Novel Bell's Inequalities for Entangled $K^0\bar{K}^0$ Pairs

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We derive new Bell's inequalities for entangled $K^0\bar{K}^0$ pairs. This requires (1) mutually exclusive setups allowing either K^0 vs $\bar{K}^0$ or K_S vs K_L detection and (2) the use of kaon regenerators. The inequalities turn out to be significantly violated by quantum mechanics, resulting in interesting tests of local realism at ϕ factories and $p\bar{p}$ machines.

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The correlations shown by the parts of certain composite systems offer one of the most counterintuitive and subtle aspects of quantum mechanics (QM). Even if these parts are far away from each other, quantum entanglement persists, giving rise to paradoxical situations as in the EPR configuration [1]. This opened the important possibility to test QM vs local realism (LR) by means of Bell's inequalities [2–4]. These tests should be performed in different and complementary branches of physics not only to avoid the loopholes encountered in photon experiments, but also because they are interesting *per se*.

EPR entangled pairs consisting of neutral kaons, i.e., of two massive hadrons, are of interest at least for two reasons. One is precisely their mass, which makes this situation quite different from the more frequently considered case of massless photons. Another reason is the strong nature of hadronic interactions, which should enhance detection efficiencies—rather poor for photons—and contribute to closing the efficiency loophole. Many recent works on kaon pairs have thus appeared [5–17]. One usually considers the maximally entangled state:

$$\Phi(t) = \frac{N(t)}{\sqrt{2}}[K^0\bar{K}^0 - \bar{K}^0K^0], \tag{1}$$

coming from ϕ-meson decays, as at the DAΦNE ϕ-factory [16], or proton-antiproton annihilations at rest. The latter have been investigated by the CPLEAR experiment [17], confirming the nonseparability of state (1). In Eq. (1), the (proper) time dependent factor $N(t) \equiv e^{-i(\lambda_S+\lambda_L)t}$, with $\lambda_{S(L)} \equiv m_{S(L)} - i\Gamma_{S(L)}/2$, accounts for kaon decays on both sides. Apart from this, the structure of state (1) is identical to that of the spin-singlet state decaying into two spin-1/2 particles, as discussed in the EPR-Bohm analysis, and to that of the photon entangled pair frequently used in experimental arrangements. In all these cases one has to deal with a bipartite system formed by two dimension-two subsystems which fly apart along a left and a right beam. There are thus clear and useful analogies among these various cases, as has been emphasized in Refs. [5,11,14].

There is, however, an important difference which reduces considerably the possibilities of Bell tests with kaons. In the two spin-1/2 (photon) case, one can measure spin projections (linear polarizations) on each one of the two distant subsystems along *any* space direction chosen at will. Individual measurement outcomes are then given by a dichotomic variable and each subsystem is projected into one of the two orthogonal states of the basis associated to that chosen direction. The same happens when measuring the strangeness S of a neutral kaon beam by forcing it to interact with dense nucleonic matter. The distinct strong interactions of the strangeness $S = \pm 1$ states, K^0 and $\bar{K}^0$, on nucleons project the initial state into one of these two orthogonal members of the strangeness basis. Such a *strangeness measurement* is then in complete analogy with the spin case. Alternatively, one can identify the K_S vs K_L components of a beam via kaon weak decays and strong interactions, as discussed in the following. The K_S and K_L states are not strictly orthogonal, $\langle K_S|K_L\rangle = 2\,\mathrm{Re}\,\epsilon/(1 + |\epsilon|^2) \simeq 3.2 \times 10^{-3}$, thus their identification cannot be exact even in principle. However, the CP-violation parameter is so small, $|\epsilon| \simeq 2.3 \times 10^{-3}$, and the decay probabilities of the two components so different ($\Gamma_S \simeq 579\Gamma_L$) that the K_S vs K_L identification can effectively work in many cases. However, apart from this (only approximate) K_S vs K_L identification and the previous (in principle exact) strangeness measurement, no other quantum-mechanical measurement with dichotomic outcomes is possible for neutral kaons. Only these two kaon quasispin directions can be used to establish Bell's inequalities. This is in sharp contrast to the spin-singlet case and, as stated, reduces the possibilities of kaon experiments [18].

The complete analogy between strangeness measurements and spin-component measurements has been exploited by many authors [5,10–12,14,15]. In the analysis by Ghirardi *et al.* [5] one considers state (1) produced by a ϕ decay and performs joint strangeness measurements at two different times on the left beam and at other two different times on the right beam. Because of strangeness oscillations in free space along both kaon paths, choosing among four different times corresponds to four different choices of measurement directions in the spin case. In this sense, there is a total analogy and the Bell's inequalities discussed in Ref. [5] are a strict consequence of LR. Unfortunately, these inequalities are never violated by QM because strangeness oscillations proceed too slowly and cannot compete with the more rapid kaon weak decays. As discussed in Refs. [14,15], the inequalities exploiting strangeness measurements at four different times can be violated by QM only if a normalization of the observables to undecayed kaons is employed. The authors of Refs. [11,12], while insisting on the convenience of performing only unambiguous strangeness measurements, have substituted the use of different times by the possibility of choosing among different kaon regenerators to be inserted along the kaon path(s). The well-known regeneration effects can be interpreted as producing rotations in the kaon quasispin space analogous to the strangeness oscillations in Ref. [5], without requiring additional time intervals. One can thus derive solid Bell's inequalities, violated by QM, for simultaneous left-right strangeness measurements. The drawback of these analyses is that, up to now, they refer to only thin regenerators and the predicted violations of Bell's inequalities (below a few percent) are hardly observable.

The alternative option, based on K_S vs K_L identification, has been proposed by Eberhard [6]. In this case, it is convenient to rewrite state (1) as

$$\Phi(t) \simeq \frac{N(t)}{\sqrt{2}}[K_S K_L - K_L K_S], \tag{2}$$

the small CP-violating effects being neglected. To observe if a neutral kaon in a beam is K_S or K_L at a given point (i.e., instant), a kaon detector is located far enough downstream from this point so that the number of undecayed K_S's reaching the detector is negligible. Since $\Gamma_L \ll \Gamma_S$, almost all K_L's can reach the detector, where they manifest by strong nuclear interactions. In a complementary way, K_S's are identified by their decays not far from that point of interest. Misidentifications and ambiguous events will certainly appear, but at an acceptably low level [6]. In Ref. [6], K_S vs K_L measurements are then performed for each one of four experimental setups. In a first setup, state (2) is allowed to propagate in free space; its normalization is lost because of weak decays, but its perfect antisymmetry is maintained. In the other three setups, thick regenerators are asymmetrically located along one beam, or along the other, or along both. An interesting inequality relating the number of K_L's detected in each experimental setup is then derived from LR. It turns out to be significantly violated by QM predictions even if the above mentioned detection uncertainties are taken into account. Unfortunately, these successful predictions have some limitations, as already discussed by the author [6]. In particular, they are valid for asymmetric ϕ factories (where the two neutral kaon beams form a small angle), whose construction is not foreseen.

The purpose of the present Letter is to derive new forms of Bell's inequalities for neutral kaons not affected by the drawbacks we have mentioned. The key point is to combine the two kinds of dichotomic measurements, K^0 vs $\bar{K}^0$ and K_S vs K_L, in various alternative experimental setups. As in Refs. [5,6,11,12,14,15], where the various measurements are either of one type or the other, one can thus derive inequalities which follow strictly from LR. The inequalities we obtain turn out to be considerably violated by QM predictions, therefore a test of LR vs QM could in principle be performed at symmetric machines in operation.

We start with neutral kaon pairs produced in ϕ decays or proton-antiproton annihilations at rest, as given by Eqs. (1) and (2). A thin regenerator is fixed, say, on the right beam very close [19] to the ϕ decay point. If the proper time Δt required by the neutral kaon to cross the regenerator is short enough, $\Delta t \ll \tau_S$, and weak decays can thus be ignored, $N(\Delta t) \simeq 1$, state (2) becomes

$$\Phi(\Delta t) \simeq \frac{1}{\sqrt{2}}[K_S K_L - K_L K_S + r K_S K_S - r K_L K_L]. \tag{3}$$

The complex parameter r characterizes the regeneration effects and is defined by [20]

$$r \equiv i\frac{\pi\nu}{m_K}(f - \bar{f})\Delta t = i\frac{\pi\nu}{p_K}(f - \bar{f})d, \tag{4}$$

where m_K is the average neutral kaon mass, p_K the kaon momentum, f ($\bar{f}$) the K^0-nucleon ($\bar{K}^0$-nucleon) forward scattering amplitude, ν the density of scattering centers of the homogeneous regenerator whose total length is d.

States (2) and (3) differ only in the terms linear in the small parameter r [$|r| = \mathcal{O}(10^{-3})$ when $d = 1$ mm [7,20]]. To enhance their difference we now allow the state (3) to propagate in free space up to a proper time T, with $\tau_S \ll T \ll \tau_L$. One thus obtains the state

$$\begin{aligned}\Phi(T) \simeq \frac{N(T)}{\sqrt{2}}[K_S K_L &- K_L K_S \\ &- r e^{-i\Delta m T} e^{(1/2)(\Gamma_S - \Gamma_L)T} K_L K_L \\ &+ r e^{i\Delta m T} e^{(1/2)(\Gamma_L - \Gamma_S)T} K_S K_S],\end{aligned} \tag{5}$$

where $\Delta m \equiv m_L - m_S$. The $K_L K_L$ component in (5) has survived against weak decays much better than the accompanying terms $K_S K_L$ and $K_L K_S$ and has thus been enhanced. On the contrary, the $K_S K_S$ component has been further suppressed and can be neglected.

The normalization of state (5) to the surviving pairs leads then to

$$\Phi = \frac{1}{\sqrt{2 + |R|^2}}[K_S K_L - K_L K_S + R K_L K_L], \tag{6}$$

where

$$R \equiv -r e^{[-i\Delta m + (1/2)(\Gamma_S - \Gamma_L)]T}. \tag{7}$$

The quantity $|R| \simeq |r| e^{(1/2)\Gamma_S T}$ is not necessarily small with an exponential factor compensating the smallness of $|r|$. The state Φ is the entangled state we are going to consider for a Bell test. It describes all kaon pairs with both left and right partners surviving up to a common proper time T. At this point alternative measurements will be performed on each one of these kaon pairs Φ.

Among the various versions of Bell's inequalities we choose to work with that by Clauser and Horne (CH) [4]. For each kaon on each beam at time T we have to perform either a strangeness measurement (K^0 vs $\bar{K}^0$) or a lifetime measurement (K_S vs K_L). Therefore, for instance, $P(K_S, \bar{K}^0)$ will stay for the joint probability to observe a K_S on the left and a $\bar{K}^0$ on the right when the appropriate experimental setup is used. Single-kaon probabilities can be expressed in terms of joint probabilities and measured with the same setups. Therefore, $P(K_S, *) \equiv P(K_S, K^0) + P(K_S, \bar{K}^0)$ will correspond to the probability of observing a K_S on the left. In these one-side probabilities the joint probabilities for the two possible outcomes on the other side are added to guarantee that both kaons have survived up to time T.

For lifetime measurements allowing for K_S vs K_L identification at time T we will apply the following strategy. One has to detect decay events taking place in free space between times T and $T + \Delta T$. The otherwise surviving kaons will be detected by a dense absorber placed at the end of this decay region. If ΔT is large enough, the undecayed kaons can be identified as K_L's with reasonable certainty, while those who decayed in free space are most probably K_S's. If, for illustration purposes, we take $\Delta T \simeq 5.5\tau_S$, 99.1% of K_L's will survive, while 99.6% of K_S's will decay. Misidentifications are thus of the order of a few per thousand, i.e., of the same order as the CP-violating effects we are systematically neglecting. Notice that being K_S or K_L is a stable property in free space; provided the decay is observed between T and $T + \Delta T$, it implies that the neutral kaon was K_S at time T already. Care has to be taken, however, to choose T large enough to guarantee the spacelike separation between left and right measurements. Locality excludes then any influence from the experimental setup encountered by one member of the kaon pair at time T on the behavior of its other-side partner between T and $T + \Delta T$. For kaon pairs from ϕ decays moving at $\beta \simeq 0.22$ this implies $T > 1.77\Delta T$, with a considerable reduction of the total kaon sample. Indeed, for our previous illustrative case one can choose $T = 2\Delta T \simeq 11\tau_S$, and only 1 in 60 000 initial events can be used, having both kaons surviving at T.

Strangeness measurements at time T are performed by exploiting the distinct K^0 and $\bar{K}^0$ strong interactions on nucleons. To this end, a thin but extremely dense sheet of matter should be placed at the corresponding distance. Most of the authors working on the subject [5,10–12,14,15] consider that such a measurement is free from ambiguities and perfectly analogous to those employed in the spin case. In principle, this is true for infinitely dense sheets behaving like perfect absorbers and thus forcing the strong interaction to occur in a short time interval around T. In practice, the efficiency of the absorbers at disposal is limited and complicates the issue [17]. It would be highly desirable to identify very efficient absorbers exploiting, for instance, the formation of baryonic resonances in low-energy $\bar{K}^0$-N reactions. If this is not enough one should proceed to introduce conventional efficiency corrections [21]. In any case, the $\bar{K}^0$-N cross sections are known to be higher than those for K^0-N, thus implying a larger $\bar{K}^0$ detection efficiency. Consequently, we will express our results in terms of $\bar{K}^0$ (rather than K^0) detection probabilities.

Notice that the two measurements described in the last two paragraphs are mutually exclusive. One can choose to measure either K_S vs K_L or K^0 vs $\bar{K}^0$. In the first case no absorber is placed at time T and nothing is learned on the strangeness of the kaon. Alternatively, if one chose to insert the absorber at time T to identify strangeness, this is achieved only for absorbed events and nothing can be ever learned on the K_S vs K_L nature of the kaon. For the undetected kaons appearing downstream of the absorber, one fails to obtain any information on their state at time T. Later they can be observed to decay soon (K_S's) or not (K_L's), but this information on their K_S vs K_L nature refers to the kaon leaving the absorber, not to that at time T, which is the one of interest.

According to the previous discussion, the requirements for deriving a Bell's inequality from LR are fulfilled: (i) A nonfactorizable or entangled state (6) is used; (ii) alternative (mutually exclusive) measurements of either K^0 vs $\bar{K}^0$ or K_S vs K_L, which correspond to noncommuting observables in quasispin space, can be chosen at will; (iii) dichotomic outcomes for each single measurement are obtained; (iv) measurement events are spacelike separated. The situation mimics perfectly that of the spin case and the very same arguments invoked by Clauser and Horne [4] can be directly used for our present purposes.

Avoiding to work with K^0 detection, because of its lower efficiency, one can write several of CH's inequalities. The most convenient ones turn out to be

$$\begin{aligned} -1 \le\ & -P(\bar{K}^0, \bar{K}^0) + P(K_S, \bar{K}^0) + P(\bar{K}^0, K_L) \\ & + P(K_S, K_L) - P(K_S, *) - P(*, K_L) \le 0\,, \\ -1 \le\ & -P(\bar{K}^0, \bar{K}^0) + P(\bar{K}^0, K_S) + P(K_L, \bar{K}^0) \\ & + P(K_L, K_S) - P(*, K_S) - P(K_L, *) \le 0\,, \end{aligned} \tag{8}$$

where each one follows from the other by just inverting left and right measurements on our left-right asymmetric state (6).

As discussed in Ref. [4], the right-hand side (homogeneous) inequalities in Eq. (8) have the advantage of being independent of the normalization of the total sample of pairs involved and are thus easier to test. Only two things remain to be seen: Does QM violate these inequalities? Is the predicted violation large enough to compensate the detection accuracy level of our procedure?

The probabilities are computed in QM by just writing state (6) in the appropriate basis. By substituting these results in the homogeneous inequalities of Eq. (8) one easily finds, respectively:

$$\frac{2 - \mathrm{Re}R + \frac{1}{4}|R|^2}{2 + |R|^2} \leq 1, \qquad \frac{2 + \mathrm{Re}R + \frac{1}{4}|R|^2}{2 + |R|^2} \leq 1, \tag{9}$$

whose only difference is the sign affecting the linear term in ReR. According to the sign of ReR, one of these two inequalities is violated if $|\mathrm{Re}R| \geq 3|R|^2/4$. The greatest violation occurs for a purely real value of R with $|R| \simeq 0.56$, for which one of the two ratios in Eq. (9) reaches the value 1.14. This 14% violating effect predicted by QM is much larger than the unavoidable inaccuracies inherent in our procedure, hence it allows, at least in principle, for a meaningful experimental test.

Values for the parameter R satisfying $|\mathrm{Im}R| \ll |\mathrm{Re}R| \simeq 0.56$, as required, are not difficult to obtain. Indeed, for kaon pairs from ϕ decays and according to the values of the regeneration parameters [7], one can use a thin beryllium regenerator 1.6 mm thick to prepare state (3), which then converts into state (6) with the desired value of R by propagating in free space up to $T \simeq 11\tau_S$, as previously considered.

In a forthcoming paper [18] we shall discuss, systematically, the other Bell's inequalities violated by QM that can be derived by exploiting the same experimental setups considered here.

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[18] A. Bramon, G. Garbarino, and N. Nowakowski (to be published).
[19] Actually, it is not strictly necessary (and practically very difficult) to place the regenerator very close to the ϕ decay point. For the following discussion the regenerator can be placed at any instant time t' after the ϕ decay: the QM state is then given again by Eqs. (1) and (2), once a normalization to undecayed kaon pairs at time t' is considered. This only affects the experimental sample of counts one can then use for the Bell test.
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[21] In order to measure the strangeness detection efficiencies of a given absorber A, it has to be inserted along one beam at the "time-of-flight distance" T together with decay detectors (and/or other absorbers) placed downstream from that point. On the other side another absorber is symmetrically placed at T and kaon pairs given by Eq. (1) are used. Events with a K^0 ($\bar{K}^0$) detected by this latter absorber in coincidence with another detection at the other side (either by the former absorber A or by the detectors downstream) are selected. First one checks that none of these events is a double K^0 ($\bar{K}^0$) event at T, in agreement with the perfect antisymmetry of the state (1). Then, the ratio of the joint detection events with a $\bar{K}^0$ (K^0) in absorber A over the total sample of joint detections gives the efficiency of A for $\bar{K}^0$ (K^0) detection in the state (1). For other states, as in our Eq. (6), the efficiencies are not necessarily the same. The supplementary hypothesis of equal efficiencies is therefore needed as in other analyses [4].