which the transverse momenta $\varepsilon_0 \iint dx\, dy\, \langle \mathbf{E} \times \mathbf{B} \rangle_x$ and $\varepsilon_0 \iint dx\, dy\, \langle \mathbf{E} \times \mathbf{B} \rangle_y$ are exactly zero.

For Laguerre-Gaussian light beams truncated by apertures, Eqs. (3) and (8) can only be evaluated numerically. Nevertheless, for all apertures, of whatever size or position, the spin angular momentum remains $\sigma\hbar$ irrespective of the choice of calculation axis and so is, as expected, intrinsic; see Fig. 1. Any beam with a helical phase front apertured symmetrically about the beam axis has zero transverse momentum and, consequently, an orbital angular momentum of $l\hbar$ per photon, independent of the axis of calculation. The orbital angular momentum of the light beam may therefore be described as intrinsic. However, when the beam is passed through an off-axis aperture, its transverse momentum is nonzero and the orbital angular momentum depends upon the choice of calculation axis and so must be described as extrinsic; see Fig. 1. An interesting result occurs when the orbital angular momentum of the apertured beam is calculated about the original beam axis. Even though the transverse momentum is nonzero, the orbital angular momentum remains $l\hbar$ per photon because r_{0x} and r_{0y} are both zero. However, it does not follow that the angular momentum of the apertured beam is intrinsic as the result does depend upon the choice of calculation axis. When any beam is apertured off axis, it is simpler and more accurate to

understand its interaction with particles by considering the components of $\mathbf{p}$ in the x-y plane. For beams with helical phase fronts, these transverse components are in the Φ direction with respect to the beam axis. It is this distinction between spin and orbital angular momentum which gives rise to differences in behavior for the interaction of light with matter.

The transfer of spin and orbital angular momentum to small particles.—The interaction of small particle with the angular momentum of a light beam has been investigated by a number of groups with the use of optical tweezers. Usually implemented by use of a high numerical-aperture microscope, optical tweezers rely on the gradient force to confine a dielectric particle near the point of highest light intensity [9]. For particles trapped on the beam axis, both the spin and orbital angular momentum have been shown to cause rotation of birefringent [10] and absorptive [11] particles, respectively. For absorbing particles, both spin and orbital angular momenta are transferred with the same efficiency so that the applied torque is proportional to the total angular momentum [12], that is $(\sigma + l)\hbar$ per photon.

In this present work we also use optical tweezers, but in this instance the particles are trapped away from the beam axis. This allows us to demonstrate the difference between particle interactions with spin and orbital angular momentum. The experimental configuration is shown in Fig. 2. Our optical tweezers are based on a 1.3 numerical aperture, $\times 100$ objective lens, configured with the trapping beam directed upwards, which allows easier access to the sample plane. This beam is generated from the 100 mW output of a commercial Nd:YLF laser transformed, using a computer generated hologram, to give a Laguerre-Gaussian mode of approximately 30 mW. The beam is circularly polarized, $\sigma = \pm 1$ with a high azimuthal mode index, $l = \pm 8$. The sign of the spin or the orbital angular momentum may be reversed by the insertion of a half–wave plate or a Dove prism, respectively [13]. The radius of maximum intensity, $r_{\max}$ of a Laguerre-Gaussian mode is given by [14],

$$r_{\max} = \sqrt{\frac{z_R l}{k}}, \qquad (9)$$

where z_R is the Rayleigh range of the beam. Even under the tight focusing associated with optical tweezers, the peak intensity ring of a Laguerre-Gaussian mode of high index l may be made several μm in diameter and, consequently, be much larger than the particles it is attempting to trap.

It is not surprising, for such conditions, that we observe the particles to be confined by the gradient force at the radius of maximum light intensity and not on the beam axis. When a birefringent particle such as a calcite fragment is trapped, and circularly polarized light is converted to linear, we observe that the particle spins about its own axis. The sense of rotation is governed by the handedness of the circular polarization.
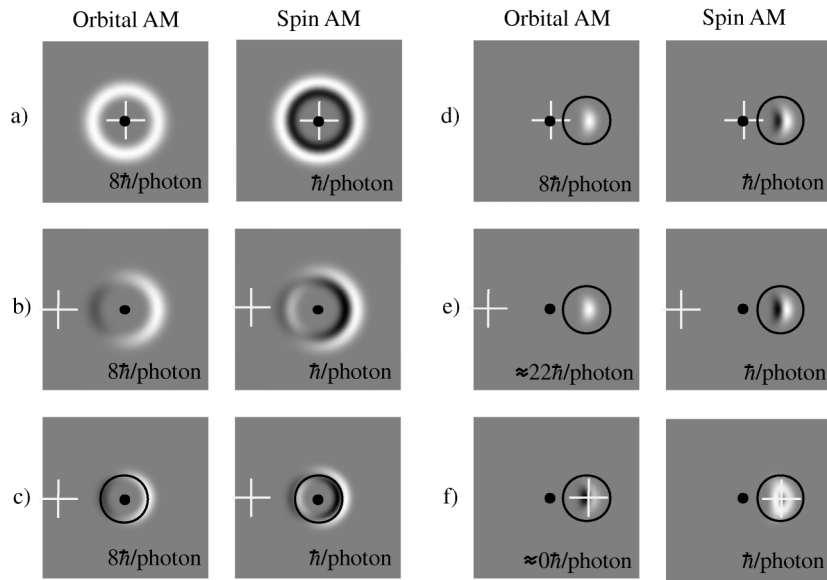
FIG. 1.   Numerically calculated local spin and orbital angular momentum densities in the direction of propagation for a $l = 8$ and $\sigma = 1$ Laguerre-Gaussian mode. A positive contribution is shown in white, gray represents zero, and black a negative contribution; the black spot marks the axis of the original beam, the white cross marks the axis about which the angular momenta are calculated and, where appropriate, the black circle marks the position of a soft edged aperture. Note that the spin angular momentum is equivalent to $\sigma\hbar$ per photon irrespective of the choice of aperture or calculation axis, whereas the orbital angular momentum is only $l\hbar$ per photon if the aperture or calculation axes coincide with the axis of the original beam.
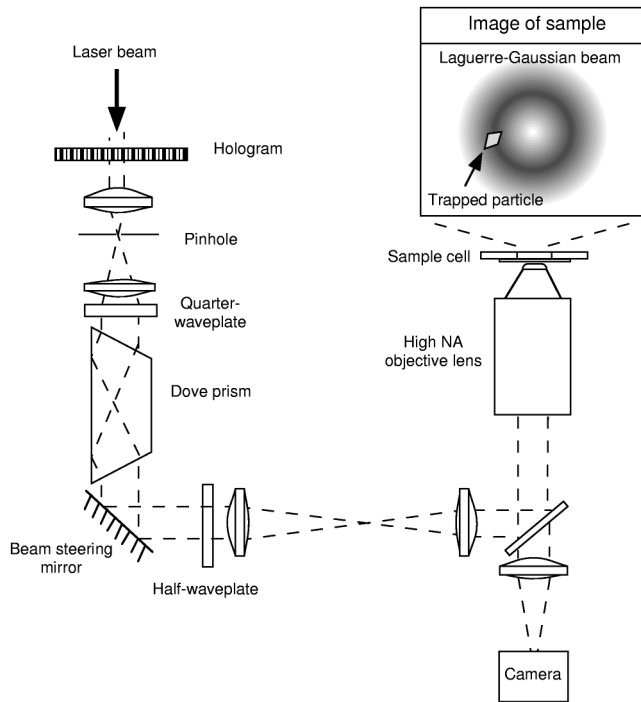

FIG. 2.   The optical tweezers use a high-order Laguerre-Gaussian beam to trap the particle in the region of maximum light intensity, away from the beam axis. The circularly polarized Laguerre-Gaussian beam is generated using a quarter-wave plate and a hologram. The sense of the orbital and spin angular momentum can be reversed by the inclusion of a half-wave plate or a Dove prism, respectively.

For small particles the force arising from the light scattering, the momentum recoil force, becomes important. For a tightly focused Laguerre-Gaussian mode, the dominant component of the scattering force lies in the direction of beam propagation. The gradient force again constrains the particle to the annulus of maximum beam intensity. However, as the intensity distribution is cylindrically symmetric, the particle is not constrained azimuthally. Because the particle is trapped off the beam axis, the inclination of the helical phase fronts and the corresponding momentum result in a tangential force on the particle. We observe that a small particle, while still contained within the annular ring of light, orbits the beam axis in a direction determined by the handedness of the helical phase fronts; see Fig. 3. We conclude that the larger calcite and small particles are interacting with intrinsic spin and extrinsic orbital angular momentum, respectively. In principle, it should be possible to observe both the orbital and spin angular momenta acting simultaneously upon the same small birefringent particle. However, our observations have been inconclusive as birefringent particles small enough for the scattering force to induce a rotation about the beam axis are typically too small to see whether they are spinning about their own axis.

This orbital and spin behavior is entirely consistent with the formulation summarized in Eqs. (7) and (8). If one considers the cross section of an off-axis trapped particle to play the rôle of an aperture, then we see that the intrinsic spin of the angular momentum creates a torque about the
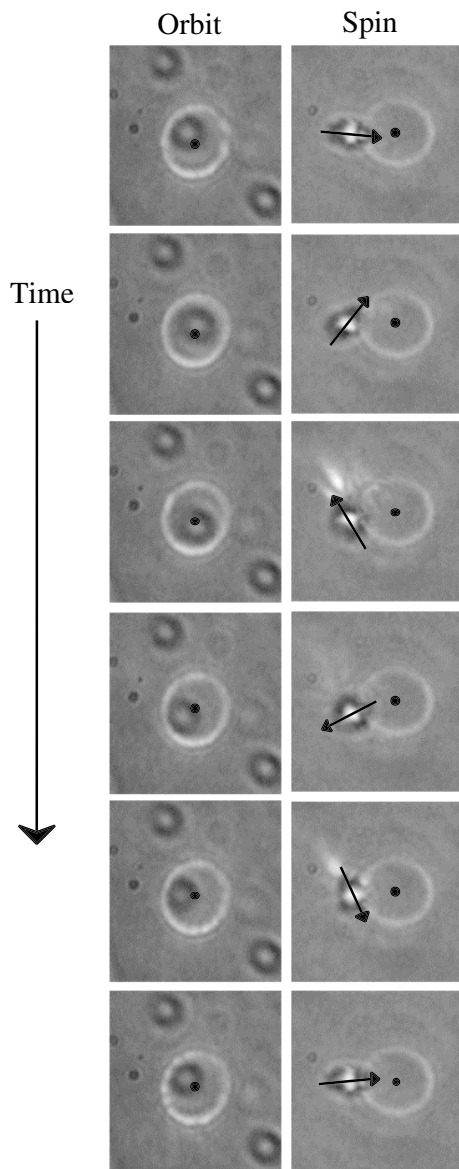
Orbit   Spin

Time

FIG. 3. Successive video frames showing particles trapped near the focus of an $l = 8$ and $\sigma = 1$ Laguerre-Gaussian mode. The left column shows particles of ≈ 1 μm diam. These particles are sufficiently small to be subject to a well-defined scattering force, allowing them to interact with the orbital angular momentum of the beam. They are set in motion, orbiting the beam axis at a frequency of ≈ 1 Hz. The right column shows a calcite fragment with a length of ≈ 3 μm and a width of about ≈ 1.5 μm, which is large enough not to interact detectably with the beam's orbital angular momentum. However, due to its birefringence it interacts with the spin angular momentum of the beam and is set spinning about its own axis at ≈ 0.3 Hz.

particle's own axis, causing it to spin. A calculation of the particle's angular momentum about an arbitrary axis shows a clear distinction between the intrinsic angular momentum associated with its spinning motion and the extrinsic angular momentum associated with its orbital motion. In this situation, orbital angular momentum is better described as the result of a linear momentum component directed at a tangent to the radius vector.

Unlike the spin angular momentum of a light beam which is always intrinsic, the z component of the orbital angular momentum can be described as intrinsic only if the z direction can be stipulated such that the transverse momentum integrated over the whole beam is zero. If an interaction is with only a fraction of the beam cross section, then the orbital angular as measured about the original axis is extrinsic.

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