Spin Textures, Screening, and Excitations in Dirty Quantum Hall Ferromagnets

S. Rapsch,[1] J. T. Chalker,[1] and D. K. K. Lee[2]

[1]*Theoretical Physics, Oxford University, Oxford OX1 3NP, United Kingdom*

[2]*Blackett Laboratory, Imperial College, Prince Consort Road, London SW7 2BW, United Kingdom*

(Received 18 September 2001; published 2 January 2002)

We study quantum Hall ferromagnets in the presence of a random electrostatic impurity potential. Describing these systems with a classical nonlinear sigma model and using analytical estimates supported by results from numerical simulations, we examine the nature of the ground state as a function of disorder strength, Δ, and deviation, $\delta\nu$, of the average Landau level filling factor from unity. Screening of an impurity potential requires distortions of the spin configuration, and in the absence of Zeeman coupling there is a disorder-driven, zero-temperature phase transition from a ferromagnet at small Δ and $|\delta\nu|$ to a spin glass at larger Δ or $|\delta\nu|$. We examine ground-state response functions and excitations.

DOI: 10.1103/PhysRevLett.88.036801 PACS numbers: 73.43.Cd, 71.10.–w, 75.10.Nr

Quantum Hall ferromagnets (QHFMs) are interesting especially as systems in which spin configurations and charge density are closely linked [1]. At small Zeeman energy and for Landau level filling factor ν close to unity, this link has the celebrated consequence that the charged quasiparticles with the lowest energy are not single electrons but Skyrmions [2]. These bound states of a minority-spin electron with one or many spin waves may be viewed classically as topological excitations of an ordered ferromagnet: in this description, the deviation of local charge density from that of a filled and ferromagnetically polarized Landau level is proportional to the topological density [3] of the spin configuration. For a clean QHFM with sufficiently small Zeeman energy, Skyrmions or anti-Skyrmions are introduced at zero temperature on varying the average filling factor from $\nu = 1$ to larger or smaller values and, at nonzero temperature, are generated thermally in pairs together with spin waves. For dirty QHFMs, coupling of an electrostatic impurity potential to the charge density offers an additional mechanism by which spin textures may arise: the consequences of such a coupling are the subject of this paper.

The interplay between disorder and exchange for QHFMs has been examined previously from several different viewpoints. Fogler and Shklovskii, building on earlier discussions, have developed a mean field treatment in the spirit of Stoner theory, finding for odd integer ν in the absence of Zeeman coupling a transition between ferromagnetic and paramagnetic ground states with increasing disorder strength [4]. They suggested that this transition should be apparent in transport measurements, in which the ferromagnetic phase is characterized by spin-resolved Shubnikov–de Haas oscillations and the paramagnet by spin-unresolved oscillations. Experimentally, a transition of this kind is observed with decreasing magnetic field strength [5], and its sharpness suggests that its origin is indeed cooperative. Within such an approach, proposed for higher Landau levels where Skyrmions are normally not stable, local moments are all collinear in the ferromagnet and vanish in the paramagnet. By contrast, near $\nu = 1$, an alternative is that a QHFM may respond to disorder mainly via the direction rather than the magnitude of its local magnetization. Some indications that this can happen come from calculations for the fully polarized ferromagnet at weak disorder. Here, a reduction in spin stiffness with increasing disorder strength has been interpreted by Green [6] as a precursor of a noncollinear phase. Moreover, even weak disorder may nucleate a dilute glass of Skyrmions (over minima in the electrostatic potential) and anti-Skyrmions (over maxima), as discussed by Nederveen and Nazarov [7]. In addition, at intermediate disorder strength both reduced and noncollinear local moments emerge from a numerical solution of Hartree-Fock theory for a model with Coulomb interactions and spatially uncorrelated disorder, by Sinova, MacDonald, and Girvin [8]. More generally, the relative importance for dirty QHFMs of local moment reduction versus the formation of spin textures will depend on the nature of disorder. In the following we focus on textures, favored by a smoothly varying impurity potential.

To this end, consider a quantum Hall system with ν close to unity, impurity potential $V(\mathbf{r})$, electron density $\rho(\mathbf{r})$, and electron-electron interaction energy $U(\mathbf{r})$. As a first step, treat screening using Thomas-Fermi theory, omitting exchange interactions and Zeeman energy. Within this approximation, developed by Efros [9] for the comparable problem in spin-polarized Landau levels when ν lies near half-integer values, the ground-state charge density at weak disorder is determined by the condition that the Hartree potential should everywhere match the chemical potential: $\mu = V(\mathbf{r}) + \int U(\mathbf{r} - \mathbf{r}')\rho(\mathbf{r}')\, d^2\mathbf{r}'$. We are concerned with circumstances in which the resulting density varies smoothly on the scale of the magnetic length, l_B, and has fluctuations from $\nu = 1$, $\delta\rho(\mathbf{r}) \equiv \rho(\mathbf{r}) - (2\pi l_B^2)^{-1}$, which are small: $|\delta\rho(\mathbf{r})| \ll \rho(\mathbf{r})$. Restoring exchange interactions under these conditions results locally in maximal ferromagnetic polarization of electron spins, with a direction that may vary in space. Denoting this direction by the three-component unit vector $\vec{S}(\mathbf{r})$, its spatial fluctuations are linked to electron density via [2,3]

$$\delta\rho(\mathbf{r}) = (8\pi)^{-1}\epsilon_{ij}\epsilon_{\alpha\beta\gamma}S^{\alpha}\partial_i S^{\beta}\partial_j S^{\gamma}. \quad (1)$$

An exchange energy is associated with such variations: combining exchange (with interaction constant J), impurity, and Hartree contributions to the total energy, and choosing for simplicity a short-range interaction $U(\mathbf{r} - \mathbf{r}') = U_0\delta(\mathbf{r} - \mathbf{r}')$, we take as our description of a dirty quantum Hall ferromagnet the configurational energy

$$\mathcal{H} = \int \left(\frac{J}{2}|\nabla\vec{S}(\mathbf{r})|^2 + V(\mathbf{r})\delta\rho(\mathbf{r}) + \frac{U_0}{2}[\delta\rho(\mathbf{r})]^2 \right) d^2\mathbf{r}\,. \quad (2)$$

Doing so, we neglect quantum fluctuations of $\vec{S}(\mathbf{r})$, as is justified in the semiclassical limit, $|\nabla\vec{S}(\mathbf{r})| \ll l_{\mathrm{B}}^{-1}$, to which we are already restricted. We choose for simplicity $V(\mathbf{r})$ Gaussian distributed with amplitude Δ and correlation length λ.

Our aim in the following is to understand the zero-temperature phase diagram of the model defined by Eq. (2), and to characterize its ground states via their response functions and excitations. As an example of a disordered electron system, it is unusual in that there is an exchange gap for single-particle excitations, even if the ground-state spin configuration $\vec{S}(\mathbf{r})$ is not ferromagnetically ordered, so that the only low-energy excitations involve collective spin modes. As an example of a ferromagnet with quenched disorder, the system is also unusual in several ways. First, the coupling to disorder leaves spin-rotation symmetry intact but breaks time-reversal symmetry, in contrast to random magnetic fields, which break both symmetries, and to random exchange interactions, which leave both symmetries intact. Second, because of the same coupling, the spin system responds to applied electric fields: we calculate the wave vector–dependent dielectric susceptibility, comparing with behavior found in more conventional disordered electron systems. Third, the link between spin and charge also endows spin waves with an electric dipole moment: we calculate the spin-wave contribution to the optical conductivity in disordered spin states, complementing Green's results [6] for the polarized ferromagnet with impurities.

We start with a simple discussion of the phase diagram. The model is characterized by two energy scales, J and Δ, and two length scales, λ and $L_{\mathrm{H}} \equiv (U_0/J)^{1/2}$. The last of these, which we call the Hartree length, plays an important role in what follows. Its significance can be made clear by comparing, for a Skyrmion of fixed shape and radius R in a clean system, the contributions to total energy from exchange and from Hartree interactions, of order J and U_0/R^2, respectively: exchange dominates on length scales large compared with L_{H}, while Hartree interactions dominate at smaller distances. We use the limit $\lambda \ll L_{\mathrm{H}}$ as a source of simplifications in analytical estimates, but take $\lambda \lesssim L_{\mathrm{H}}$ in numerical simulations.

Examine first the ground state for $\langle\delta\rho\rangle = 0$ as a function of Δ/J. At weak disorder ($\Delta \lesssim J$) it can be pictured as a dilute glass of Skyrmions and anti-Skyrmions, discussed in Ref. [7]. This reflects the existence of a threshold [6–8], defined by $|V(\mathbf{r})| = 4\pi J$: in most parts of the system (roughly, those where $|V(\mathbf{r})|$ is below threshold) ferromagnetic order is essentially unaffected by the impurity potential [10]. More precisely, for any $\vec{S}(\mathbf{r})$ one has $|\nabla\vec{S}(\mathbf{r})|^2 \geq 8\pi|\delta\rho(\mathbf{r})|$ and hence $\mathcal{H} \geq \int[4\pi J|\delta\rho(\mathbf{r})| + V(\mathbf{r})\delta\rho(\mathbf{r})]d^2\mathbf{r}$, which for $V(\mathbf{r})$ below threshold everywhere implies that the ground state is the perfectly aligned ferromagnet with $\delta\rho(\mathbf{r}) = 0$. At stronger disorder ($\Delta \gg J$), by contrast, screening is almost perfect at short distances and $\delta\rho(\mathbf{r}) \simeq -V(\mathbf{r})/U_0$. Corrections to such screening arise at and beyond the scale L_{H}, where exchange is important. We can summarize the effect of exchange by dividing the system into regions of area L_{H}^2, finding for each such area the integral $Q \equiv -\int V(\mathbf{r})/U_0\, d^2\mathbf{r}$, and adjusting the total screening charge within every region to the integer value closest to Q. We argue that these integers are predominantly zero in a ferromagnetic phase, and predominantly nonzero in a phase without ferromagnetic order. To see this, consider a well-ordered ferromagnetic phase, in which $S(\mathbf{r})$ as a function of $\mathbf{r}$ has small fluctuations around a global direction of magnetization. In this case, the net topological charge in each region has magnitude much less than one. Conversely, in a phase without such order, unit topological charge accumulates in a region of linear size given by the ferromagnetic correlation length. The phase boundary for the ferromagnet is therefore located by setting $\langle Q^2\rangle^{1/2} \sim 1$. Since each area of size λ^2 contributes to Q a charge of magnitude $\lambda^2\Delta/U_0$ with random sign, $\langle Q^2\rangle^{1/2} \sim \lambda L_{\mathrm{H}}\Delta/U_0$ yielding $\Delta_c \sim U_0/(\lambda L_{\mathrm{H}}) = J(L_{\mathrm{H}}/\lambda)$. For $\Delta > \Delta_c$ the ground state has no ferromagnetic order; because within a classical zero-temperature description spins are frozen, we identify this phase as a spin glass.

Consider next the ground state at fixed $\Delta < \Delta_c$, as a function of $\langle\delta\rho\rangle$. Charge is introduced into the system for $\langle\delta\rho\rangle > 0$ as Skyrmions of size R. In the presence of Hartree energy alone, R is divergent, but fluctuations of an impurity potential establish an optimal size, by generating random potential wells. For $R \gg \lambda$ these have an average depth $\Delta(\lambda/R)$ (the case $R \lesssim \lambda$ is treated in Ref. [7]). The contributions to the Skyrmion energy from these two sources are of the order of U_0/R^2 and $-\Delta\lambda/R$, respectively; minimizing their sum, we find $R \sim U_0/(\Delta\lambda) = L_{\mathrm{H}}(\Delta_c/\Delta)$. (Note that, since $R > L_{\mathrm{H}}$ for $\Delta < \Delta_c$, screening as discussed in the previous paragraph does not alter this argument.) We expect ferromagnetic order to persist with increasing $\langle\delta\rho\rangle$ until such Skyrmions overlap, so the phase boundary lies at $\langle \delta\rho \rangle_c \sim L_{\mathrm{H}}^{-2}(\Delta/\Delta_c)^2$. In this way we arrive at the schematic phase diagram shown in the inset to Fig. 1.

In order to test these arguments, we have studied ground states of a lattice model numerically. In outline, we replace Eq. (2) with a Heisenberg model on a square lattice which has nearest-neighbor ferromagnetic interactions of strength J and a topological charge defined on each elementary plaquette. These charges contribute to the total energy by a local Hartree interaction with coefficient U_0, and by their interaction with plaquette potentials, independently and uniformly distributed in $[-\Delta, \Delta]$. Thus $\lambda = 1$ in units of the lattice spacing. We find low-energy states using a simulated annealing procedure similar to that of Ref. [11], and can identify states that are equivalent under a global rotation by comparing total energies and also charge distributions. Using system sizes up to 56^2 spins and annealing with 10^6 Monte Carlo steps per spin, for a given disorder realization in successive runs we repeatedly reach states from a small set: we take the lowest of these in energy to be the ground state. In the ground state for each disorder realization we calculate the site-averaged magnetization, $M = |\langle \vec{S}(\mathbf{r}) \rangle|$. We also determine the ground-state susceptibility χ, from the response to a Zeeman field applied in the direction of the magnetization; the ground-state spin stiffness ρ_{S}, from the energy cost of long-wavelength twists imposed on the spin configuration; and the wave vector–dependent compressibility, from the linear response of $\delta\rho(\mathbf{r})$ to a periodic potential added to $V(\mathbf{r})$. Further details of our methods will be described elsewhere [12].

Representative results are shown in Fig. 2. Taking $\langle \delta\rho \rangle = 0$ and $U_0 = 8\pi J$, we present M, χ, and ρ_{S} as a function of disorder strength, $\Delta/4\pi J$, using 40^2 spins and an average over three disorder realizations. Three regimes are evident from the behavior of M. For $\Delta/4\pi J < 1$, the ground state is a fully polarized ferromagnet, because we use a bounded disorder distribution. For $1 < \Delta/4\pi J \lesssim 2.5$, the ground state is a partially polarized ferromagnet (though with an almost saturated magnetization for $1 < \Delta/4\pi J \lesssim 1.8$). And for $2.5 \lesssim \Delta/4\pi J$, the ground state is, we argue, a spin glass, with a small nonzero M arising as a finite-size effect. In support of this interpretation, a large peak in χ indicates a phase transition at $\Delta/4\pi J \approx 2.5$. Moreover, the distinction between phases is illustrated by their spin stiffness, specified in full by a 3×3 symmetric tensor. We plot the eigenvalues of this in Fig. 2. For the fully polarized ferromagnet, rotations about the magnetization direction do not alter the spin configuration, and so one eigenvalue is zero while the other two are degenerate. For the partially polarized ferromagnet, all three eigenvalues are nonzero, with two remaining degenerate. And at $2.5 \lesssim \Delta/4\pi J$ magnetic isotropy and finite spin stiffness in the spin glass are illustrated by the fact that all three eigenvalues are approximately degenerate and nonzero.

Repeating such calculations for $\langle \delta\rho \rangle > 0$, we determine the phase diagram shown for $U_0 = 4\pi J$ in the main panel of Fig. 1, which is qualitatively similar to that in the inset. The maximum density range over which disorder may stabilize the ferromagnet, $|\langle \delta\rho \rangle| \lesssim 10^{-2}$ in units of charge per plaquette, is strikingly narrow.

We now turn to the dielectric response of the partially polarized ferromagnet and spin glass, characterized at zero frequency by the wave vector–dependent dielectric susceptibility $\chi_{\mathrm{e}}(q)$ or by the compressibility $\kappa(q)$, with $\kappa(q) = q^2\chi_{\mathrm{e}}(q)\epsilon_0/e^2$. To fix definitions, take $V(\mathbf{r}) \rightarrow V(\mathbf{r}) + V_1\cos(\mathbf{q}\cdot\mathbf{r})$ in Eq. (2): the ground-state electron density changes according to $\delta\rho(\mathbf{r}) \rightarrow \delta\rho(\mathbf{r}) + \delta\rho_1(\mathbf{r})$ and the disorder-averaged linear response is $\langle \delta\rho_1(\mathbf{r}) \rangle = -\kappa(q)V_1\cos(\mathbf{q}\cdot\mathbf{r})$. We can apply to $\kappa(q)$ the approach used in our discussion of the phase diagram: the Hartree length L_{H} again plays an important role, within our model

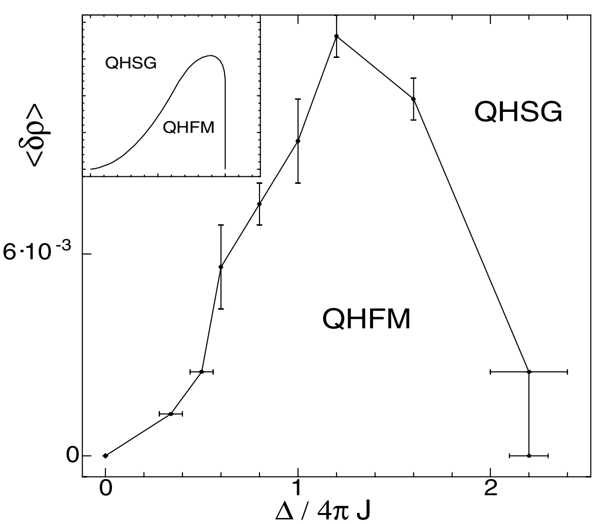

FIG. 1. Phase diagram for $L_{\mathrm{H}} = \sqrt{4\pi}\,\lambda$, obtained from simulations. Inset: phase diagram for $L_{\mathrm{H}} \gg \lambda$, obtained from analytical arguments described in the text.

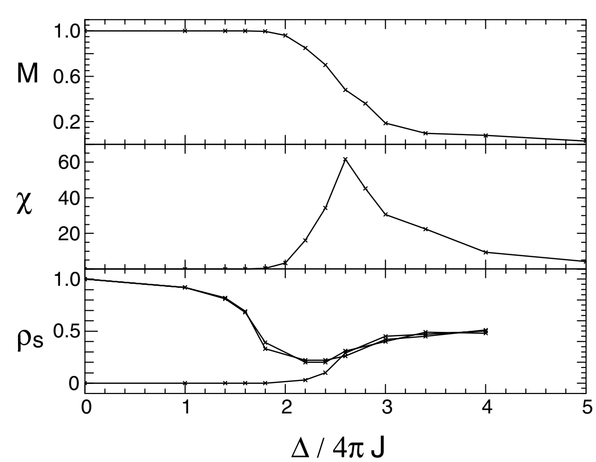

FIG. 2. Ground-state properties as a function of disorder strength: magnetization M, susceptibility χ, and eigenvalues ρ_{S} of the spin stiffness tensor.

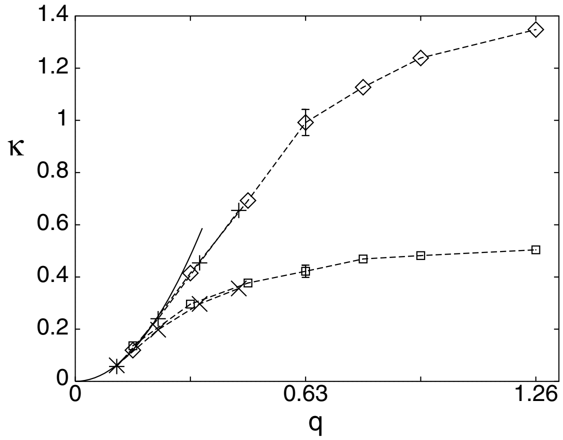

FIG. 3. Compressibility $\kappa(q)$ as a function of wave vector q for systems with $J = 1$ and $U_0 = 4\pi$ ($\Diamond$, $+$) or $U_0 = 8\pi$ ($\Box$, $\times$); full line: quadratic fit at small q.

problem in which Coulomb interactions are omitted. For $q \gg L_{\mathrm{H}}^{-1}$, exchange may be neglected and $\kappa(q) \simeq U_0^{-1}$. More generally, we anticipate the scaling form $\kappa(q) = U_0^{-1} f(L_{\mathrm{H}} q)$, with $f(x)$ constant at large x. For $q \ll L_{\mathrm{H}}^{-1}$ we expect that exchange dominates and hence that $\kappa(q)$ should be independent of U_0 in this regime. In turn this implies for the scaling function $f(x) \propto x^2$ at small x, so that $\kappa(q) \propto (qL_{\mathrm{H}})^2 U_0^{-1} \equiv Jq^2$ for $q \ll L_{\mathrm{H}}^{-1}$. To summarize, the response is that of a metal [$\kappa(q)$ constant] at large wave vectors, and that of an insulator [$\chi_{\mathrm{e}}(q)$ constant] at small wave vectors [13].

We have tested these arguments using the simulation methods outlined above. Results for $\kappa(q)$ are displayed in Fig. 3, where we compare behavior in systems with $J = 1$ and $U_0 = 4\pi$ or $U_0 = 8\pi$, combining for each case data from lattices of size 40^2 and 56^2 in order to maximize wave-vector resolution. The distinction is clear between a U_0-independent $\kappa(q)$, quadratic in q, at small q, and a q-independent $\kappa(q)$, varying roughly as U_0^{-1}, at large q.

Finally, we consider optical conductivity $\sigma(\omega)$ at frequency ω. A spin glass in the absence of Zeeman energy is expected [14] to support three degenerate Goldstone modes. Their dispersion is linear at small frequencies with speed $c = (\rho_{\mathrm{S}}/\chi)^{1/2}$. In an QHFM they have an electric dipole moment, which arises because they propagate in a noncollinear ground state. Their contribution to $\sigma(\omega)$ is determined by a product of the mean square dipole moment with the density of states. We find [12]

$$\sigma(\omega) \sim \frac{e^2}{h}\left(\frac{\omega\xi}{c}\right)^2 \frac{\hbar\omega}{J}, \tag{3}$$

where ξ is the spin correlation length. We estimate, for example, $\sigma(\omega) \sim 10^{-5} e^2/h$ at 10 GHz, using $c \sim 10^4\ \mathrm{ms}^{-1}$ (obtained from our numerical simulations), $J = 40$ K, and $\xi = 10^{-7}$ m. Variable-range hopping would mask this contribution to $\sigma(\omega)$ but should be absent if spins locally have maximal polarization throughout the sample.

We are grateful for discussions with N. R. Cooper and S. L. Sondhi. The work was supported in part by the EPSRC under Grant No. GR/J78327 (J. T. C.), and by the Royal Society (D. K. K. L.).

[1] For an introduction to quantum Hall ferromagnets, see S. M. Girvin, in *Topological Aspects of Low Dimensional Systems,* Proceedings of the Les Houches Summer School, Session LXIX, edited by A. Comtet, T. Jolicoeur, and S. Ouvry (Les Editions de Physique, Paris, 1999).
[2] S. L. Sondhi, A. Karlhede, S. A. Kivelson, and E. H. Rezayi, Phys. Rev. B **47**, 16 419 (1993).
[3] For an introduction, see R. Rajaraman, *Solitons and Instantons* (North-Holland, Amsterdam, 1989).
[4] M. M. Fogler and B. I. Shklovskii, Phys. Rev. B **52**, 17 366 (1995); T. Ando and Y. Uemura, J. Phys. Soc. Jpn. **37**, 1044 (1973).
[5] M. A. Paalanen *et al.,* Phys. Rev. B **25**, 5566 (1982); D. R. Leadley *et al.,* Phys. Rev. B **58**, 13 036 (1998).
[6] A. G. Green, Phys. Rev. B **57**, R9373 (1998); Phys. Rev. Lett. **82**, 5104 (1999).
[7] A. J. Nederveen and Y. V. Nazarov, Phys. Rev. Lett. **82**, 406 (1999).
[8] J. Sinova, A. H. MacDonald, and S. M. Girvin, Phys. Rev. B **62**, 13 579 (2000).
[9] A. L. Efros, Solid State Commun. **65**, 1281 (1988); **70**, 253 (1989).
[10] One should ask whether even a *dilute* glass of Skyrmions and anti-Skyrmions would destroy ferromagnetic magnetic order, since the spin deviation due to an isolated Skyrmion falls off with distance only as r^{-1}. In fact, we argue [12] that in such a glass the $U(1)$ degrees of freedom for different Skyrmions are correlated and long-range ferromagnetic order is preserved.
[11] M. Hale, O. Schwindt, and T. Weidig, Phys. Rev. E **62**, 4333 (2000).
[12] S. Rapsch, J. T. Chalker, and D. K. K. Lee (to be published).
[13] More precisely, we expect this to be the linear response at low but nonzero frequency, arising from only small changes in $\vec{S}(\mathbf{r})$. A finite value for $\kappa(0)$, related in the usual way to the thermodynamic density of states, requires $\mathcal{O}(1)$ changes in $\vec{S}(\mathbf{r})$, possible only in an extreme low-frequency regime.
[14] B. I. Halperin and W. M. Saslow, Phys. Rev. B **16**, 2154 (1977); S. L. Ginzburg, Sov. Phys. JETP **48**, 756 (1978).