Two-Proton Widths of ^{12}O, ^{16}Ne, and Three-Body Mechanism of Thomas-Ehrman Shift

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Two-proton decays of ^{12}O and ^{16}Ne ground states are studied in a three-body model. We have found that the two-proton widths for the states should be much less than the existing experimental values (about 10 times for ^{12}O and about 100 times for ^{16}Ne). We also have found that the structure of these states differs significantly from the mirror isobaric analog states (IAS): breaking of isobaric symmetry is at the level of tens of percents. Together with a corresponding decrease of the Coulomb energy, this effect can be considered as a three-body mechanism of the Thomas-Ehrman shift.

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Although predicted many years ago [1], two-proton radioactivity is still a complicated and controversial problem from both experimental and theoretical sides. A novel theoretical approach to the two-proton radioactivity problem has been developed in [2]. Together with ^{6}Be, the ^{12}O and ^{16}Ne nuclei are the only known light two-proton emitters and they have methodological importance for understanding this phenomenon in general. We are going to demonstrate that these nuclei deserve also special attention as representing an interesting form of nuclear dynamics. In some approximations these nuclei, as well as the mirror ^{6}He, ^{12}Be, and ^{16}C nuclei, can be considered in a three-body core + N + N model.

The properties of the ^{12}O ground state (g.s.) are E_T = 1.82(12) MeV, Γ = 400(250) keV [3] or E_T = 1.79(4) MeV, Γ = 580(200) keV [4] (E_T is the energy above two-proton threshold). There is a controversy about the widths [5–8]. For example, Barker insists that an upper limit for the width of ^{12}O g.s. is about 100 keV [6] and for diproton emission the upper limit is only 5 keV [8]. Our calculations correspond well to these estimates.

The situation in ^{16}Ne is very similar to that in ^{12}O, but as yet it has attracted only a little attention [9]. The experimental data is E_T = 1.35(8) MeV, Γ = 200(100) keV [3] or E_T = 1.40(2) MeV, Γ = 110(40) keV [10]. Just these values are enough to raise some questions: the corresponding characteristics of the well studied ^{6}Be g.s. E_T = 1.37 MeV and Γ = 92 keV are very close to them, but the Coulomb interaction of protons with ^{14}O core is 4 times stronger in ^{16}Ne. The exponential character of penetrability dependence implies a very rough estimate $\Gamma_{^{16}\mathrm{Ne}} \approx \Gamma^4_{^{6}\mathrm{Be}}/(2\gamma^2_{\mathrm{WL}})^3 \approx 0.02$–$3$ eV if we assume that the reduced width in ^{6}Be is close to the Wigner limit γ^2_{WL}. A solution of this paradox was suggested in the qualitative discussion in [9], where the experimental widths of ^{6}Be and ^{16}Ne were used to infer a considerable difference in their structures. In our studies, based on firm three-body calculations of ^{6}Be and ^{16}Ne, we derive the width which is larger than in the simple estimate above, but still much lower than the experimental value.

We find that the peculiar nuclear dynamics in ^{12}O and ^{16}Ne leads to a strong (tens of percents) breaking of isobaric symmetry. This can be considered as a three-body mechanism for the Thomas-Ehrman shift (TES). The original idea of TES [11] applies to core + N mirror nuclei. When the proton drip line is approached, the wave function (WF) of the nucleon in the last shell becomes wider due to larger penetration to the classically forbidden region, and correspondingly the Coulomb energy decreases. This decrease is most pronounced for the s-wave states, which leads to a relative shift of levels in the mirror partners. The recent papers [12,13] give a more general view of TES. Paper [12] speculates about the possible "indirect" mechanism of the TES due to a modification of the residual nuclear interaction, and considers also core + N + N systems. The definition of the TES in [13] as the difference between perturbative and actual Coulomb shifts for the isobaric analog states (IAS) in mirror nuclei is physically close to the original and is applicable not only to core + N systems.

Studying ^{12}O and ^{16}Ne nuclei, we have found that not only has the WF expanded, but also the structure of the states has changed significantly compared to isobaric states in ^{12}Be and ^{16}C. While ^{12}Be and ^{16}C have a strong s, p, d configuration mixing for the valence nucleons, the s-wave components of their proton-rich analogs have markedly increased. The qualitative reasons for that are clear, as the variational WF benefits most from the decrease of the Coulomb energy in the channels where the barriers are lowest, and the WF has the best opportunity for radial expansion. This mechanism differs significantly from the ordinary perception of TES as a purely "geometric" effect, connected only with shape of the WF.

The large isobaric symmetry breaking and a novel mechanism of TES which we have found are not restricted to just these two systems. The effects could be expected in heavier, even Z, systems as well. The combination of

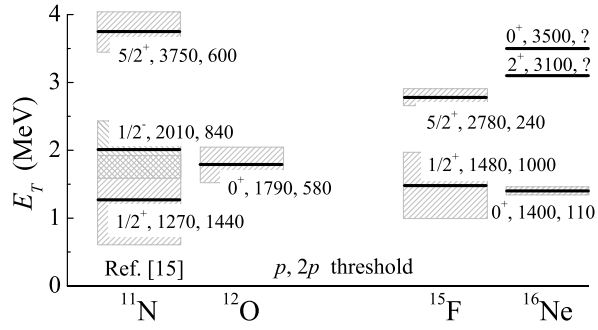

FIG. 1. States observed in ^{12}O, ^{16}Ne and their "core + proton" subsystems: J^π, decay energy (keV), width (keV).

these features and the uncertain experimental situation makes these systems of broader interest.

Models.—We use two types of three-body models, both based on the hyperspherical harmonic (HH) method. (i) In Ref. [2] we developed a theoretical three-body model ["source function" (SF) model] suited to studies of two-proton emission and three-body decays of narrow states. In this model we first find a "box" WF for a finite domain and then use it as a source, to construct a decaying WF with outgoing asymptotic. The Pauli principle (PP) between the valence protons and the core is approximately taken into account by use of repulsive cores in the orbitals with occupied states. (ii) Alternatively we make the three-body bound state and continuum ($3 \to 3$ scattering) calculations as in [14] (marked as PP below). In this model the Pauli principle is taken into account by projecting out forbidden states.

The ^{12}O and ^{16}Ne nuclei do not fit rigorously to the definition of the true three-body decay (in terms of [1]): formally there are sequential decay branches via the ground state in ^{11}N with $E = 1.27$ MeV and via the tail of the g.s. in ^{15}F with $E = 1.48$ MeV (see Fig. 1). However, these are wide s-wave states, whose widths of $\Gamma = 1.44$ MeV and $\Gamma = 1$ MeV are comparable with the decay energies. This makes the systems still well suited for studies in the framework of the HH method [2].

Potentials.—The Coulomb potential of the uniformly charged sphere with radius r_c is used. The nuclear potentials are l dependent. The nuclear part of pp interaction was taken a simple central potential, acting only in s wave: $V(r) = -31 \exp[-(r/1.8)^2]$. In the core-$p$ subsystem we used the Woods-Saxon (WS) potential:

$$V(r) = V_c^l f(r) + V_{ls}^l (\mathbf{l} \cdot \mathbf{s})(\lambda_\pi^2/r)[df(r)/dr], \qquad f(r) = \{1 + \exp[(r - R)/a]\}^{-1}. \tag{1}$$

The use of pairwise potentials fitted to experimental data provides a sensible description of ^{12}O and ^{16}Ne. However, the three-body decay width strongly depends on the separation energy of protons. As in [14], for fine-tuning of the resonance energy we use a collective three-body potential which depends only on hyperradius ρ:

$$V_3(\rho) = V_3^0[1 + (\rho/\rho_0)^3]^{-1}. \tag{2}$$

This potential has a plausible three-body asymptotic behavior $\propto \rho^{-3}$ and practically does not distort the WF. The parameter ρ_0 can be taken in the range 3–5 fm.

^{12}O ground state.—For PP calculations we used the ^{10}C + p potential (e) from [15], which was obtained by analyzing the strength function for elastic scattering. For SF calculations this potential was modified as follows: (i) an equivalent potential was introduced in the s wave by adding the WS core with $a_{\rm core} = 0.53$ fm, $R_{\rm core} = 1.393$ fm, $V^0_{\rm core} = 200$ MeV, and (ii) the p-wave potential was refitted with inverted volume ls forces to push the $p_{3/2}$ state (which is already occupied in ^{10}C) high into continuum: $V_c^1 = 30$ MeV, $V_{ls}^1 = 65.3$ MeV. Both the original and modified potentials reproduce well the $1/2^+$, $1/2^-$, $5/2^+$ states in ^{11}Be and ^{10}C + p scattering data of [15].

All calculations were done with $K_{\rm max} = 20$–24 to get convergence. The ^{12}O g.s. is overbound by about 0.5–0.8 MeV. We use a collective potential (2) to obtain the right resonance energy, and then the mirror state is calculated with the same nuclear potentials. After this is done we obtain in the SF model $\Gamma = 66^{+17}_{-11}$ keV (where the uncertainties are connected with experimental uncertainty of the resonance energy in ^{12}O). This is within the upper limit of 100 keV, estimated by Barker [6], but much less than the reported experimental values. The PP model yields a very similar value of $\Gamma \approx 60$ keV. The width of ^{12}O is dominated by the partial width of the first WF component (see Table I), which is entirely s^2. If we assume that the width scales with the weight of the s^2 component of the WF, we can estimate an upper limit of $\Gamma \approx 100$ keV.

In Table I the partial decompositions of the WFs of ^{12}O, ^{16}Ne, and the mirror states are given. We discuss the ^{16}Ne

TABLE I. Partitions of ^{12}O, ^{16}Ne g.s., and mirror states WFs (in percents) obtained in the SF model. Weights for continuum WF are obtained by integration up to $\rho = 20$ fm. Γ are partial widths for ^{12}O and ^{16}Ne. K is the hypermoment, S is spin, and l_x is angular momentum between protons.

K S l_x	^{12}Be	^{12}O	Γ (keV)	^{16}C	^{16}Ne	Γ (eV)
0 0 0	35.5	66.6	56.9	33.9	54.1	695
2 0 0	19.3	12.1	0.86	6.36	5.13	8.55
2 1 1	21.9	9.98	0.06	0.18	0.14	6×10^{-3}
4 0 2	0.16	0.58	6.92	0.81	0.49	107
4 0 0	18.3	9.38	0.51	47.9	32.9	7.81
4 1 1	3.57	1.54	1×10^{-4}	9.67	5.98	1×10^{-3}

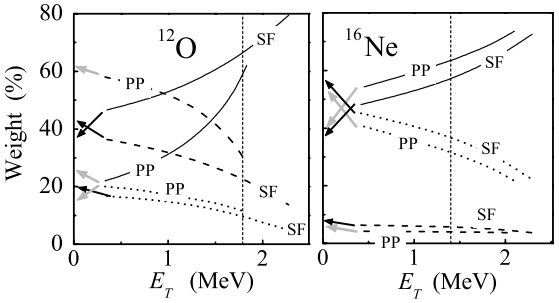

FIG. 2. The weights of single-particle configurations for valence protons if the binding is artificially changed. Solid, dashed, and dotted curves stand for s, p, and d waves. Arrows show the weights of corresponding components in the mirror IAS g.s. WFs at the correct binding energies.

case later, but because of qualitative similarity the results for $A = 16$ nuclei are given in parallel. The structures of mirror states differ significantly. We presume that the reason of such a lack of isobaric symmetry is the barrier-top character of the ^{12}O ground state. When we move from the neutron-rich to the proton-rich side of the isobar, the weight is shifted to the s-wave component of WF, as this component first achieves the top of the barrier and thus benefits most from the opportunity to expand and reduce the Coulomb energy. Strong mixing of components in the WF is essential for the shift, as it allows even small changes in interactions to lead to significant changes in the weights of the components.

To have insight into the dynamics we investigated the energy dependence of the effect. Figure 2 shows what happens with the structure, if we artificially change the resonance energy using the V_3^0 parameter in Eq. (2). The isobaric symmetry is clearly recovering when the state moves to the subbarrier region. The weights obtained in SF and PP calculations differ significantly for ^{12}Be. This reflects the high sensitivity of the ^{12}Be structure to the details of interaction [7]. However, it does not influence the conclusion about the qualitative effects: the s-wave component of WF is growing rapidly in both models, as we move the system higher in the continuum. Note that the relative weights of the g.s. components in ^{12}Be obtained in the SF model correspond well to the recent experimental results [16].

The Coulomb shift between the g.s. of ^{12}Be and ^{12}O is reproduced very well in our calculations (Fig. 3). The TES can be defined [13] as the difference between the perturbative value of the Coulomb shift [upper curves, $\Delta E_{\rm pert}(^{12}{\rm O}) = \langle ^{12}{\rm Be}|\sum_{i=1,2,3} V_{\rm coul}^{(i)}|^{12}{\rm Be}\rangle$] and the actual shift (lower curves). $\langle ^{12}{\rm Be}|$ consists of the spatial part of ^{12}Be WF, while charges correspond to ^{12}O. We see that the TES is growing as we push the state out of the well and that this increase is correlated with the increase in the s-wave component weight shown in Fig. 2.

^{16}Ne ground state.—All the kinds of estimates we have done for ^{16}Ne give a width of the ^{16}Ne g.s. 1–2 orders of magnitude smaller than the experimental value of 110 keV. Figure 4 shows the results of quasiclassical estimates for simultaneous emission and diproton models (as they were used in [2]). The latter provides a reliable upper limit for the width. For the experimental energy of the ^{16}Ne g.s. the experimental width can be obtained only with unrealistically large values of channel radius $r_{ch} \sim 9$ fm.

For the three-body calculations, we use potential (1) with parameters $a = 0.53$, $R = 2.89$, $r_c = 3.82$ fm, $V_c^0 = -56.2$, $V_c^1 = -10$, $V_c^2 = -53.45$, $V_{ls}^1 = -11$, $V_{ls}^2 = 11$ MeV. Also, for SF calculations, a WS core was added in the s wave with parameters $a_{\rm core} = 0.53$ fm, $R_{\rm core} = 1.7$ fm, $V_{\rm core}^0 = 144$ MeV. This potential reproduces well the properties of $1/2^+$ and $5/2^+$ states in ^{15}F and ^{15}C. The p-wave component of the potential is not well defined as there are no low-lying single-particle negative parity states in ^{15}F and ^{15}C. We used the most negative value of V_c^1 which does not contradict the experimental spectra.

With the above potentials we obtain very reasonable spectra of ^{16}C and ^{16}Ne. The behavior of weights and Coulomb shifts is qualitatively very similar to the ^{12}O case (see Figs. 2 and 3 and Table I). The 0^+ and 2^+ states in ^{16}C are at $E_T = -6.12$ and $E_T = -3.42$ MeV

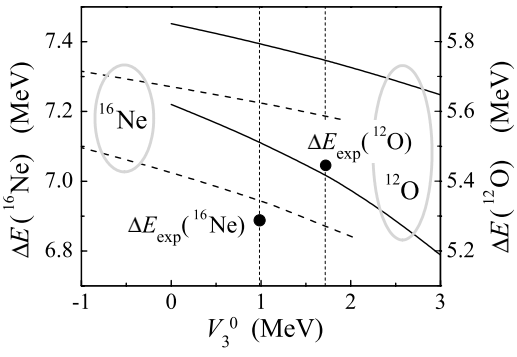

FIG. 3. Coulomb energies for pairs ^{12}Be–^{12}O (solid curves) and ^{16}C–^{16}Ne (dashed curves). Upper curves show perturbative Coulomb shift, while lower ones show actually calculated. Values of V_3^0 at which the experimental resonance energies are reproduced are marked by vertical dashed lines; dots on the lines shows the experimental values of Coulomb shifts.

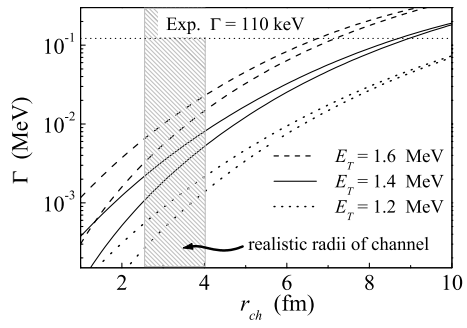

FIG. 4. Width of the ^{16}Ne g.s. as a function of channel radius r_{ch}. Solid, dashed, and dotted curves correspond to E_T equal 1.4, 1.6, and 1.2 MeV, respectively. The curves of the same style are diproton (upper curve) and simultaneous (lower curve) emission estimates.

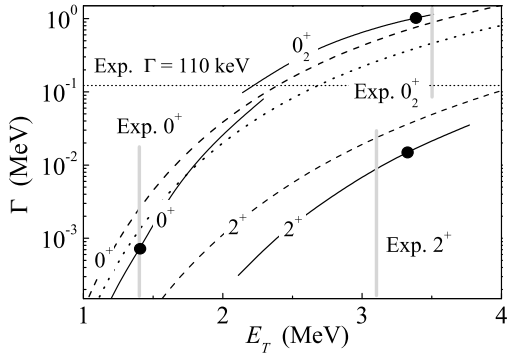

FIG. 5. Widths of ^{16}Ne 0^+ and 2^+ states as a function of energy. Solid, dashed, and dotted curves show correspondingly the three-body results, diproton estimates, and simultaneous emission estimates ($r_{ch} = 3$ fm). Experimental positions of states are indicated by vertical lines. Calculated positions of 0^+ and 2^+ states are shown by dots.

(experimental values are -5.468 and -2.441 MeV). If we adjust them to experimental energies we obtain the second 0^+ in ^{16}C at $E_T = -1.61$ and in ^{16}Ne we have 0^+ states at $E_T = 1.44$, 3.38 MeV and a 2^+ state at $E_T = 3.33$ MeV (all consistent with experiment; see Fig. 5).

The energy dependence of the widths for the states in ^{16}Ne is shown in Fig. 5. We predict the width of the g.s. to be $\Gamma = 0.8^{+2.3}_{-0.65}$ keV (the uncertainty is due to g.s. energy uncertainty). In paper [17] a first excited state in ^{16}Ne, found at excitation energy 2.1 MeV, was identified as a second 0^+ rather than 2^+. In our calculations the 0^+ and 2^+ states have practically the same energy (see also the analysis in [12]) but their widths differ by 2 orders of magnitude. The width of the state could be a good indicator of whether it is 2^+ or second 0^+.

In the SF model we can study energy distributions of the decay products. Figure 6 shows the distributions for decays of ^{12}O and ^{16}Ne g.s. We can see that actual three-body decay mechanism is neither diproton nor sequential: there are wide distributions, which can be better described in terms of "democratic decay" [18].

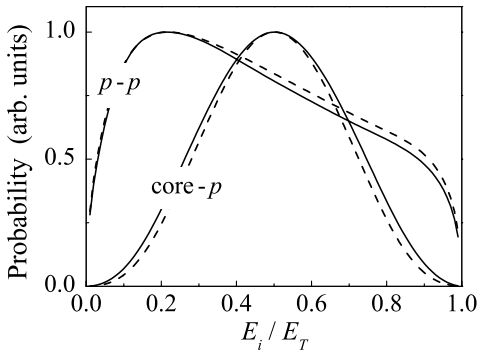

FIG. 6. Energy distributions for p-p and core-p subsystems for ^{12}O (solid line) and ^{16}Ne (dashed line).

Conclusion.— In this Letter we have studied the structure and two-proton decays of the ^{12}O and ^{16}Ne ground states. There are two issues, which deserve also the further attention of both theoreticians and experimentalists. (i) We have found an interesting three-body mechanism of isobaric symmetry breaking and relate it to the TES effect. The effect is large (tens of percents) and should be experimentally observable. In the phenomenological shell model analysis [12] it was suggested that modification of the residual nuclear interaction is required to describe TES in nuclei around ^{16}O. In our three-body model we describe well the Coulomb shifts for ^{12}Be–^{12}O and ^{16}C–^{16}Ne pairs (see Fig. 3). However, we find that responsible for this is a peculiar three-body effect (the increase of s^2 component in the ^{12}O and ^{16}Ne WFs compared to their mirror partners). (ii) In this Letter the two-proton widths of ^{12}O and ^{16}Ne are calculated for the first time. There are large discrepancies between the calculated and experimental widths, which are too serious to be attributed to uncertainties of the theoretical models. The results of different model approaches are quite close to each other and all far from experiment. We expect the width for ^{12}O to be about 60 keV, and for ^{16}Ne about 1 keV. For a specified resonance energy the obtained limits on the widths are reliable and should not be ignored when interpreting the experimental results. This issue is of considerable importance for the theory of two-proton radioactivity in general.

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