Maximal Violation of Bell's Inequalities for Continuous Variable Systems

Zeng-Bing Chen,[1] Jian-Wei Pan,[1,2] Guang Hou,[1] and Yong-De Zhang[3,1]

[1]*Department of Modern Physics, University of Science and Technology of China, Hefei, Anhui 230027, People's Republic of China*

[2]*Institut für Experimentalphysik, Universität Wien, Boltzmanngasse 5, 1090 Wien, Austria*

[3]*CCAST (World Laboratory), P.O. Box 8730, Beijing 100080, People's Republic of China*

(Received 10 September 2001; published 14 January 2002)

We generalize Bell's inequalities to biparty systems with continuous quantum variables. This is achieved by introducing the Bell operator in perfect analogy to the usual spin-$1/2$ systems. It is then demonstrated that two-mode squeezed vacuum states display quantum nonlocality by using the generalized Bell operator. In particular, the original Einstein-Podolsky-Rosen states, which are the limiting case of the two-mode squeezed vacuum states, can maximally violate Bell's inequality due to Clauser, Horne, Shimony, and Holt. The experimental aspect of our scheme is briefly considered.

DOI: 10.1103/PhysRevLett.88.040406 PACS numbers: 03.65.Ud, 03.65.Ta, 03.67.–a

In their famous paper [1], Einstein, Podolsky, and Rosen (EPR) introduced two striking aspects of quantum mechanics into physics: quantum entanglement and quantum nonlocality. The relationship between them has then been a source of great theoretical interest. These fundamental issues play an essential role in the modern understanding of quantum phenomena. However, further studies of quantum nonlocality and entanglement, especially those providing quantitative tests of quantum mechanics versus local realism in the form of Bell's inequalities [2–4], used mainly Bohm's version [5] of the EPR entangled states instead of the original EPR states with continuous degrees of freedom. In recent years, the later has attracted much attention. The preparation of the EPR-type states for photons was investigated both theoretically [6,7] and experimentally [8,9]. However, as noticed in Refs. [8,9], the generalization of Bell's inequalities to quantum systems with continuous variables (CVs) is a challenging issue.

In the burgeoning field of quantum information theory [10,11], EPR entanglement and quantum nonlocality are also of practical importance. The fascinating nonlocal correlations can be exploited to perform classically impossible tasks. While most of the concepts in quantum information theory were initially developed for quantum systems with discrete quantum variables, quantum information protocols (e.g., quantum teleportation [12], quantum error correction [13], quantum computation [14], entanglement purification [15], and cloning [16]) of CVs have also been proposed very recently.

Quantum nonlocality for position-momentum variables associated with the original EPR states was analyzed recently [4,17,18]. Using the Wigner function approach [19], Bell [4,17] has argued that the original EPR states will not exhibit nonlocality, because its Wigner function is positive everywhere, and as such will allow a hidden variable description of the system. By sharp contrast, it was demonstrated in recent publications [18] that the Wigner function of the two-mode squeezed vacuum states (the "regularized" EPR states), though positive definite, provides a direct evidence of the nonlocal character of the states. The demonstration is based on the fact that the Wigner function can be interpreted as a correlation function for the joint measurement of the parity operator. By making use of the parity of coherent states as one of the observables, Yurke and Stoler have also presented a proposal for observing the local realism violation with squeezed states [20]. Using homodyning with weak coherent fields and photon counting, a recent experiment [21] reported the observed violation of the Bell-type inequalities by the regularized EPR states produced in a pulsed nondegenerate optical parametric amplifier (NOPA), confirming the theoretical prediction in Refs. [18,22].

There is a crucial point implied in Ref. [18]: A state does not have to violate all possible Bell's inequalities to be considered quantum nonlocal; a given state is nonlocal when it violates *any* Bell's inequality. This point has been also stressed in Ref. [23]. Thus the degree of quantum nonlocality that we can uncover crucially depends not only on the given quantum state but also on the "Bell operator" [24]. In their demonstration of quantum nonlocality of the NOPA states by means of the phase-space formalism, Banaszek and Wódkiewicz (BW) used the Bell operator based on the joint parity measurements [18]. However, it still remains to be answered whether or not the original EPR states can maximally violate Bell's inequalities within the BW formalism. Moreover, the violation of Bell's inequalities uncovered by BW depends upon the magnitude of the displacement in phase space, an unsatisfactory feature. Thus the challenging problem of generalizing Bell's inequalities to quantum systems with CVs is only *partially* solved in Ref. [18].

In this paper, we generalize Bell's inequalities to the CV cases for biparty systems. We then show that the original EPR states, which are the limiting case of the NOPA states, can maximally violate Bell's inequality due to Clauser, Horne, Shimony, and Holt [3], called the Bell-CHSH inequality in the following. In contrast to the BW formalism (using the phase-space approach) and the proposal by Grangier *et al.* [22] (using the homodyne detection scheme), here we show an interesting and direct analogy between Bell's inequalities for both discrete-variable and CV cases; the correlation functions to be measured for observing the violation of the Bell-CHSH inequality are also analogous for the two cases.

To this end, we need to introduce a Bell operator suitable for the present purpose. First, let us recall some well-known results of the Bell-CHSH inequality for two-qubit systems (e.g., spin-1/2 systems). In the two-qubit case, the Bell operator reads [24]

$$\begin{aligned}\mathcal{B}_{\text{qubit}} = (\mathbf{a}\cdot\sigma_1)\otimes(\mathbf{b}\cdot\sigma_2) + (\mathbf{a}\cdot\sigma_1)\otimes(\mathbf{b}'\cdot\sigma_2) \\ + (\mathbf{a}'\cdot\sigma_1)\otimes(\mathbf{b}\cdot\sigma_2) - (\mathbf{a}'\cdot\sigma_1)\otimes(\mathbf{b}'\cdot\sigma_2),\end{aligned} \tag{1}$$

where σ_j is the Pauli matrix for the jth ($j = 1, 2$) qubit; $\mathbf{a}$, $\mathbf{a}'$, $\mathbf{b}$, and $\mathbf{b}'$ are four unit three-dimensional vectors. We can easily derive [24,25]

$$\mathcal{B}_{\text{qubit}}^2 = 4I_{2\times2} + 4[(\mathbf{a}\times\mathbf{a}')\cdot\sigma_1]\otimes[(\mathbf{b}\times\mathbf{b}')\cdot\sigma_2], \tag{2}$$

where $I_{2\times2}$ is the identity operator for the qubit systems. As a result, the expectation value of $\mathcal{B}_{\text{qubit}}^2$ with respect to a two-qubit state satisfies $\langle\mathcal{B}_{\text{qubit}}^2\rangle \le 4 + 4 = 8$, implying that $|\langle\mathcal{B}_{\text{qubit}}\rangle|$ with respect to any two-qubit state is bounded by $2\sqrt{2}$, known as the Cirel'son bound [26].

Now for a single-mode light field, we can introduce the following "pseudospin" operators for photons (perhaps the pseudospin operators have been introduced in literature somewhere we are unaware of):

$$\begin{aligned} s_z &= \sum_{n=0}^{\infty}[|2n+1\rangle\langle 2n+1| - |2n\rangle\langle 2n|], \\ s_- &= \sum_{n=0}^{\infty}|2n\rangle\langle 2n+1| = (s_+)^\dagger, \end{aligned} \tag{3}$$

where $|n\rangle$ are the usual Fock states. The operator $s_z = -(-1)^N$ (N is the number operator), where $(-1)^N$ is the parity operator; s_+ and s_- are the "parity-flip" operators. In terms of the creation ($a^\dagger$) and annihilation operators (a), s_- can also be written as $s_- = [I + (-1)^N] \times [2\sqrt{N+1}]^{-1}a$, where I is the identity operator. It is interesting to note that $(1/\sqrt{N+1})a = e^{i\vartheta}$, with ϑ known as the Susskind-Glogower phase operator [27]. We can easily check that

$$[s_z, s_\pm] = \pm 2 s_\pm, \qquad [s_+, s_-] = s_z. \tag{4}$$

The commutation relations in Eq. (4) are identical to those of the spin-1/2 systems. Therefore the pseudospin operator $\hat{\mathbf{s}} = (s_x, s_y, s_z)$, where $s_x \pm i s_y = 2s_\pm$, can be regarded as a counterpart of the spin operator σ. It is a kind of spin operator acting upon the parity space of photons, and thus can be called the "parity spin" of photons. The quadrature amplitudes of a single-mode light field correspond to the usual position and momentum operators, accompanied with the position-momentum uncertainty. The fact that one can define the parity spin as in (3) might imply a new intrinsic uncertainty for photons (and other bosons).

Now choosing an arbitrary vector living on the surface of a unit sphere $\mathbf{a} = (\sin\theta_a\cos\varphi_a, \sin\theta_a\sin\varphi_a, \cos\theta_a)$ [$\theta_a(\varphi_a)$ being the polar (azimuthal) angle of $\mathbf{a}$], we have

$$\mathbf{a}\cdot\hat{\mathbf{s}} = s_z\cos\theta_a + \sin\theta_a(e^{i\varphi_a}s_- + e^{-i\varphi_a}s_+). \tag{5}$$

Analogously, $\mathbf{a}$ may be interpreted as the "direction" along which we measure the parity spin $\hat{\mathbf{s}}$. The commutation relations in Eq. (4) lead to

$$(\mathbf{a}\cdot\hat{\mathbf{s}})^2 = I. \tag{6}$$

Equation (6) means that the outcome of the measurement of the Hermitian operator $\mathbf{a}\cdot\hat{\mathbf{s}}$ (with eigenvalues ±1) is 1 or -1. The above observations show that there exists a *perfect* analogy between the CV systems and the usual spin-1/2 systems. Thus all types of Bell's inequalities derived for the latter have their counterpart in the former.

In particular, for two-mode light fields, we define the Bell operator as

$$\begin{aligned}\mathcal{B}_{\text{CHSH}} = (\mathbf{a}\cdot\hat{\mathbf{s}}_1)\otimes(\mathbf{b}\cdot\hat{\mathbf{s}}_2) + (\mathbf{a}\cdot\hat{\mathbf{s}}_1)\otimes(\mathbf{b}'\cdot\hat{\mathbf{s}}_2) \\ + (\mathbf{a}'\cdot\hat{\mathbf{s}}_1)\otimes(\mathbf{b}\cdot\hat{\mathbf{s}}_2) - (\mathbf{a}'\cdot\hat{\mathbf{s}}_1)\otimes(\mathbf{b}'\cdot\hat{\mathbf{s}}_2).\end{aligned} \tag{7}$$

Here $\mathbf{a}$, $\mathbf{a}'$, $\mathbf{b}$, and $\mathbf{b}'$ are four unit vectors as before; $\hat{\mathbf{s}}_1$ and $\hat{\mathbf{s}}_2$ are defined as in Eq. (3). Then local realistic theories impose the following Bell-CHSH inequality [3]:

$$|\langle\mathcal{B}_{\text{CHSH}}\rangle| \le 2, \tag{8}$$

where $\langle\mathcal{B}_{\text{CHSH}}\rangle$ is the expectation value of $\mathcal{B}_{\text{CHSH}}$ with respect to a given quantum state of CVs. Equation (8) represents the Bell-CHSH inequality of quantum systems with CVs. Interestingly, our generalization of Bell's inequalities to CV systems is realized via joint measurements on discrete (dichotomic) observable $\hat{\mathbf{s}}$, in a perfect analogy to the usual joint measurements on spins. Within this scheme, the *correlation function* reads $E(\mathbf{a},\mathbf{b}) = \langle(\mathbf{a}\cdot\hat{\mathbf{s}}_1)\otimes(\mathbf{b}\cdot\hat{\mathbf{s}}_2)\rangle$.

By using $(\mathbf{a}\cdot\hat{\mathbf{s}}_1)^2 = (\mathbf{b}\cdot\hat{\mathbf{s}}_2)^2 = (\mathbf{a}'\cdot\hat{\mathbf{s}}_1)^2 = (\mathbf{b}'\cdot\hat{\mathbf{s}}_2)^2 = I$ [see Eq. (6)] and the commutation relations in Eq. (4), it can been shown, similarly to the two-qubit case, that

$$\begin{aligned}\mathcal{B}_{\text{CHSH}}^2 &= 4I - [\mathbf{a}\cdot\hat{\mathbf{s}}_1, \mathbf{a}'\cdot\hat{\mathbf{s}}_1]\otimes[\mathbf{b}\cdot\hat{\mathbf{s}}_2, \mathbf{b}'\cdot\hat{\mathbf{s}}_2] \\ &= 4I + 4[(\mathbf{a}\times\mathbf{a}')\cdot\hat{\mathbf{s}}_1]\otimes[(\mathbf{b}\times\mathbf{b}')\cdot\hat{\mathbf{s}}_2].\end{aligned} \tag{9}$$

Consequently,

$$\langle\mathcal{B}_{\text{CHSH}}^2\rangle \le 4 + 4 = 8, \tag{10}$$

which again implies that $|\langle\mathcal{B}_{\text{CHSH}}\rangle|$ with respect to any quantum state of CVs is bounded by $2\sqrt{2}$. When $|\langle\mathcal{B}_{\text{CHSH}}\rangle| = 2\sqrt{2}$ for a given state, we say that the Bell-CHSH inequality (8) is maximally violated by the state. In the following, we will use the Bell-CHSH inequality (8) to uncover the nonlocality of the NOPA states as well as of the original EPR states.

The NOPA process represents a nonlinear interaction of two quantized modes (denoted by the corresponding annihilation operators a_1 and a_2) in a nonlinear medium with a strong classical pump field. In this process, the NOPA can generate the two-mode squeezed vacuum states, i.e., the NOPA states [6,7]:

$$|\mathrm{NOPA}\rangle = e^{r(a_1^\dagger a_2^\dagger - a_1 a_2)}|00\rangle = \sum_{n=0}^{\infty} \frac{(\tanh r)^n}{\cosh r}|nn\rangle, \tag{11}$$

where $r > 0$ is known as the squeezing parameter and $|nn\rangle \equiv |n\rangle_1 \otimes |n\rangle_2 = \frac{1}{n!}(a_1^\dagger)^n (a_2^\dagger)^n |00\rangle$. The NOPA states $|\mathrm{NOPA}\rangle$ are the optical analog of the EPR entangled states in the limit of infinite squeezing. Thus the EPR's argument can be tested experimentally with the parametric amplifier [8,9]. The squeezed-state entanglement of $|\mathrm{NOPA}\rangle$ is also essential in the teleportation of continuous quantum variables [12].

Using Eqs. (3), (5), and (11) we derive

$$\langle \mathcal{B}_{\mathrm{CHSH}} \rangle = E(\theta_a, \theta_b) + E(\theta_a, \theta_{b'}) + E(\theta_{a'}, \theta_b) - E(\theta_{a'}, \theta_{b'}), \tag{12}$$

where the correlation function,

$$E(\theta_a, \theta_b) = \langle \mathrm{NOPA}| s_{\theta_a}^{(1)} \otimes s_{\theta_b}^{(2)} |\mathrm{NOPA}\rangle = \cos\theta_a \cos\theta_b + K(r) \sin\theta_a \sin\theta_b, \tag{13}$$

$$s_{\theta_a}^{(j)} \equiv s_{jz} \cos\theta_a + s_{jx} \sin\theta_a, \tag{14}$$

with $K(r) \equiv \tanh(2r) \leq 1$. In deriving Eq. (12), we have set all azimuthal angles to be zero without affecting the following discussion. Choosing $\theta_a = 0$, $\theta_{a'} = \pi/2$, and $\theta_b = -\theta_{b'}$, we have

$$\langle \mathcal{B}_{\mathrm{CHSH}} \rangle = 2(\cos\theta_b + K \sin\theta_b). \tag{15}$$

For this specific setting, the maximum of $\langle \mathcal{B}_{\mathrm{CHSH}} \rangle$ is

$$\langle \mathcal{B}_{\mathrm{CHSH}} \rangle_{\max} = 2\sqrt{1 + K^2}, \tag{16}$$

when $\theta_b = \tan^{-1} K$. Thus, the NOPA states always violate the Bell-CHSH inequality (8) provided that $r \neq 0$. Meanwhile, the degree of quantum nonlocality uncovered here is *uniquely* determined by the squeezing parameter r; the parameter K may be reasonably regarded as a quantitative measure of quantum nonlocality. Compared with Ref. [18], here we do not rely on the phase-space formalism.

The NOPA states $|\mathrm{NOPA}\rangle$ can also be written as [18]

$$|\mathrm{NOPA}\rangle = \sqrt{1 - \tanh^2 r} \times \int dq \int dq' \, g(q, q'; \tanh r) \, |qq'\rangle, \tag{17}$$

where $g(q, q'; x) \equiv 1/\sqrt{\pi(1 - x^2)} \exp\{-(q^2 + q'^2 - 2qq'x)/[2(1 - x^2)]\}$ and $|qq'\rangle \equiv |q\rangle_1 \otimes |q'\rangle_2$, with $|q\rangle$ being the usual eigenstates of the position operator. Since $\lim_{x\to 1} g(q, q'; x) = \delta(q - q')$, one has $\lim_{r\to\infty} \int dq \times \int dq' \, g(q, q'; \tanh r) \, |qq'\rangle = \int dq |qq\rangle = |\mathrm{EPR}\rangle$, which is exactly the original EPR states. Thus, in the infinite squeezing limit, $|\mathrm{NOPA}\rangle|_{r\to\infty}$ become the original, normalized EPR states, for which we have

$$\langle \mathcal{B}_{\mathrm{CHSH}} \rangle_{\max} = 2\sqrt{2}, \tag{18}$$

by noting $K(r \to \infty) = 1$ and choosing $\theta_a = 0$, $\theta_{a'} = \pi/2$, and $\theta_b = -\theta_{b'} = \pi/4$ in Eq. (12). This remarkable result indicates that *the normalized version of the original EPR states can maximally violate the Bell-CHSH inequality* (8).

Having shown *theoretically* the violation of the Bell-CHSH inequality by the NOPA states, an important question arises as to what physical measurements are necessary to test *experimentally* quantum mechanics versus local realism within the present scheme. For this purpose, it is sufficient to consider how to measure $s_{\theta_a}^{(j)}$ in Eq. (14) so that the correlation function $E(\theta_a, \theta_b)$ can be obtained. Thus, in the following, we discuss possible schemes of measuring $s_\theta \equiv s_z \cos\theta + s_x \sin\theta$ for an arbitrary single-mode state of CVs. Quantum mechanically, s_θ represents an observable and thus can be measured in principle. But measuring it in practice is nontrivial.

In Ref. [28], a scheme is presented to measure an arbitrary motional observable of a trapped ion. It may be used if one plans to test the present Bell-CHSH inequality with two trapped ions in entangled motional states. Here we consider the case where the entangled states of CVs are prepared within two spatially separated high quality cavities [23], each of which resonantly interacts with a sequence of N two-level atoms. It suffices to consider only one of the cavities characterized by the annihilation operator a of the cavity field. The atom-cavity interaction is the usual Jaynes-Cummings Hamiltonian. Assuming the interaction time of each atom is t_I, the unitary evolution operator of the total atom-cavity system, in the interaction picture, is [29]

$$U_N(t_I) = e^{-igt_I(a^\dagger \sigma_N + a\sigma_N^\dagger)} \cdots e^{-igt_I(a^\dagger \sigma_1 + a\sigma_1^\dagger)}, \tag{19}$$

where $\sigma_i = |g\rangle_{ii}\langle e|$ ($|g\rangle_i$ and $|e\rangle_i$ are the ground and excited states, respectively, of the ith atom), and g is the atom-cavity coupling strength. Under the condition that trapping states are absent, the property of "asymptotic completeness" can be proved [29]:

$$\lim_{N\to\infty} U_N^\dagger(t_I) (A \otimes I_N) U_N(t_I) = I \otimes M_A, \tag{20}$$

which means that every observable A (e.g., s_θ) of the cavity field fully develops into the corresponding observable M_A of atoms in the asymptotic limit. Here, I_N is the unit operator in the Hilbert space of the N atoms. Reversing the interaction time t_I in (20), the asymptotic completeness should be still valid, and reads

$$\lim_{N\to\infty} W_N(t_I)(A \otimes I_N)W_N^\dagger(t_I) = I \otimes M_A, \qquad (21)$$

$$W_N(t_I) \equiv U_N^\dagger(-t_I), \qquad (22)$$

providing that A and M_A are time independent. Using the asymptotic completeness (21), the expectation value of A with respect to a cavity field state $|f\rangle$ is

$$\begin{aligned}\langle f|A|f\rangle &= \langle N|\langle f|A \otimes I_N|f\rangle|N\rangle \\ &= \lim_{N\to\infty}\langle N|\langle f|W_N^\dagger(I \otimes M_A)W_N|f\rangle|N\rangle, \qquad (23)\end{aligned}$$

with $|N\rangle$ being the initial state of atoms. Equation (23) implies that, to measure $\langle f|A|f\rangle$, one can send the N atoms across the cavity in sequence; in the limit $N \to \infty$, $\langle f|A|f\rangle$ is fully determined by measuring the observable of atoms M_A only.

However, it is impossible to handle practically an infinite number of atoms as above. Fortunately, with a finite number N of atoms, the above strategy can still yield the desired result with high accuracy, in fact, approaching 100% exponentially fast in N [29]. In this respect, it is also important to choose $|N\rangle$ properly to obtain optimal accuracy. Since states of atoms can be manipulated and measured with current technology with good accuracy [10], the strategy described here, though still experimentally challenging, offers a feasible way to measure the correlation function $E(\theta_a, \theta_b)$ with acceptable accuracy. But a more elegant method of measuring the correlation function $E(\theta_a, \theta_b)$ is highly desirable.

In summary, we have defined a new Bell operator for biparty systems with CVs. In this way Bell's inequalities have been generalized to CV systems. It is then demonstrated that the NOPA states display quantum nonlocality by using the Bell operator. In the limiting case of infinite squeezing, the NOPA states reduce to the original normalized EPR states which are shown to maximally violate the Bell-CHSH inequality. A strategy to approximately test the Bell-CHSH inequality has been proposed. The present work reveals a perfect analogy between the CV systems and the usual spin-$1/2$ systems. This fact opens up the possibility that, in terms of the parity spin, the CV systems may be exploited to do quantum information tasks (e.g., quantum teleportation [30]) as if they were usual qubits. Since the parity spin operator acts as a collective operator, we expect that our method might be robust against photon losses. Moreover, the present formulation enables us to derive all types of Bell's theorems for CV systems. For instance, the extension of our result to the Greenberger-Horne-Zeilinger theorem [31] for multiparty quantum systems with CVs is also possible and straightforward [32].

We thank K. Banaszek and Z. Y. Jeff Ou for valuable discussions. This work was supported by the National Natural Science Foundation of China and by the Chinese Academy of Sciences.

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