Electron Entanglement via a Quantum Dot

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This Letter presents a method of electron entanglement generation. The system under consideration is a single-level quantum dot with one input and two output leads. The leads are arranged such that the dot is empty, single-electron tunneling is suppressed by energy conservation, and two-electron virtual cotunneling is allowed. Such a configuration effectively filters the singlet-state portion of a two-electron input, yielding a nonlocal spin-singlet state at the output leads. Coulomb interaction mediates the entanglement generation, and, in its absence, the singlet state vanishes. This approach is a four-wave mixing process analogous to the photon entanglement generated by a $\chi^{(3)}$ parametric amplifier.

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Identical quantum particles are inherently indistinguishable, and this oftentimes leads to nonclassical behavior such as entanglement in quantum mechanical systems. The Einstein-Podolsky-Rosen (EPR) state [1] is an interesting example of two-particle entanglement, because it has potential use in secure quantum communication protocol [2], quantum information processing [3], and fundamental tests of quantum mechanics [4]. Photons in nonlinear media interact to produce polarization-entangled EPR pairs and have been used in experimental demonstrations of quantum state teleportation [5], quantum nondemolition measurements [6], and violations of Bell's inequality [7]. Although entanglement with ions [8] and between atoms and cavity field modes has been demonstrated [9], to our knowledge, there have yet to be any experimental demonstrations specifically utilizing EPR pair-type entangled electrons. Recently, there have been several proposals to generate [10–14], detect [15–19], and characterize entangled electrons [11,20]. Electrons have been demonstrated to have long spin dephasing times in semiconductors [21]. In addition, the quantum optics tools [22,23], for example, an electron waveguide [24], beam splitter [25,26], intensity interferometer [26,27], and collision analyzer [17,25], required to detect entangled electrons have been demonstrated in two-dimensional electron gas systems. Furthermore, the lossless nature of electrons and the noiseless property of a cryogenic Fermi source may provide experimental advantages, for example, high detection efficiency [8,18], over their photon counterparts.

In this Letter, we consider a means to generate entangled EPR pairs from a two-electron mixed-state input using a three-port quantum dot (Fig. 1a) operating in the coherent tunneling regime [28,29]. The dot consists of a single input lead and two output leads, with an energy band diagram shown in Fig. 1b. The lead arrangement is such that the dot is empty. Conceptually, there are two key factors to the successful operation of this entangler. The first is that the leads are nondegenerate and of relatively narrow width in energy, thus acting as "energy filters." Single-electron tunneling does not conserve energy and is forbidden. However, the lead energies can be arranged such that two-electron cotunneling events do conserve energy, and thus the lowest order contribution to the tunneling current is two-electron virtual cotunneling through the dot. The second is that double occupancy of the dot incurs an on-site Coulomb energy. The Coulomb interaction mediates electron entanglement in this system, as a nonlinear medium does for photon entanglement. With the Coulomb interaction turned off, the singlet and triplet states destructively interfere. However, in the presence of Coulomb interaction, the singlet-state destructive interference becomes imperfect, while the triplet state destructive interference remains complete. This leads to a net singlet-state amplitude at the output of the dot.

The system is analyzed within the interaction picture using the Anderson Hamiltonian with an on-site Coulomb energy term U. We consider only a single, spin-degenerate energy level for the dot, and there are no single-electron excitations within the dot.

$$\hat{H}_{\mathrm{And}} = \hat{H}_0 + \hat{V},$$

$$\hat{H}_0 = \sum_{\eta,k,\sigma} \varepsilon_{\eta,k} \hat{a}^\dagger_{\eta,k,\sigma} \hat{a}_{\eta,k,\sigma} + \sum_{\sigma} \varepsilon_{\mathrm{d}} \hat{c}^\dagger_{\sigma} \hat{c}_{\sigma} + U \hat{n}_\uparrow \hat{n}_\downarrow,$$

$$\hat{V} = \sum_{\eta,k,\sigma} (V_\eta \hat{a}^\dagger_{\eta,k,\sigma} \hat{c}_\sigma + \mathrm{H.c.}), \tag{1}$$

where $\eta \in \{\mathrm{L}, \mathrm{R}_1, \mathrm{R}_2\}$ is the lead label, k is the lead electron momentum, $\sigma \in \{\uparrow, \downarrow\}$ is the electron spin, V_η

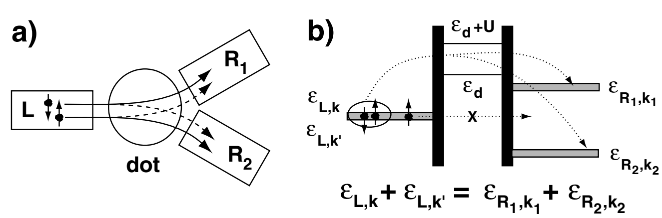

FIG. 1. (a) Three-port quantum dot. (b) Energy band diagram for three-port quantum dot with nondegenerate energy leads which act as energy filters.

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is the overlap matrix element between the dot and the lead states, $\hat{a}$ ($\hat{a}^\dagger$) is the annihilation (creation) operator for the lead electrons, $\hat{c}$ ($\hat{c}^\dagger$) is the annihilation (creation) operator for the dot electrons, and $\hat{n}_\sigma \equiv \hat{c}_\sigma^\dagger \hat{c}_\sigma$ is the dot electron number operator. The dot energy levels, ε_d and $\varepsilon_\mathrm{d} + U$ shown in Fig. 1, are taken to be off-resonance with the leads. The left lead energy is below its quasi-Fermi level so that the lead is full of electrons. The right leads are empty. In addition, we set $\varepsilon_\mathrm{d} = 0$, thereby referencing all energies to ε_d. The lead energies $\varepsilon_{\eta,k}$ and the charging energy U are left as parameters which can be adjusted to consider different dot configurations.

In all cases, the three-port quantum dot is biased such that single-electron tunneling from the left lead to the right lead is suppressed, that is, $\varepsilon_{\mathrm{L},k} \neq \varepsilon_{\mathrm{R}_1,k_1} \neq \varepsilon_{\mathrm{R}_2,k_2}$. However, two-electron virtual cotunneling does conserve energy; that is, the initial energy $\varepsilon_\mathrm{i} \equiv \varepsilon_{\mathrm{L},k} + \varepsilon_{\mathrm{L},k'}$ equals the final energy $\varepsilon_\mathrm{f} \equiv \varepsilon_{\mathrm{R}_1,k_1} + \varepsilon_{\mathrm{R}_2,k_2}$, requiring one electron from the left lead to go to lead R_1 and the other to go to lead R_2. This is the energy conserving process considered throughout this Letter.

We introduce the notation E_L, Δ_L, and Δ_R to parametrize $\varepsilon_{\mathrm{L},k}$, $\varepsilon_{\mathrm{L},k'}$, $\varepsilon_{\mathrm{R}_1,k_1}$, and $\varepsilon_{\mathrm{R}_2,k_2}$ and simplify the presentation: $E_\mathrm{L} \equiv \frac{1}{2}(\varepsilon_{\mathrm{L},k} + \varepsilon_{\mathrm{L},k'}) = \frac{1}{2}(\varepsilon_{\mathrm{R}_1,k_1} + \varepsilon_{\mathrm{R}_2,k_2})$, $\Delta_\mathrm{L} \equiv \frac{1}{2}(\varepsilon_{\mathrm{L},k} - \varepsilon_{\mathrm{L},k'})$, and $\Delta_\mathrm{R} \equiv \frac{1}{2}(\varepsilon_{\mathrm{R}_1,k_1} - \varepsilon_{\mathrm{R}_2,k_2})$. The two-electron initial state has energies $\varepsilon_{\mathrm{L},k}, \varepsilon_{\mathrm{L},k'} = E_\mathrm{L} \pm \Delta_\mathrm{L}$ within the left lead, and the two-electron final state has energies $\varepsilon_{\mathrm{R}_1,k_1}, \varepsilon_{\mathrm{R}_2,k_2} = E_\mathrm{L} \pm \Delta_\mathrm{R}$ in the right leads. Suppressing single-electron tunneling requires that $\Delta_\mathrm{L} < \Delta_\mathrm{R}$ for any Δ_L and Δ_R.

In terms of a perturbation expansion in the tunneling matrix element V, the lowest-order contribution to the two-electron cotunneling current from the left lead to the right leads is $\mathcal{O}(V^4)$. Given the assumption that only one dot energy level is relevant in this quantum dot, all higher-order terms contribute to either the self-energy of the electrons or are higher orders of two-electron cotunneling. Therefore, a two-electron initial state is used in this model,

$$|\phi_\mathrm{i}\rangle = \hat{a}_{\mathrm{L},k,\sigma}^\dagger \hat{a}_{\mathrm{L},k',\sigma'}^\dagger |0\rangle, \tag{2}$$

where $|0\rangle$ is the zero-particle state of this model system. This initial state is an arbitrary selection of two electrons from the entire left lead ground state at $T \approx 0$ K. We use the transition matrix (T matrix) formalism [30] to calculate the transition amplitude $\langle \phi_\mathrm{f} | \hat{T}(\varepsilon_\mathrm{i}) | \phi_\mathrm{i} \rangle$ between this initial state $|\phi_\mathrm{i}\rangle$ and a final state $|\phi_\mathrm{f}\rangle$ to order $\mathcal{O}(V^4)$, where $\hat{T}(\varepsilon_\mathrm{i}) = \hat{V} + \hat{V}\frac{1}{\varepsilon_i - \hat{H}_0}\hat{T}(\varepsilon_\mathrm{i})$ is the transition operator. The transition matrix can be used to find the transition rate $\omega_{i\to f} = (2\pi/\hbar)|\langle \phi_\mathrm{f}|\hat{T}(\varepsilon_\mathrm{i})|\phi_\mathrm{i}\rangle|^2 \delta(\varepsilon_\mathrm{f} - \varepsilon_\mathrm{i})$, from which one can calculate, for example, the current.

We consider first the case of a spin-up and spin-down electron tunneling through the dot to the output leads as indicated in Fig. 1. The initial state is Eq. (2) with $\sigma = \uparrow$ and $\sigma' = \downarrow$. The time-ordering operator in the perturbative expansion [30] leads to 12 unique time orderings by which these two electrons can virtually cotunnel from the left lead to the right leads. We group them into six paths, each with two time orderings: a "direct" time ordering as shown in Fig. 2, and an "exchange" time ordering due to the exchange of the output leads R_1 and R_2. This interchange of R_1 and R_2 introduces a minus sign due to the electron commutation relation. Each path is presented in terms of its singlet transition amplitude $T_s^{(i)} \equiv \langle s|\hat{T}|\phi_\mathrm{i}\rangle$ and triplet transition amplitude $T_t^{(i)} \equiv \langle t|\hat{T}|\phi_\mathrm{i}\rangle$, where i indicates the path number and we have used the shorthand notation $|s\rangle, |t\rangle \equiv (1/\sqrt{2})(\hat{a}_{\mathrm{R}_1\uparrow}^\dagger \hat{a}_{\mathrm{R}_2\downarrow}^\dagger \mp \hat{a}_{\mathrm{R}_1\downarrow}^\dagger \hat{a}_{\mathrm{R}_2\uparrow}^\dagger)|0\rangle$ to indicate the singlet and triplet states. The factor $V_\mathrm{L}^{*2} V_{\mathrm{R}_1} V_{\mathrm{R}_2}/(E_\mathrm{L}^2 - \Delta_\mathrm{R}^2)$ is common to all terms and omitted until the end. We obtain

$$T_s^{(\mathrm{I})} = \frac{\sqrt{2}(-\Delta_\mathrm{L} E_\mathrm{L} + \Delta_\mathrm{R}^2)}{(E_\mathrm{L} - \Delta_\mathrm{L})(\Delta_\mathrm{L}^2 - \Delta_\mathrm{R}^2)}, \quad T_t^{(\mathrm{I})} = \frac{-\sqrt{2}\,\Delta_\mathrm{R}}{\Delta_\mathrm{L}^2 - \Delta_\mathrm{R}^2},$$

$$T_s^{(\mathrm{II})} = \frac{\sqrt{2}\,E_\mathrm{L}}{(E_\mathrm{L} - \Delta_\mathrm{L})(2E_\mathrm{L} - U)}, \quad T_t^{(\mathrm{II})} = \frac{-\sqrt{2}\,\Delta_\mathrm{R}}{2E_\mathrm{L} - U},$$

$$T_s^{(\mathrm{III})} = \frac{\sqrt{2}\,E_\mathrm{L}}{(E_L - \Delta_\mathrm{L})(2E_\mathrm{L} - U)}, \quad T_t^{(\mathrm{III})} = \frac{+\sqrt{2}\,\Delta_\mathrm{R}}{2E_\mathrm{L} - U},$$

$$T_s^{(\mathrm{IV})} = \frac{\sqrt{2}(\Delta_\mathrm{L} E_\mathrm{L} + \Delta_\mathrm{R}^2)}{(E_\mathrm{L} + \Delta_\mathrm{L})(\Delta_\mathrm{L}^2 - \Delta_\mathrm{R}^2)}, \quad T_t^{(\mathrm{IV})} = \frac{+\sqrt{2}\,\Delta_\mathrm{R}}{\Delta_\mathrm{L}^2 - \Delta_\mathrm{R}^2},$$

$$T_s^{(\mathrm{V})} = \frac{\sqrt{2}\,E_\mathrm{L}}{(E_\mathrm{L} + \Delta_\mathrm{L})(2E_\mathrm{L} - U)}, \quad T_t^{(\mathrm{V})} = \frac{+\sqrt{2}\,\Delta_\mathrm{R}}{2E_\mathrm{L} - U},$$

$$T_s^{(\mathrm{VI})} = \frac{\sqrt{2}\,E_\mathrm{L}}{(E_\mathrm{L} + \Delta_\mathrm{L})(2E_\mathrm{L} - U)}, \quad T_t^{(\mathrm{VI})} = \frac{-\sqrt{2}\,\Delta_\mathrm{R}}{2E_\mathrm{L} - U}.$$

The transition amplitudes from paths I and IV do not incur a Coulomb energy U, while the remaining transition amplitudes do incur a Coulomb energy U during the virtual tunneling event. Because of the coherent nature of the virtual tunneling process, the six paths will interfere to produce the total transition amplitude to the final-state singlets, $T_s = \sum_{i=1}^6 T_s^{(i)}$, and final-state triplets, $T_t = \sum_{i=1}^6 T_t^{(i)}$.

The triplet transition amplitudes destructively interfere; paths I and IV, paths II and III, and paths V and VI

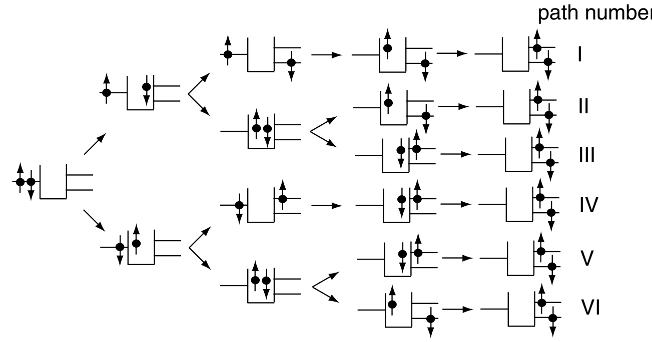

FIG. 2. Multiple paths by which two electrons can virtually cotunnel from the left lead through an empty dot to the two right leads.

have triplet contributions which cancel, independent of the Coulomb energy U. Only singlet transition amplitudes remain, and the singlet contributions along paths II and III and along paths V and VI are identical. Combining the U-independent (I and IV) and the U-dependent (II, III, V, and VI) singlet amplitudes yields

$$\sum_{i\in\{\mathrm{I,IV}\}} T_s^{(i)} = -\sqrt{2}\,\frac{2E_\mathrm{L}}{E_\mathrm{L}^2 - \Delta_\mathrm{L}^2}, \tag{3}$$

$$\sum_{i\in\{\mathrm{II,III,V,VI}\}} T_s^{(i)} = \sqrt{2}\,\frac{2E_\mathrm{L}}{E_\mathrm{L}^2 - \Delta_\mathrm{L}^2}\,\frac{2E_\mathrm{L}}{2E_\mathrm{L} - U}\,. \tag{4}$$

In the absence of the Coulomb energy U, the singlet transition amplitudes in the U-dependent paths destructively and completely interfere with those in the U-independent paths. However, the presence of the Coulomb energy U scales the singlet contribution of the U-dependent paths. The interference between the U-dependent and U-independent paths remains destructive, but it is incomplete, leaving a residual singlet amplitude at the output. Reinserting the common factor $V_\mathrm{L}^{*2}V_{\mathrm{R}_1}V_{\mathrm{R}_2}/(E_\mathrm{L}^2 - \Delta_\mathrm{R}^2)$, the transition amplitudes are

$$\langle s|\hat{T}|\phi_\mathrm{i}\rangle = \sqrt{2}\,\frac{V_\mathrm{L}^{*2}V_{\mathrm{R}_1}V_{\mathrm{R}_2}2E_\mathrm{L}}{(E_\mathrm{L}^2 - \Delta_\mathrm{R}^2)(E_\mathrm{L}^2 - \Delta_\mathrm{L}^2)}\,\frac{U}{2E_L - U}\,, \tag{5}$$

$$\langle t|\hat{T}|\phi_\mathrm{i}\rangle = 0. \tag{6}$$

The remaining case is the virtual co-tunneling of two electrons with the same spin. The input state is Eq. (2) with $\sigma = \sigma'$. Because of the Pauli exclusion principle, only paths I and IV can contribute to the output amplitude. Since there are no singlet amplitudes in the same-spin case and the triplet amplitudes destructively interfere for paths I and IV, there is no same-spin cotunneling contribution at the output,

$$\langle\uparrow\uparrow|\hat{T}|\phi_\mathrm{i}\rangle = \langle\downarrow\downarrow|\hat{T}|\phi_\mathrm{i}\rangle = 0\,. \tag{7}$$

The physical interpretation of the mathematics leading to Eq. (5) can be explained in the following way. Two electrons (with possibly different energies) virtually tunnel through the quantum dot along six different paths (12 unique time orderings). These different paths interfere with each other. The interference for the triplet states is destructive and complete. This is due to the Fermi statistics and the indistinguishability between the direct and exchange time orderings for the U-independent paths (I, IV) and the U-dependent paths (II, III and V, VI). For example, due to indistinguishability within the U-independent paths (I, IV), two electrons in the left lead will tunnel along these paths to yield direct ($\hat{a}^\dagger_{\mathrm{R}_1\uparrow}\hat{a}^\dagger_{\mathrm{R}_2\downarrow}$) and exchange ($\hat{a}^\dagger_{\mathrm{R}_2\uparrow}\hat{a}^\dagger_{\mathrm{R}_1\downarrow}$) output states with the same energy-dependent coefficient. The electron commutation relation immediately leads to a singlet state $\frac{1}{\sqrt{2}}(\hat{a}^\dagger_{\mathrm{R}_1\uparrow}\hat{a}^\dagger_{\mathrm{R}_2\downarrow} - \hat{a}^\dagger_{\mathrm{R}_1\downarrow}\hat{a}^\dagger_{\mathrm{R}_2\uparrow})$. The same holds for the U-dependent paths (II, III) and (V, VI). Only singlet states remain, resulting in the U-independent and U-dependent singlet transition amplitudes in Eqs. (3) and (4), respectively. By the same type of argument, one can show that the interference is also destructive and complete for the same-spin triplet transition amplitudes in Eq. (7). We note that in any case, there is no requirement to "mix" the U-dependent and U-independent paths to get the complete destructive interference of the triplet states.

However, this "nonmixing property" or system symmetry is broken in the case of the singlet states. The interference between the singlet transition amplitudes in Eq. (3) and Eq. (4) is also destructive in nature, but complete only when the Coulomb energy $U = 0$. For $U \neq 0$, the destructive interference becomes imperfect, and a residual singlet state remains at the output. One perspective is that U scales the singlet contribution in Eq. (4). In this Letter, we have taken $E_\mathrm{L} < \varepsilon_\mathrm{d} = 0$ as shown in Fig. 1, so that the scaling factor $2E_\mathrm{L}/(2E_\mathrm{L} - U) \in (0, 1]$ for $U \in [0, \infty)$. (We consider elsewhere [31] the resonant enhancement due to $E_\mathrm{L} \rightarrow U/2$.) In this case, increasing U acts to suppress virtual passage along the paths II, III, V, VI, by making the energy penalty for these virtual processes larger. In the limit $U = 0$, the singlet transition amplitudes in Eqs. (3) and (4) are equal in magnitude and of opposite parity; they cancel when summed. However, $U \neq 0$ breaks this symmetry by suppressing the transition amplitudes leading to Eq. (4), resulting in the net singlet contribution in Eq. (5). As $U \rightarrow \infty$, only the U-independent singlet transition amplitudes in Eq. (3) contribute to the net singlet state in Eq. (5).

The conclusion is that virtual cotunneling of two electrons through an empty quantum dot in the presence of Coulomb interaction filters out the singlet-state component [the Hamiltonian in Eq. (1) conserves spin], generating entangled-electron spin-singlet states at the output. This quantum dot system exhibits Coulomb-mediated wave mixing [32] ; the optical analog is a $\chi^{(3)}$ parametric amplifier, a four-wave mixing process in which two input photons interact within the $\chi^{(3)}$ nonlinear medium and may generate an entangled pair of output photons. The $\chi^{(3)}$ nonlinearity mediates the entanglement in the photon case, whereas it is the nonlinear Coulomb interaction as manifest in the on-site Coulomb energy U in the electron case. We note that the additional mechanisms required for photon-polarization entanglement are usually engineered through, for example, the appropriate choice of phase-matching conditions and postselection, while the additional mechanism for electron-spin entanglement, Fermi statistics, is intrinsic to the quantum dot system. Although we limited this discussion to a quantum dot system, we note that the principle of Coulomb-mediated entanglement (see also Ref. [10]) is applicable to other fermion systems which demonstrate Coulomb charging behavior.

Finally, an experimental demonstration of an electron entangler requires a means to generate and a means to detect the entanglement. This system requires a quantum dot with tight confinement, for example, through side gating with low surface depletion, to reach the empty dot limit discussed in this Letter. This requirement might be relaxed to allow dots with even numbers of electrons. In any case, the dot level broadening and temperature must be small compared with the Coulomb energy, the dot level separation, and the output lead energy separation. Realization of the energy filters might be achieved using additional quantum dots [14] or superlattice structures [33]. We note here that the energy filter broadening must be tight enough to function as an energy filter, yet large enough that the dwell time is small compared with the dephasing time of the EPR state. Single-electron current leaks will depend largely on the actual design of the energy filters, but are composed of two dominant contributions. First is single-electron virtual tunneling through the dot and the energy filters, and second is phonon-assisted single-electron tunneling. Although we would ideally desire a pure EPR-pair state at the output, most correlation effects (for example, those used in detection schemes) will still be visible in the limit that the single-electron and two-electron cotunneling contributions to the current are of the same order. Finally, experimental verification of electron entanglement might be achieved through an electron bunching/antibunching experiment [16,17], or spin-correlation measurements and, ultimately, a Bell's inequality test [18]. In both cases, standard noise measurement techniques could be adopted to infer the degree and type of entangled state produced. We address elsewhere [31] the issues of experimental implementation in more detail.

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