Hard Scattering and Gauge/String Duality

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We consider high-energy fixed-angle scattering of glueballs in confining gauge theories that have supergravity duals. Although the effective description is in terms of the scattering of strings, we find that the amplitudes are hard (power law). This is a consequence of the warped geometry of the dual theory, which has the effect that in an inertial frame the string process is never in the soft regime. At small angle we find hard and Regge behaviors in different kinematic regions.

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The idea that large-N QCD can be recast as a string theory has been a tantalizing goal since the original proposal of 't Hooft [1]. At low energy, strings give a natural representation of confinement, but the high energy behavior has always presented a fundamental challenge: gauge theory amplitudes are hard, while string theory amplitudes are soft. Thus, the ordinary critical string theory must be modified.

This subject has taken an interesting turn with Maldacena duality [2,3]. The original duality was for conformal theories, but various perturbations produce gauge/string duals with a mass gap, confinement, and chiral symmetry breaking [4–7]. While these theories have QCD-like behavior at low energy, they also differ from QCD at high energy. They are not asymptotically free; rather, the 't Hooft coupling must remain large at all energies in order to obtain a useful string dual. Still, as QCD is itself a nearly conformal field theory at high energies, many of their qualitative features should be similar.

In this paper we will address the following puzzle: in these theories, hadronic amplitudes are well described at large 't Hooft parameter as the scattering of strings, for which the high energy behavior is soft. How does the dual string theory generate the hard behavior of the gauge theory?

Let us first explain the amplitudes to be considered. Conformal field theories, the subject of the original Maldacena duality, do not have an S matrix. (There has been some discussion of the *ten-dimensional* S matrix of the dual string theory, and its representation in terms of gauge theory correlators [8].) However, once conformal symmetry is broken and a mass gap produced, the theory has an ordinary four-dimensional S matrix. We will then study the $2 \rightarrow m$ scattering of closed strings, corresponding to exclusive glueball scattering, at large energy $\sqrt{s}$ and fixed angles. There is a simple dimensional prediction for exclusive amplitudes of low-lying hadrons in QCD [9,10] and other asymptotically free confining theories: they scale as

$$s^{2-(1/2)n}, \qquad n = \sum_{i=1}^{m+2} n_i\,, \tag{1}$$

where the sum runs over all initial and final hadrons, and n_i is the minimum number of hard constituents in the ith hadron. Importantly, n_i is also the *twist* τ_i (dimension minus spin) of the lowest-twist operator that can create the ith hadron: the interpolating fermion and gauge field strength operators (and scalars, if present) each have minimum twist one. (The covariant derivative, with twist zero, creates a minimum of zero partons.) We will recover this result in theories with dual string descriptions and no identifiable partons.

For a *conformal* gauge theory, the dual string spacetime is the product of AdS_5 with a transverse five-dimensional space X:

$$ds^2 = \frac{r^2}{R^2}\,\eta_{\mu\nu}\,dx^\mu\,dx^\nu + \frac{R^2}{r^2}\,dr^2 + R^2\,ds_X^2\,. \tag{2}$$

The anti–de Sitter (AdS) radius R is related to the string coupling, number of colors, and string scale by $R^4 \sim gN\alpha'^2$. The key feature here is the warping, the fact that the normalization of the four-dimensional metric is a function of the radius r. The conserved momentum, corresponding to invariance under translation of x^μ, is $p_\mu = -i\partial/\partial x^\mu$. On the other hand, the momentum in local inertial coordinates, for a state roughly localized in the transverse coordinate r, is

$$\tilde{p}_\mu = \frac{R}{r}\,p_\mu\,. \tag{3}$$

For states with a characteristic ten-dimensional scale $\tilde{p} \sim R^{-1}$, the four-dimensional energy is

$$p \sim \frac{r}{R^2}\,. \tag{4}$$

Thus the gauge theory physics is encoded in a holographic way, with low-energy states at small r and high energy at large r [2,3,11]. This gravitational redshift is, for example, the essence of the Randall-Sundrum proposal for the origin of the weak/Planck hierarchy [12]: a single ten-dimensional scale $\tilde{p}$ can give rise to many four-dimensional scales, at different values of r.

For duals to nonconformal gauge theories the spacetime is of the approximate form (2) at large r, but differs at small r. (In cascading gauge theories [6], the geometry evolves logarithmically even at large r; we will ignore this correction in the present paper.) In particular, if there is a mass gap then the gravitational redshift has a nonzero lower bound, unlike the conformal case where it vanishes at $r = 0$. Define the scale Λ to be set by the mass of the lightest glueball state; on the string side the holographic relation (4) determines that the lower cutoff on the geometry is of the order of

$$r_{\min} \sim \Lambda R^2. \tag{5}$$

The string tension $\hat{\alpha}'$ in the gauge theory is not Λ^{-2} but $(gN)^{-1/2}\Lambda^{-2}$. Note that

$$\sqrt{\alpha'}\,\tilde{p} = \sqrt{\hat{\alpha}'}\,p\left(\frac{r_{\min}}{r}\right) \le \sqrt{\hat{\alpha}'}\,p\,,$$

so at $r \sim r_{\min}$ the momenta in units of the string tension are the *same* in the string theory and the confining gauge theory, with the string theory's momenta decreasing as r increases. Thus high-energy scattering in the gauge theory may in principle involve high-, medium-, or low-energy scattering in the string theory.

Glueballs correspond to closed string states; to be specific let us initially consider the dilaton Φ. This will start in some state

$$\Phi = e^{ip\cdot x}\psi(r,\Omega)\,, \tag{6}$$

where Ω are coordinates on the transverse space. The scale of variation of the factor ψ is set by the AdS radius R. The scale of variation of the exponential depends on r as in Eq. (3). We will see that the dominant r is such that the exponential varies on the string scale $\sqrt{\alpha'}$. In the regime of large 't Hooft parameter where this duality is useful, $R \gg \sqrt{\alpha'}$ and so the variation of Φ in the transverse directions can be treated as slow. The amplitude can then be treated as an essentially ten-dimensional scattering taking place at a point in the transverse dimensions, integrated coherently over this transverse position:

$$\mathcal{A}(p) = \int dr\, d^5\Omega \sqrt{-g}\, \mathcal{A}_{\text{string}}(\tilde{p}) \prod_{i=1}^{m+2} \psi_i(r,\Omega)\,. \tag{7}$$

The essential feature here is that the local scattering amplitude $\mathcal{A}_{\text{string}}$ depends on the momenta $\tilde{p}$ seen by a local inertial observer, and not the fixed global momenta p. The amplitude is dominated by the radius

$$r_{\text{scatt}} \sim R\sqrt{\alpha'}\,p \sim (\sqrt{\hat{\alpha}'}\,p)r_{\min}\,, \tag{8}$$

where the local momenta are of order $1/\sqrt{\alpha'}$. At smaller radii the inertial momenta are large compared to the string scale, and so the integral is damped by the softness of high-energy string scattering. At larger radii the integral is damped by the wave functions $\psi_i(r)$.

At large r the wave function factorizes,

$$\psi(r,\Omega) = Cf(r/r_{\min})g(\Omega)\,, \tag{9}$$

where g is a normalized harmonic in the angular directions. In the AdS case such a factorized form holds at all r, while in the nonconformal case the small-r (IR) dynamics will in general induce mixing between different harmonics. In Eq. (9) we have written only the dominant large-r term, which scales as

$$f \to (r/r_{\min})^{-\Delta}, \tag{10}$$

where Δ is the conformal dimension of the lowest-dimension operator that creates this state [3]. The integral (A4) becomes $C^2R^4 \int dr\, rf^2(r/r_{\min}) = 1$; this is dominated by $r \sim r_{\min}$ and so $C \propto 1/R^2 r_{\min}$. Finally, the amplitude for $m + 2$ scalars in 10 dimensions is dimensionally

$$\mathcal{A}_{\text{string}}(\tilde{p}) = g^m \alpha'^{2m-1} F(\tilde{p}\sqrt{\alpha'})\,. \tag{11}$$

We will not need the detailed form of the dimensionless function $F(\tilde{p}\sqrt{\alpha'})$, but for completeness we note that for $m = 2$ it has the familiar Virasoro-Shapiro form

$$F(\tilde{p}\sqrt{\alpha'}) = \left[\prod_{x=s,t,u} \frac{\Gamma(-\alpha'\tilde{x}/4)}{\Gamma(1 + \alpha'\tilde{x}/4)}\right] K(\tilde{p}\sqrt{\alpha'})\,, \tag{12}$$

with K being a kinematic factor of order $\tilde{p}^8$ [13].

Assembling all factors, we have in the large-r region

$$\mathcal{A}(p) \sim \frac{g^m \alpha'^{2m-1}}{R^{2m+2} r_{\min}^{m+2}} \int dr\, r^3 \left(\frac{r_{\min}}{r}\right)^{\Delta} F(\tilde{p}\sqrt{\alpha'})\,, \tag{13}$$

where $\Delta = \sum_{i=1}^{m+2} \Delta_i$. This is dominated by $r \sim r_{\text{scatt}}$, where the argument of F is of the order of 1, and so

$$\mathcal{A}(p) \sim \frac{(gN)^{(1/4)(\Delta-2)}}{N^m \Lambda^{m-2}} \left(\frac{\Lambda}{p}\right)^{\Delta-4}. \tag{14}$$

This is our main result. The scaling of this amplitude with energy is precisely as in QCD, Eq. (1), with the identification $n_i = \Delta_i$. We have focused on the dilaton, a scalar in both ten dimensions and four dimensions, but the result holds for all four-dimensional scalars independent of the ten-dimensional spin: it depends only on the scaling of the wave function in tangent space, which depends only on the conformal dimension. For states with four-dimensional spin σ, the boost of the wave function contributes an extra factor of p^{σ}. Therefore in the energy dependence the dimension Δ_i is replaced by the *twist* $\tau_i = \Delta_i - \sigma_i$, as in QCD.

The coupling dependence in QCD is [9]

$$\mathcal{A}(p) \sim \frac{(gN)^{(1/2)(n-2)}}{N^m \Lambda^{m-2}} \left(\frac{\Lambda}{p}\right)^{n-4}, \tag{15}$$

where we have substituted $g_{\text{YM}}^2 \sim g$. Comparing with the string result (14), the agreement of the N dependence is standard [1] and the agreement of the Λ dependence is dimensional, but curiously the dependence on the 't Hooft parameter can be obtained by simply replacing

$gN \to (gN)^{1/2}$. Similar effects have been seen in other contexts, e.g., the strength of the Coulomb force [16].

The energy dependence (1), (14) is initially surprising from the string point of view, but it can be derived from conformal invariance, with one additional assumption. Suppose the scattering takes place in a single hard process; then we can replace this process with an effective Lagrangian. Terms relevant for the process at hand will be a product of $m + 2$ gauge-invariant operators,

$$O(p^{4-\Delta_a}) \prod_{i=1}^{m+2} \mathcal{O}_{a_i}^{(i)}, \qquad \Delta_a = \sum_{i=1}^{m+2} \Delta_{a_i}. \qquad (16)$$

Here $(a_1, \ldots, a_{m+2})$ index all sets of operators. The matrix element of $\mathcal{O}_{a_i}^{(i)}$ to create the ith external state is proportional to $p^{\sigma_{a_i}}$, so this operator contributes an energy dependence of the order of

$$O(p^{4-\tau_a}), \qquad \tau_a = \sum_{i=1}^{m+2} \tau_{a_i}, \qquad \tau_{a_i} = \Delta_{a_i} - \sigma_{a_i}, \qquad (17)$$

where σ_{a_i} is the spin of $\mathcal{O}_{a_i}$. Thus the operator with lowest twist dominates the high-energy behavior. This operator also dominates the wave function at large r, hence reproducing the string result. Note also that the dictionary from gravity to gauge theory identifies the wave function $\psi(r)$ with that amplitude for the corresponding conformal operator to create the given state [14].

The assumption of a single hard process is nontrivial. It is possible in the string theory because of the warped geometry, wherein a string at large r has a characteristic four-dimensional size proportional to r^{-1}. In QCD, processes with independent hard scattering at separate spacetime points (so-called pinch singularities) are formally dominant at large s, but are believed to be suppressed by Sudakov (color bremsstrahlung) effects [10], which make a purely exclusive process unlikely. These processes, and other hard partonic effects, are not easy to see in the dual string theory; they may be absent at large 't Hooft coupling.

In $\mathcal{N} = 1^*$ [5], one can interpolate between large and small gN. The dilaton with transverse angular momentum L corresponds to the operator $\mathrm{Tr}(F_{\mu\nu}^2 \phi^L)$, which creates $n = L + 2$ partons at small gN but has twist $\tau = \Delta = L + 4$ [3,15] in seeming disagreement with $n = \tau$. The point is that this state generically mixes with that created by the spin-two components of $\mathrm{Tr}(F_{\mu\nu}F_{\sigma\rho}\phi^L)$, corresponding to fluctuations of the four-dimensional metric, for which $n = \tau = L + 2$. Incidentally, for $L \geq 2$, these states also mix with those created by $\mathrm{Tr}(\phi^L)$, which corresponds to fluctuations of the transverse metric; this has twist $n = \tau = L$ [3,15], and generically dominates at high energy (though in some cases selection rules may suppress these mixings).

Excited hadrons can be created by nonchiral operators, which have dimension/twist of order $(gN)^{1/4}$. However, these massive string states will generally mix with light states through the coupling to the background curvature; although suppressed at large gN, this allows a wavefunction component of low twist to rapidly dominate the scattering even if its normalization is subleading in gN.

Finally, let us analyze the $2 \to 2$ process in more detail when $0 < -t \ll s$ [17]. For this we do need the detailed form of the amplitude (12) in the region $\tilde{s} \gg |\tilde{t}|, \alpha'^{-1}$:

$$F(\tilde{p}\sqrt{\alpha'}) = F(\alpha'\tilde{s}, \alpha'\tilde{t}) \approx (\alpha'\tilde{s})^{2+(1/2)\alpha'\tilde{t}} \frac{\Gamma(-\frac{1}{4}\alpha'\tilde{t})}{\Gamma(1 + \frac{1}{4}\alpha'\tilde{t})}. \qquad (18)$$

We have used the mass-shell relation $s + t + u = 0$; Kaluza-Klein effects give mass to the glueball and so shift the Regge intercept, but only at order $(gN)^{-1/2}$. Inserting this form into the amplitude (13), and noting $\tilde{s}/\tilde{t} = s/t$, we may usefully rewrite it as an integral over $\nu = \alpha'|\tilde{t}|$.

$$\frac{\sqrt{gN}}{N^2(\hat{\alpha}'|t|)^{(1/2)\Delta-2}} \int_0^{\hat{\alpha}'|t|} d\nu\, \nu^{(1/2)\Delta-3} F\left(\nu \frac{s}{|t|}, \nu\right). \qquad (19)$$

The integral, a function of Δ, $s/|t|$, and $\hat{\alpha}'t$, can only depend on $\hat{\alpha}'$ if the integrand is large near its upper end point, where $r \sim r_{\min}$. The dominant ν dependence comes from terms

$$d\nu\, \nu^{\Delta/2-2}(s/|t|)^{-(1/2)\nu}, \qquad (20)$$

and so the dominant value of ν is

$$\nu_0 = \alpha'|\tilde{t}_0| \sim \min\left[\frac{\Delta - 4}{\ln(s/|t|)}, \hat{\alpha}'|t|\right] \qquad (21)$$

(if $\hat{\alpha}'|t| \ll 1$ then replace $s/|t|$ with $\hat{\alpha}'s$). Thus if $\hat{\alpha}'|t| < (\Delta - 4)/\ln(s/|t|)$ we find Regge behavior in the gauge theory amplitude

$$\mathcal{A}(p) \sim (\hat{\alpha}'s)^{2+(1/2)\hat{\alpha}'t} \qquad (22)$$

with a negative shift in the Regge intercept of order $1/\sqrt{gN}$. However, when $\hat{\alpha}'|t| > (\Delta - 4)/\ln(s/|t|)$,

$$\mathcal{A}(p) \sim s^2|t|^{-\Delta/2}[\ln(s/|t|)]^{1-(1/2)\Delta}, \qquad (23)$$

where terms of order $1/\ln s$ in the exponents have been dropped, as well as energy-independent prefactors. Thus, for any t, when s is sufficiently large one finds Regge behavior is lost, and instead one finds an inverse power of the small angle.

These last results require some caution. The fact that Regge behavior is seen mainly for $\hat{\alpha}'t \sim 1$ is consistent with QCD data [18,19]. But the transition from (22) to (23), which involves $\alpha'\tilde{s} \gg \alpha'|\tilde{t}| \sim 1$, is complicated by the fact that multiloop amplitudes can be important [20]. For moderate values of N, one may expect a region of multi-Pomeron exchange—and indeed there is some evidence that QCD exhibits this physics [19]. Furthermore, there is a Froissart-Martin unitarity bound on the total cross section (unlike ordinary string theories, whose massless particles give an infinite total cross section) which is violated by the tree-level result at rather low energies. Neither here nor in QCD [19] is it understood how this bound is satisfied. We believe these and similar issues are worthy of future study.

In conclusion, we have found that the high-energy behavior in confining gauge/gravity duals is remarkably QCD-like. Since the scattering takes place when the momentum invariants are of order the string scale, this effect involves physics beyond the supergravity approximation. The fifth dimension, whose importance has repeatedly been emphasized by Polyakov [21], plays a pivotal role, as does the softness of high-energy string theory. We believe that our results provide some clue as to the connection with perturbative field theory.

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Appendix: QFT in curved spacetime.—Consider a scalar field of mass m^2 in a $(4 + k)$-dimensional spacetime

$$ds^2 = e^{2A(y)}\eta_{\mu\nu}\,dx^\mu\,dx^\nu + g_{\perp mn}(y)\,dy^m\,dy^n. \quad \text{(A1)}$$

The canonical commutator is

$$[\Phi(\mathbf{x}, y), \dot{\Phi}(\mathbf{x}', y')] = i\,\frac{e^{2A}}{\sqrt{-g}}\,\delta^3(\mathbf{x} - \mathbf{x}')\delta^k(y - y') = \frac{i}{e^{2A}\sqrt{g_\perp}}\,\delta^3(\mathbf{x} - \mathbf{x}')\delta^k(y - y'). \quad \text{(A2)}$$

Expand

$$\Phi(x, y) = \sum_\alpha \int \frac{d^3\mathbf{k}}{(2\pi)^3}\,\frac{1}{2k_0}\,[a_\alpha(\mathbf{k})e^{ik\cdot x}\psi_\alpha(y) + \text{H.c.}]. \quad \text{(A3)}$$

The $\psi_\alpha(y)$ are eigenmodes of $-e^{-2A}(\nabla_\perp^2 + m^2)$, whose eigenvalue μ_α^2 is the four-dimensional mass squared, $-k^2 = \mu_\alpha^2$. The oscillators are covariantly normalized, $[a_\alpha(\mathbf{k}), a_\beta^\dagger(\mathbf{k}')] = (2\pi)^3\delta(\mathbf{k} - \mathbf{k}')\delta_{\alpha\beta}$. It follows that the modes are normalized

$$\int d^k y\, e^{2A}\sqrt{g_\perp}\,\psi_\alpha(y)\psi_\beta^*(y) = \delta_{\alpha\beta}. \quad \text{(A4)}$$

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