# Measuring Double-Parton Distributions in Nucleons at Proton-Nucleus Colliders

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(Received 11 June 2001; published 2 January 2002)

We predict a strong enhancement of multijet production in proton-nucleus collisions at collider energies, as compared to a naive expectation of a cross section $\propto A$. The study of the process would allow one to measure, for the first time, the double-parton distribution functions in a nucleon in a model-independent way and hence to study both the longitudinal and the transverse correlations of partons.

DOI: 10.1103/PhysRevLett.88.031801 PACS numbers: 13.85.Hd, 25.40.Ve

The systematic studies of hard inclusive processes during the past two decades have led to a pretty good understanding of the single-parton densities in nucleons. However, very little is known about multiparton correlations in nucleons which could provide a new tool for discriminating between different models of nucleons. Such correlations may be generated, for example, by the fluctuations of the transverse size of the color field in the nucleon leading, via color screening, to correlated fluctuations of the densities of gluons and quarks. A related source of correlations is QCD evolution, since a selection of a parton with a given $x, Q^2$ may lead to a local (in transverse plane) enhancement of the parton density at different $x$ values.

It was already recognized more than two decades ago [1] that the increase of parton densities at small $x$ leads to a strong increase in the probability of nucleon-nucleon collisions, where two or more partons of each projectile experience pairwise-independent hard interactions. Although the production of multijets through the double-parton scattering mechanism was investigated in several experiments [2,3] at $pp$, $p\bar{p}$ colliders, the interpretation of the data was hampered by the need to model both the longitudinal and the transverse partonic correlations at the same time. The aim of this Letter is to point out that the near future perspectives to study proton-nucleus collisions at Relativistic Heavy Ion Collider (RHIC), as well as the plans for $pA$ collisions at CERN Large Hadron Collider (LHC), provide a feasible opportunity to study separately the longitudinal and transverse partonic correlations in the nucleon, as well as to check the validity of the underlying picture of multiple collisions.

The simplest case of a multiparton process is the double-parton collision. Since the momentum scale $p_t$ of a hard interaction corresponds to much smaller transverse distances $\sim 1/p_t$ in coordinate space than the hadronic radius, in a double-parton collision the two interaction regions are well separated in the transverse space. Also in the c.m. frame, pairs of partons from the colliding hadrons are located in pancakes of thickness $\leq (1/x_1 + 1/x_2)/p_{\rm c.m.}$. Thus two hard collisions occur practically simultaneously as soon as $x_1, x_2$ are not too small and there is no cross talk between two hard collisions. A consequence is that the different parton processes add incoherently in the cross section. The double-parton scattering cross section, being proportional to the square of the elementary parton-parton cross section, is therefore characterized by a scale factor with a dimension of the inverse of a length squared. The dimensional quantity is provided by the nonperturbative input to the process, namely, by the multiparton distributions. In fact, because of the localization of the interactions in transverse space, the two pairs of colliding partons are aligned in such a way that the transverse distance between the interacting partons of the target hadron is practically the same as the transverse distance between the partons of the projectile. The double-parton distribution is therefore a function of two momentum fractions and of their transverse distance, and it can be written as $\Gamma(x, x', b)$. Actually $\Gamma$ also depends on the virtualities of the partons, $Q^2, Q'^2$, though to make the expressions more compact we will not write explicitly this $Q^2$ dependence. Hence the double-parton scattering cross section for the two "two → two" parton processes $\alpha$ and $\beta$ in an inelastic interaction between hadrons $a$ and $b$ can be written as

$$\sigma_D(\alpha, \beta) = \frac{m}{2} \int \Gamma_a(x_1, x_2; b) \hat{\sigma}_\alpha(x_1, x_1') \hat{\sigma}_\beta(x_2, x_2') \times \Gamma_b(x_1', x_2'; b)\, dx_1 dx_1' d\, x_2 d\, x_2' d^2 b\,, \quad (1)$$

where $m = 1$ for indistinguishable parton processes and $m = 2$ for distinguishable parton processes. Note that, though the factorization approximation of Eq. (1) is generally accepted in the analyses of the multijet processes and appears natural based on the geometry of the process, no formal proof exists in the literature. As we will show below, the study of the $A$ dependence of this process will allow one to perform a stringent test of this approximation.

In the case of $NN$ scattering, one cannot proceed further without making some simplifying assumptions about transverse correlations of partons in nucleons. Our key observation is that the introduction of a new large transverse scale, the nucleus radius, allows us to separate the effects of the transverse and longitudinal parton correlations. Essentially, we can express the function $\Gamma_A(x_1, x_2, b)$ through $\Gamma_N(x_1, x_2, b)$ and the distribution of nucleons in the nucleus, practically without any extra model assumption. Here, to simplify the discussion, we neglect small nonadditive effects in the parton densities, which is a reasonable approximation for $0.02 \leq x \leq 0.5$. In this case we have to take into account only $b$-space correlations of partons in individual nucleons. One has therefore two different contributions to the double-parton scattering cross section. The first one, $\sigma_1^D$, which is represented in Fig. 1a, is the same as for the nucleon target (the only difference being the enhancement of the parton flux), and the corresponding cross section is

$$\sigma_1^D = \sigma_D \int d^2B\, T(B) = A\sigma_D . \tag{2}$$

where $T(B)$ is the nuclear thickness, as a function of the impact parameter of the hadron-nucleus collision $B$.

The contribution to the term in $G_A(x_1', x_2', b)$ due to the partons originated from different nucleons of the target (Fig. 1b) can be calculated *solely* from the geometry of the problem by observing that the nuclear density does not change within a transverse scale $\langle b \rangle \ll R_A$.

The two simplest methods are to use the Abramovsky-Gribov-Kancheli cutting rules [4] or the technique in [5]. We can write, for two indistinguishable parton processes,

$$\sigma_2^D = \frac{1}{2} \int G_N(x_1, x_2) \hat{\sigma}(x_1, x_1') \hat{\sigma}(x_2, x_2') G_N(x_1') \times G_N(x_2')\, dx_1 dx_1' dx_2 dx_2' \int d^2B\, T^2(B), \tag{3}$$

where $G_N(x_1, x_2) = \int d^2b\, \Gamma_N(x_1, x_2; b)$, while $x_i$ are nucleon and $x_i'$ are nuclear parton fractions. Notice that a distinctive feature is that, differently from the case of $NN$ interactions, no transverse scale factor related to the nucleon scale is present in $\sigma_2^D$. The correct dimensionality is provided by the nuclear thickness function, which appears in $\sigma_2^D$ at the second power. The two contributions $\sigma_1^D$ and $\sigma_2^D$ are therefore characterized by a different dependence on the atomic mass number of the target. The

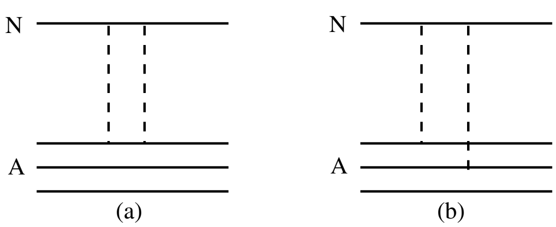


FIG. 1. Two contributions to the "four → four" process in $pA$ scattering. The dashed lines represent hard interactions.

$A$ dependence of the two terms is in general a function of the values of the momentum fractions and of the virtuality scale of the $2 \to 2$ interactions. The simplest situation is in the kinematical regime where shadowing corrections to the nuclear structure function can be neglected. $\sigma_1^D$ is then proportional to $A^1$ and $\sigma_2^D$ to $A^{1.5}$. (Note that the nuclear surface effects lead to a faster dependence of $\int T^2(B)\, d^2B$ on $A$, for $A \leq 240$, than the naive expectation $A^{4/3}$). The presence of two terms with distinctive $A$ dependence [and comparable magnitude for a wide range of $x$; see Eq. (6) below] will allow one to separate them with ease experimentally and also to check in the course of such an analysis the factorization approximation of Eq. (1).

To estimate the relative importance of $\sigma_1^D$ and $\sigma_2^D$, and only to that purpose, we used the CDF analysis [3], where all correlations in fractional momenta have been neglected, so that one can write $\Gamma_N$ in a factorized form as a product of two parton densities $G_N(x)$ and of a function of the interparton transverse distance $b$: $\Gamma_N(x, x', b) = G_N(x)G_N(x')F(b)$. (In a sense this could be considered as merely a convenient parametrization of the experimental data.) With these simplifications, one obtains

$$\sigma_D(\alpha, \beta) = \frac{m}{2} \frac{\sigma_\alpha \sigma_\beta}{\sigma_{\text{eff}}}, \tag{4}$$

where $\sigma_\alpha$ and $\sigma_\beta$ are the inclusive cross sections for the two processes $\alpha$ and $\beta$ in the hadron-hadron interactions. Under the factorization assumption the whole new information on the hadron structure can be reduced to a single quantity with dimensions of a cross section, $\sigma_{\text{eff}}$, which was measured by CDF to be

$$\sigma_{\text{eff}} = 14.5 \pm 1.7^{+1.7}_{-2.3} mb . \tag{5}$$

Within the accuracy and in the limited kinematic range accessible to the experiment (0.01–0.40 for the photon + jet scattering, 0.002–0.20 for the dijet scattering) no evidence was found of an $x$ dependence of $\sigma_{\text{eff}}$, supporting the simplest uncorrelated picture of the interaction. However, the absolute value of the cross section is significantly larger (by a factor $\geq 2$) than the cross section one would obtain within the factorization hypothesis, when assuming that the transverse distribution of partons reflects the matter distribution in the nucleon needed to obtain the value of the nucleon nonsingle-diffractive cross section measured by CDF (for extensive discussions, see [6]). So the CDF data actually appear to indicate the presence of parton-parton correlations in a nucleon, though one cannot distinguish whether they are solely due to transverse correlations or to a combination of longitudinal and transverse correlations.

Substituting Eq. (4) into Eq. (2) and the factorized form of $\Gamma_N$ in Eq. (3), we can estimate the relative importance of $\sigma_1^D$ and $\sigma_2^D$:

$$\frac{\sigma_2^D}{\sigma_1^D} = \frac{\int T^2(B)\, d^2B}{A} \sigma_{\text{eff}} \approx 0.45 \times \left(\frac{A}{10}\right)^{0.5}_{|A \geq 10|} . \tag{6}$$

Here we evaluated $\int T^2(B)\, d^2B$ by using the standard experimentally determined Fermi step parametrization of the nuclear matter densities [7]. One can see from Eq. (6) that for heavy nuclei, which are available at RHIC, the second term will constitute about 70% of the cross section and hence the study of the $A$ dependence of the four jet production will allow a straightforward separation of the two contributions to the cross section. (Note also that the cross section of the two partons $\rightarrow$ four jet process, which constitutes a background to the four $\rightarrow$ four processes, depends linearly on $A$, so that its contribution may be disentangled by studying the $A$ dependence of the cross section.) It is worth emphasizing that, if the small value of $\sigma_{\text{eff}}$ is due to the correlation of the longitudinal distributions, the relative contribution of the second term would be further enhanced. Considering that the enhancement of $\sigma_D$ in $p\bar{p}$ collisions is roughly by a factor 2 as compared with the naive expectation, one would expect in this case an additional enhancement of $\sigma_2^D$ by a factor $\sim\sqrt{2}$.

One could question whether the soft particle production background may create more serious problems in $pA$ scattering than in $NN$ scattering. It appears that this problem can be avoided by choosing $x_i > x_i'$, $x_i \geq 0.1$ and selecting a kinematics close to 90° in the c.m. of the partonic collisions. In this case the jets are produced predominantly in the proton fragmentation region, where the soft hadron multiplicity in $pA$ collisions is smaller than in $NN$ collisions.

In order to extend the analysis to the $x_i' \leq 0.01$ kinematics, one needs to take into account the shadowing effects in the nuclear parton densities. Here we restrict our discussion to the case of the leading twist parton shadowing, which is a pretty safe approximation for $p_t \geq 5-7$ GeV$/c$ and $x \geq 10^{-3}$, which is the minimal cut used on $p_t$ to be able to observe the jets. (For recent estimates of the kinematics where blackbody/unitarity effects may become important, see [8].) In the case where only one of the nuclear partons is in the shadowing region, the ratio of $\sigma_2^D/\sigma_1^D$ is modified only by the dependence of the shadowing on the nuclear impact factor, given the different selection of impact parameters in two terms $\sigma_1^D$ and $\sigma_2^D$ [the integration with measure $T^2(B)$ leads in fact to a somewhat smaller average $B$ as compared with the integration with measure $T(B)$]. We have performed a numerical estimate of this effect within the leading twist approximation of [9] and we found that even for $A = 240$ this effect leads to a decrease of $\sigma_2^D/\sigma_1^D$ by $\leq$10%. The effect could be in any case studied experimentally by investigating the single hard scattering as a function of $A$ in the same kinematics. When both $x_i'$s are in the shadowing region the evaluation of the effect is more model dependent, though it still appears to be rather small. In any case such kinematics is more appropriate for the study of the dynamics of the nuclear shadowing and hence is not directly related to the subject of this Letter. Note also that pushing such measurements into a kinematical region close to the blackbody limit would create additional problems since, due to the increase of the transverse momenta of the nuclear partons, the pairwise azimuthal correlation, which allows easy identification of the double-parton collision events, would become weaker and weaker. Note also that we argued in the beginning that processes which may violate factorization should be amplified when $x_i, x_i'$ become smaller. Hence the study of the $A$ dependence in this kinematics would provide an additional test of the factorization approximation.

Summarizing, the study of the $A$ dependence of the double-parton scattering will allow us to separate two contributions to the cross section—due to scattering off one and two nucleons of the nucleus. Because of the large nuclear size, $R_A \gg R_N$, the $\sigma_2^D$ term provides a model-independent measurement of the double-parton densities in nucleons—while no such model-independent measurement is possible with proton targets. At the same time the comparison of the two terms will allow a practically model-independent determination of the transverse separation between two partons [modulus a possible small effect due to a different transverse separation of $\Gamma(x_1, x_2, b)$ and $\Gamma(x_1', x_2', b)$] as well as checking the factorization approximation.

Obviously one can also consider three parton collisions. In contrast to the case of the double collisions it is more difficult for the available range on nuclei to extract the triple-parton distribution without making simplifying hypotheses. This is because the triple scattering process originates due to three different mechanisms, corresponding to the number of target nucleons involved. While the terms with one (Fig. 2a) and three (Fig. 2c) target nucleons are analogous to the contributions already considered for the double scattering, the contribution with two different target nucleons (Fig. 2b) is different. In the latter case, in fact, the integration on the transverse coordinates of the interacting partons involves at the same time two partons of the projectile and two partons of the target. The simplest possibility is that the longitudinal and transverse degrees of freedom can still be factorized; in this case the integration over the transverse partonic coordinates gives as a result a factor with dimensions of the inverse of a cross section. We call the new dimensional quantity $\sigma_{\text{eff}}'$ and a naive expectation would be that its value is not much different from $\sigma_{\text{eff}}$. The different contributions to the triple scattering cross section are therefore

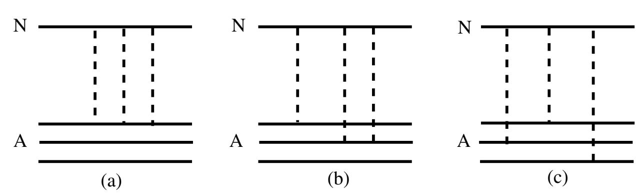


FIG. 2. Three contributions to the "six $\rightarrow$ six" process in $pA$ scattering. The dashed lines represent hard interactions.

$$
\begin{aligned}
\sigma_1^T &= \sigma_T \int d^2B\, T(B) = A\sigma_T\,, \\
\sigma_2^T &= \frac{1}{3!}\int G(x_1,x_2,x_3)\hat{\sigma}(x_1,x_1')\hat{\sigma}(x_2,x_2')\hat{\sigma}(x_3,x_3')\,dx_1dx_1'dx_2dx_2'dx_3dx_3' \\
&\quad\times [G(x_1',x_2')G(x_3') + G(x_2',x_3')G(x_1') + G(x_1',x_3')G(x_2')]\int d^2\,BT^2(B)\frac{1}{\sigma_{\rm eff}'}\,, \\
\sigma_3^T &= \frac{1}{3!}\int G(x_1,x_2,x_3)\hat{\sigma}(x_1,x_1')G(x_1')G(x_2')G(x_3')\hat{\sigma}(x_2,x_2')\hat{\sigma}(x_3,x_3')\,dx_1dx_1'dx_2dx_2'dx_3dx_3'\int d^2B\,T^3(B)\,,
\end{aligned}
\tag{7}
$$

where $\sigma_T$ is the triple-parton scattering cross section on a nucleon target. The second term provides additional information about correlations of partons in nucleons while the third term measures triple-parton density in nucleons. If we assume that the integral over transverse coordinates in dimension scale for $\sigma_1^T$, $\sigma_2^T$ are approximately $\sigma_{\rm eff}^{-2}$ and $\sigma_{\rm eff}^{-1}$ we can estimate that the relative importance of the three terms, for $A \geq 10$, is approximately

$$\sigma_1^T{:}\sigma_2^T{:}\sigma_3^T = 1{:}1.45(A/10)^{0.5}{:}0.25(A/10)\,. \tag{8}$$

This estimate indicates that the $A$ dependence of $\sigma^T$ is much stronger than for $\sigma^D$, with the scattering off several nucleons already becoming important for light nuclei. The $\sigma_3^T$ term is likely to become comparable to the other terms for heavy nuclei, so, in principle, an accurate study of the $A$ dependence would allow us to measure all three terms separately and hence determine the triple-parton density in a nucleon in a model-independent way. Obviously, one would need LHC energies and a large acceptance FELIX-type [10] detector to be able to study such reactions.

One can go a step further and try to get information about global characteristics of the nucleon as a function of the values of the flavor, $x$'s, etc., of the probed partons. Actually, the number of the secondaries produced gives an indication of the actual transverse size of the projectile, so that one would expect to observe a relatively smaller population of sea quarks and gluons in events with few secondaries, while, in that case, the momentum carried by the valence should be larger than average. Hence one might start by studying the correlation between soft characteristics of the events (the simplest for RHIC would be the number of neutrons in the zero angle calorimeter) and the momentum fraction $x$ of the projectile parton in a single hard collision [11]. A next step would be to compare the single and double hard scattering events for fixed $x_1$, say $x_1 \sim 0.2$, while increasing the value of $x_2$. If, when selecting two fast partons in a nucleon, one selects configurations with a small size, one would expect, for example, that the number of knockout nucleons would decrease when $x_2$ increases. Overall, such studies would allow one to obtain unique information about the three-dimensional structure of the nucleon.

We thank L. Frankfurt for useful discussions. We thank the Department of Energy's Institute for Nuclear Theory at the University of Washington for hospitality and the Department of Energy for partial support during the completion of this work. This work was partially supported by the Italian Ministry of University and of Scientific and Technological Researches (MURST) by the Grant No. COFIN99.

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[1] C. Goebel, F. Halzen, and D. M. Scott, Phys. Rev. D **22**, 2789 (1980); N. Paver and D. Treleani, Nuovo Cimento A **70**, 215 (1982); B. Humpert, Phys. Lett. **131B** 461 (1983); B. Humpert and R. Odorico, Phys. Lett. **154B**, 211 (1985); T. Sjostrand and M. Van Zijl, Phys. Rev. D **36**, 2019 (1987).
[2] AFS Collaboration, T. Akesson *et al.,* Z. Phys. C **34**, 163 (1987); UA2 Collaboration, J. Alitti *et al.,* Phys. Lett. B **268**, 145 (1991); CDF Collaboration, F. Abe *et al.,* Phys. Rev. D **47**, 4857 (1993).
[3] CDF Collaboration, F. Abe *et al.,* Phys. Rev. Lett. **79**, 584 (1997); Phys. Rev. D **56**, 3811 (1997).
[4] V. A. Abramovsky, V. N. Gribov, and O. V. Kancheli, Sov. J. Nucl. Phys. **18**, 308 (1973).
[5] L. Bertocchi and D. Treleani, J. Phys. G **3**, 147 (1977).
[6] G. Calucci and D. Treleani, Phys. Rev. D **60**, 054023 (1999).
[7] G. D. Alkhazov, S. L. Belostotsky, and A. A. Vorobev, Phys. Rep. **42**, 89 (1978).
[8] L. Frankfurt, V. Guzey, M. McDermott, M. Strikman, L. Frankfurt, V. Guzey, M. McDermott, and N. Strikman, hep-ph/0104252; L. Frankfurt, V. Guzey, and M. Strikman, J. Phys. G **27**, R23 (2001).
[9] L. Frankfurt and M. Strikman, Eur. Phys. J. A **5**, 293 (1999).
[10] FELIX Collaboration, E. Lippmaa *et al.*, Report No. CERN-LHCC-97-45.
[11] L. Frankfurt and M. Strikman, Nucl. Phys. **B250**, 143 (1985).