Generic Entanglement Generation, Quantum Statistics, and Complementarity

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A general and an arbitrarily efficient scheme for entangling the spins (or any spinlike degree of freedom) of two independent uncorrelated identical particles by a combination of two particle interferometry and which way detection is formulated. It is shown that the same setup could be used to identify the quantum statistics of the incident particles from either the sign or the magnitude of measured spin correlations. Our setup also exhibits a curious complementarity between particle distinguishability and the amount of generated entanglement.

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Recent years have witnessed a great surge of interest in the applications of entanglement [1–3]. In this context, it is important to explore efficient and general ways of preparing entanglement. Most known mechanisms for obtaining entangled states [3–7] are dependent on the *specific nature* of the systems involved. Here we propose a very *general* scheme for entangling the spins (or any spinlike degree of freedom) of two particles of *any* type (bosons or fermions) by a combination of two particle interferometry and which way detection. The fractional yield of entangled pairs for a given number of input pairs can be arbitrarily increased by a recursive procedure using just one beam splitter and two detectors. The main application of our setup will be in entangling material objects such as neutrons, electrons, atoms, or macromolecules. This will enable testing quantum nonlocality through separate measurements on far separated massive particles. A salient feature of our setup is the fact that two independent identical particles *do not need to interact directly* for getting entangled. They need only to interact individually with beam splitters and detectors, and their *indistinguishability* can be exploited to yield entanglement. This is thus useful for entangling those particles which interact weakly (or do not interact) with other particles of the same species. Another advantage of our setup is that the disentangled initial state of the independent particles can be prepared by classical communications without any nonlinearity as opposed to the states in the current methods [3,4]. We show that the same setup allows *identification of the quantum statistics* of the incident particles from either the presence/absence of entanglement or the sign of measured spin correlations, depending upon whether the two particle input state is unpolarized or oppositely spin polarized, as opposed to previous tests based on particle number measurements [8–12]. It also exhibits a complementarity between particle distinguishability and the amount of entanglement produced. This complementarity involving "which particle" information in two particle interference differs fundamentally from the usual form involving "which way" information in single particle interference [13–16]. Our work also suggests a curious *dual* to standard entanglement in the context of second quantization.

We will first present a preliminary setup by modifying an interference process used in the last stage of production of polarization entangled photons through down-conversion [4,17] which works with 50% efficiency (successful cases being identifiable by appropriate detector clicks). Figure 1 depicts the setup composed of a beam splitter with input channels A and B, output channels C and D and which-channel detectors P_C in C and P_D in D. These detectors are assumed to be *nonabsorbing* and are able to *determine the path without disturbing the spin* (this is possible since position and spin commute; feasibility will be discussed later). Now consider two identical

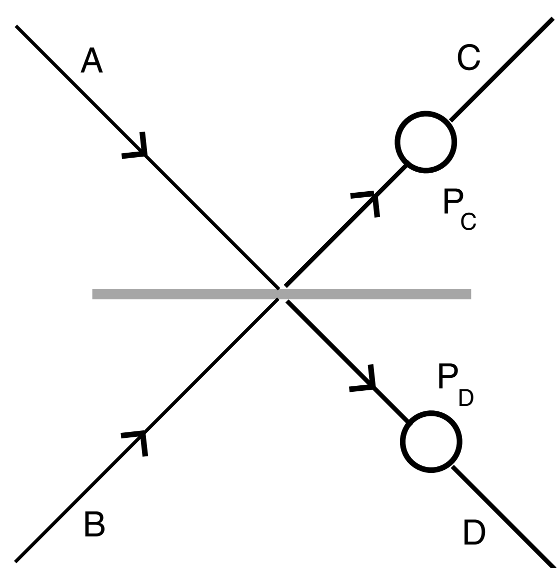

FIG. 1. A preliminary setup consisting of a beam splitter (input paths A and B, output paths C and D) and absorptionless path detectors P_C and P_D which do not disturb the spin. When a pair of identical particles with opposite spins are incident on the first beam splitter, one from arm A and the other from arm B, then corresponding to a coincidence in P_C and P_D, a spin entangled state is generated.

particles in different spin states (say $|\uparrow\rangle$ and $|\downarrow\rangle$) incident simultaneously on the beam splitter from arms A and B as shown in Fig. 1. This state, in second quantized notation, is described as $a^\dagger_{A\uparrow}a^\dagger_{B\downarrow}|0\rangle$ where $|0\rangle$ is the vacuum state and $a^\dagger_{A\uparrow}$ and $a^\dagger_{B\downarrow}$ are creation operators for $\uparrow$ spin in path A and $\downarrow$ spin in path B, respectively. We will label the state concisely as $|A\uparrow;B\downarrow\rangle$. For fermions $|A\uparrow;B\downarrow\rangle = -|B\downarrow;A\uparrow\rangle$, and for bosons $|A\uparrow;B\downarrow\rangle = |B\downarrow;A\uparrow\rangle$. The transformation done by the beam splitter is [12,18]

$$|A\uparrow;B\downarrow\rangle \rightarrow \frac{1}{\sqrt{2}}\left\{\frac{1}{\sqrt{2}}(|D\uparrow;C\downarrow\rangle \pm |D\downarrow;C\uparrow\rangle)\right\} + \frac{i}{2}(|C\uparrow;C\downarrow\rangle + |D\uparrow;D\downarrow\rangle), \qquad (1)$$

where the $+$ sign stands for fermions and the $-$ sign stands for bosons. After the detectors click, the combined state of the particles and the detectors is

$$\frac{1}{\sqrt{2}}\left\{\frac{1}{\sqrt{2}}(|D\uparrow;C\downarrow\rangle \pm |D\downarrow;C\uparrow\rangle)\right\}|P_C^*\rangle|P_D^*\rangle \oplus \frac{i}{2}(|C\uparrow;C\downarrow\rangle|P_C^*\rangle|P_D\rangle \oplus |D\uparrow;D\downarrow\rangle|P_C\rangle|P_D^*\rangle), \qquad (2)$$

where $\{|P_C\rangle, |P_D\rangle\}$ and $\{|P_C^*\rangle, |P_D^*\rangle\}$ are the unexcited and excited (corresponding to detection of one or more particles) detector states, respectively. In the above $\oplus$ has been used to indicate the lack of *coherence* between orthogonal detector states. When the detectors are found in the state $|P_C^*\rangle|P_D^*\rangle$ (coincidence), the state of the particles is projected onto $\frac{1}{\sqrt{2}}(|D\uparrow;C\downarrow\rangle \pm |D\downarrow;C\uparrow\rangle)$. The spin part of this state can be rewritten in the first quantized notation (using the paths as particle labels) as $|\psi^\pm\rangle_{CD} = \frac{1}{\sqrt{2}}(|\uparrow\rangle_D|\downarrow\rangle_C \pm |\downarrow\rangle_D|\uparrow\rangle_C)$ (spin entangled state). It is fully legitimate to use the paths as particle labels because the particles are *identical* (the same labeling is used for photon pairs exiting a parametric down converter [4]).

In the above description, we have made two significant changes to the scheme used for photons. First is the presence of the special detectors, which could be easier to design for massive particles. For photons [4,17], such detectors are absent and *after* the path measurements, the resource of entanglement is *not available* for applications. Our detectors help us to obtain a useful *source* of spin entangled particles. The other difference with the photonic scheme is that the incident state $|A\uparrow;B\downarrow\rangle$ is *not* entangled. In contrast, the down-conversion based schemes [4] use a *momentum entangled* incident state of the two photons. We merely require the particles to impinge on the beam splitter at the same instant of time; any *prior entanglement* is *not* necessary for this. For example, if atoms from two independent sources are velocity selected, they can be made to pass through the same region at the same time as in recent cavity QED experiments [19]. By removing the necessity of nonlinearity for generating the incident state, we thus enlarge the scope of the method to cover all types of particles.

We now describe our full scheme, in which the efficiency can be arbitrarily increased. First we consider the addition of two more beam splitters to the setup as shown in Fig. 2 with the four exit paths E, F, G, and H being incorporated with which-path detectors P_E, P_F, P_G, and P_H, respectively. If the state $|A\uparrow;B\downarrow\rangle$ is incident on the first beam splitter, then the final combined state of the particles and the detectors is

$$\begin{aligned}
&\ominus (|G\uparrow;E\downarrow\rangle \pm |G\downarrow;E\uparrow\rangle)|P_E^*\rangle|P_F\rangle|P_G^*\rangle|P_H\rangle \oplus i(|H\uparrow;E\downarrow\rangle \pm |H\downarrow;E\uparrow\rangle)|P_E^*\rangle|P_F\rangle|P_G\rangle|P_H^*\rangle \\
&\oplus i(|G\uparrow;F\downarrow\rangle \pm |G\downarrow;F\uparrow\rangle)|P_E\rangle|P_F^*\rangle|P_G^*\rangle|P_H\rangle \oplus (|H\uparrow;F\downarrow\rangle \pm |H\downarrow;F\uparrow\rangle)|P_E\rangle|P_F^*\rangle|P_G\rangle|P_H^*\rangle \\
&\ominus (|F\uparrow;E\downarrow\rangle \mp |F\downarrow;E\uparrow\rangle)|P_E^*\rangle|P_F^*\rangle|P_G\rangle|P_H\rangle \ominus (|H\uparrow;G\downarrow\rangle \mp |H\downarrow;G\uparrow\rangle)|P_E\rangle|P_F\rangle|P_G^*\rangle|P_H^*\rangle \\
&\ominus i\{|E\uparrow;E\downarrow\rangle|P_E^*\rangle|P_G\rangle \ominus |G\uparrow;G\downarrow\rangle|P_E\rangle|P_G^*\rangle\}|P_F\rangle|P_H\rangle \oplus i\{|F\uparrow;F\downarrow\rangle|P_F^*\rangle|P_H\rangle \oplus |H\uparrow;H\downarrow\rangle|P_F\rangle|P_H^*\rangle\}|P_E\rangle|P_G\rangle,
\end{aligned}$$

where the upper/lower signs stand for fermions/bosons and $\oplus$ and $\ominus$ indicate the lack of coherence. The above expression indicates that there will be coincidence between a pair of detectors in 75% of the cases. In each of these cases, a spin entangled state will be generated along the corresponding pair of exit channels. For example, for fermions, if P_E and P_G click, $|\psi^+\rangle_{GE}$ is produced, and if P_H and P_G click, $|\psi^-\rangle_{GH}$ is produced. Based on the knowledge of the detector clicks, all the different entangled states can be converted to a desired entangled state by applying spin dependent phases along appropriate paths. Note that in only 25% of the cases detector clicks will result in a disentangled state.

The above improvement in success probability stems from the fact that the extra pair of beam splitters not only maps the entangled part $|\psi^\pm\rangle_{DC}$ of the state after the first beam splitter to entangled final parts, but also maps 50% of the disentangled parts [5] $|C\uparrow;C\downarrow\rangle$ and $|D\uparrow;D\downarrow\rangle$ to entangled final parts. We can easily double the number of output channels by subdividing each of the existing outputs by beam splitters and at each stage the entangled fraction increases. If we subdivide in this manner to obtain 2^N outputs, and put detectors only in these final exit paths, the fractional yield of entangled pairs is $1 - 1/2^N$. For $N = 7$, this exceeds 99%.

We now describe an interesting *feedback scheme* to reduce the resources required in our proposal, while still retaining the arbitrarily high efficiency. In the setup of Fig. 1, when the output is found disentangled (i.e., P_C and P_D do not click in coincidence), it is fed back again into the *same* beam splitter. This procedure could be repeated successively to increase arbitrarily the efficiency of entanglement generation. Here the probability of failure

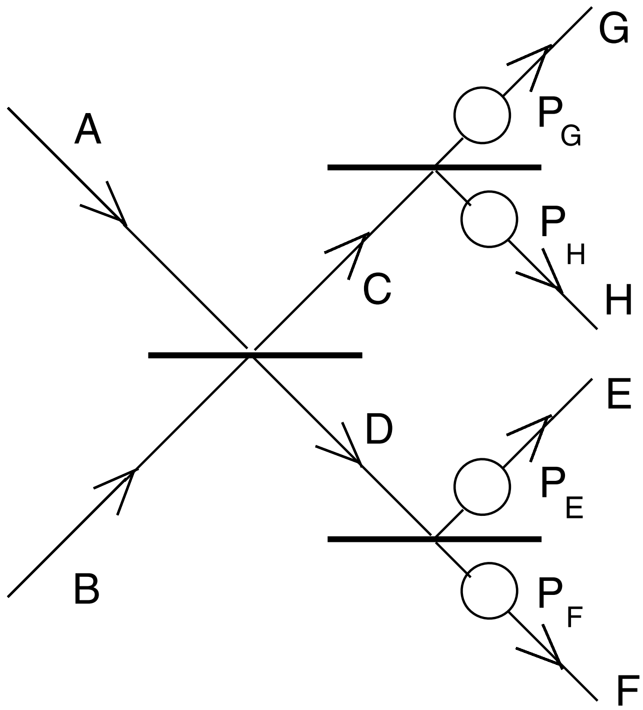

FIG. 2. An improved version of the setup. A and B are the input paths, E, F, G, and H are the output paths, and P_E, P_F, P_G, and P_H are absorptionless path detectors which do not disturb the spin. A pair of identical particles with opposite spins are incident on the first beam splitter, one from arm A and the other from arm B. For coincidence between any pair of detectors, which happens in 75% of the cases, a spin entangled state is emitted along the corresponding pair of paths.

decreases exponentially (2^{-N}) with the number N of feedback rounds, while the required resources (a single beam splitter and a pair of detectors) remain unchanged.

The next issue of the paper is the identification of quantum statistics through spin correlation measurements. Consider Fig. 1 once again and the incident state $|A\uparrow; B\downarrow\rangle$. If there is a detector coincidence, then the unitary operation $|\uparrow\rangle \to \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle), |\downarrow\rangle \to \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle)$ is applied to the spins of particles in each of the output channels. The spins are then measured in the $\{|\uparrow\rangle, |\downarrow\rangle\}$ basis. There will be *perfect correlation* between the spin measurement outcomes in the two paths for *fermions* and *perfect anticorrelation* for *bosons*. The sign of the spin correlation can thus be used to identify quantum statistics of the incident particles. This differs from earlier schemes which rely on particle number measurements for testing statistics [8–12].

In our setup (Fig. 1), if each particle is fed in the random spin state $\rho_M = |\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|$, the difference between bosons and fermions is manifested through the *presence* or *absence* of entanglement in a given experiment. For detector coincidence, a maximally entangled state $|\psi^-\rangle_{CD}$ is created for bosons and a disentangled state $(1/2)|\uparrow_C\uparrow_D\rangle\langle\uparrow_C\uparrow_D| + (1/2)|\downarrow_C\downarrow_D\rangle\langle\downarrow_C\downarrow_D| + (1/\sqrt{2})|\psi^-\rangle_{CD}\langle\psi^-|_{CD}$ results for fermions. The difference between bosons and fermions can thus be demonstrated through Bell's inequalities. The sufficiency of ρ_M implies that no prior spin correlation between the incident particles is necessary for entanglement generation using bosons.

We now describe a curious complementarity between particle distinguishability and entanglement in our scheme. The complementarity of "which channel" information with fringe contrast in single particle interference is much discussed [13–16]. In two particle interferometry, which channel information is naturally replaced by which particle information. The particles impinging on our setup (Fig. 1) are indistinguishable apart from their spins (which we choose not to measure as we intend to create a spin entangled state). Now suppose the particles were partially or fully distinguishable through some other observable such as energy or momentum or any nonspin internal degree of freedom. For example, suppose the incident state is $|A\uparrow S_1; B\downarrow S_2\rangle$, with $|\langle S_1|S_2\rangle| = a \leq 1$. Then the two particle state produced due to detector coincidence is $\frac{1}{\sqrt{2}}(|D\uparrow S_1; C\downarrow S_2\rangle \pm |D\downarrow S_2; C\uparrow S_1\rangle)$. The spin state of the particles (in the first quantized notation) is

$$\rho = \tfrac{1}{2}(|\uparrow_C\downarrow_D\rangle\langle\uparrow_C\downarrow_D| + |\downarrow_C\uparrow_D\rangle\langle\downarrow_C\uparrow_D| \pm |a|^2|\uparrow_C\downarrow_D\rangle\langle\downarrow_C\uparrow_D| \pm |a|^2|\downarrow_C\uparrow_D\rangle\langle\uparrow_C\downarrow_D|). \quad (3)$$

Note that in the second quantized notation all the degrees of freedom belong to the *same* Hilbert space (are created from the same vacuum). But they become elements of *distinct* Hilbert spaces when we proceed to the first quantized notation. For the above state, a certain entanglement measure called concurrence [20] is $\mathbf{E} = |a|^2$. The probability of successful discrimination between the states $|S_1\rangle$ and $|S_2\rangle$ (which is a measure of particle distinguishability) is $\mathbf{D} = 1 - |a|^2$. Thus we have, in analogy with Englert's relation in single particle interference [16], the following testable complementarity relation:

$$\mathbf{E} + \mathbf{D} = 1. \quad (4)$$

The concurrence $\mathbf{E}$ for ρ can be inferred by measuring the expectation value of the Bell-CHSH operator $\hat{a}\hat{b} + \hat{a}\hat{b}' + \hat{a}'\hat{b} - \hat{a}'\hat{b}'$ on the two particles (labeled by their paths C and D) with $\hat{a} = \sigma_x^C, \hat{a}' = \sigma_y^C, \hat{b} = \frac{1}{\sqrt{2}}(\sigma_x^D + \sigma_y^D)$, $\hat{b}' = \frac{1}{\sqrt{2}}(\sigma_x^D - \sigma_y^D)$ and dividing the result by $\pm 2\sqrt{2}$.

Equation (4) also helps us to estimate the amount of entanglement generated by our scheme if $|S_1\rangle$ and $|S_2\rangle$ are Gaussian wave-packet states of the incident particles arriving at the beam splitter. If the wave packets have width σ, velocity v, and a time delay Δt with respect to each other, the entanglement is $\mathbf{E} = \exp(-v^2\Delta t^2/2\sigma^2)$.

We now suggest an unexplored ramification of entanglement in the context of second quantization. One needs *at least two quantum numbers* associated with the creation operators (the spin and output channel labels in our case) to meaningfully describe an entangled state. In a suitable quantum state such as $\frac{1}{\sqrt{2}}(|D\uparrow; C\downarrow\rangle \pm |D\downarrow; C\uparrow\rangle)$, when one of the labels (the channel labels C and D in our case) denotes the identity of the particle, the other degree of freedom (the spin in our case) appears *entangled.*

However, the above state can *equally well* be rewritten in first quantized notation as $|D_\uparrow C_\downarrow\rangle + |C_\uparrow D_\downarrow\rangle$ (for both bosons and fermions), where the spins have been used to label the particles and their paths are entangled. Devising schemes to detect this *dual form of entanglement* (via the *reversal* of labels) is currently under way [21].

Finally we discuss the feasibility of our scheme. The basic ingredients are beam splitters, the capability of performing two particle interference, and which way detectors that keep the internal degree of freedom to be entangled undisturbed. Beam splitters are available for photons, electrons [22], neutrons [23], atoms [14,18,24], and macromolecules [25]. Two particle interferometry is feasible with photons [9,10]. Its realizability with electrons has received much attention [11,22]. For atoms, one could use the recently fabricated beam splitters for guided atoms [24] to study two particle interferometry. Next is the question of the special type of "which way" detectors required for our scheme. A theoretical model of absorptionless path detectors that keep spin unaffected has been considered in the context of quantum state reduction [26]. Such detectors have already been fabricated for electrons (based on the effects of electric fields) for a complementarity experiment [15], and there are also proposals for simple variants of such detectors [27]. Detectors of the required type have been suggested for photons (based on crossed phase modulation) [28], and for neutrons (based on momentum transfer) [29]. For atoms, one can implement our scheme by placing cavities in the arms C and D to act as our which way detectors. One would have to use an atom with hyperfine ground levels $|g_1\rangle$, $|g_2\rangle$, $|g_3\rangle$, and $|g_4\rangle$ which can be made to interact with a cavity field in Fock state $|n\rangle$ to undergo transitions $|g_1\rangle|n\rangle \rightarrow |g_2\rangle|n+1\rangle$ and $|g_3\rangle|n\rangle \rightarrow |g_4\rangle|n+1\rangle$ [30]. Then, with an incident state $|Ag_1; Bg_3\rangle$, appropriate transitions in cavities will result in the entangled state $\frac{1}{\sqrt{2}}(|g_2\rangle_D|g_4\rangle_C \pm |g_4\rangle_D|g_2\rangle_C)$ when both cavities are found in the state $|n+1\rangle$. For macromolecules, one can choose any two independent degrees of freedom, one for entangling and the other for path detection.

To summarize, we have presented a scheme of arbitrarily high efficiency for entangling two particles of *any type*. This is important, as entangled states of objects such as neutrons, electrons, or macromolecules are yet to be prepared. Our scheme provides strong motivation for developing two particle interferometry for various systems in tandem with absorptionless which way detectors. That the *same* setup can be used to test quantum statistics through entanglement-induced spin correlations and probe complementarity in two-particle interference enhance the significance of our scheme. Our work also suggests potential connections between entanglement, quantum statistics, and complementarity, which call for further study (see, for example, Ref. [31]).

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