Competition of Periodic and Homogeneous Modes in Extended Dynamical Systems

B. Dressel,[1] A. Joets,[2] L. Pastur,[3] W. Pesch,[1] E. Plaut,[4] and R. Ribotta[2]
[1]*Physikalisches Institut der Universität Bayreuth, D-95440 Bayreuth, Germany*
[2]*Laboratoire de Physique des Solides, Université Paris-Sud-CNRS, F-91405 Orsay cedex, France*
[3]*Istituto Nazionale di Ottica Applicata, Largo Fermi, 6, I-50125 Firenze, Italy*
[4]*LEMTA, INPL-UHP-CNRS, 2 avenue de la Forêt de Haye, F-54504 Vandœuvre cedex, France*
(Received 12 February 2001; revised manuscript received 24 October 2001; published 27 December 2001)

Despite their simple structure, spatially homogeneous modes can participate directly in pattern-formation processes. This is demonstrated by new experimental and theoretical results for thermo- and electroconvection in planar nematic liquid crystals, where two distinct homogeneous modes, twist and splay distortions of the director field, emerge. Their nonlinear excitation is due to certain spontaneous symmetry-breaking bifurcations.

DOI: 10.1103/PhysRevLett.88.024503 PACS numbers: 47.54.+r, 47.20.Ky, 47.20.Lz, 82.70.Dd

The transition to spatiotemporal complexity in driven systems is typically traced back to the spatial and temporal periodicity of the linear exponentially growing modes, their nonlinear couplings and saturation [1]. Recently also the importance of *spatially homogeneous* modes with zero wave vector for the pattern selection in chemical systems has been emphasized [2]. Another example is homogeneous shear flow in inclined layer convection [3]. It is understandable that homogeneous modes have attracted less attention so far. They are more difficult to identify directly by standard optical methods. Moreover, they are often associated with spontaneous symmetry-breaking bifurcations in the nonlinear regime, which can be easily overlooked in the theoretical analysis as well.

In order to study the possible scenarios associated with multiple homogeneous modes, thermally or electrically driven convection in nematic liquid crystals (nematics) provides interesting model systems [4,5]. The mean orientation of the elongated molecules of nematics is described by their director field $\hat{\mathbf{n}}$. The coupling of director distortions to the other fields (e.g., velocity, temperature, charge density) allows for new efficient convection mechanisms typical for intrinsically anisotropic fluids [4–6]. Another advantage of nematics is that $\hat{\mathbf{n}}$ can be monitored optically and oriented through electromagnetic fields. Such effects are, for instance, exploited in liquid crystal displays.

Since the first systematic investigations of convection in nematics in the 1970s, the use of a homogeneous magnetic field as a secondary control parameter has contributed considerably to the analysis of the underlying mechanisms [6–9]. In the case of planar nematic convection where $\hat{\mathbf{n}} = \hat{\mathbf{x}}$ at the cell boundaries, a planar magnetic field $\mathbf{H} = H\hat{\mathbf{x}}$ tends to hinder director rotations off the $\hat{\mathbf{x}}$ direction on the *linear* level and the convection threshold does increase rapidly with H. Our new results yet prove that *two* homogeneous modes, twist and splay (Fig. 1), surprisingly with a director component perpendicular to $\mathbf{H}$, control the secondary bifurcation sequences in the *nonlinear* regime above onset. Before only the importance of a homogeneous twist has been realized [10,11], which is, however, more difficult to visualize in the experiments than the new splay mode in Figs. 2 and 3. The theoretical analysis of the complicated interplay between the twist and splay modes is condensed into Fig. 4.

We focus mainly on nematic thermoconvection which is obtained when the temperature difference ΔT across a nematic layer between two horizontal plates at $z = \pm d/2$ exceeds a critical value ΔT_c [5,12,13]. In addition, we present preliminary results in nematic electroconvection where an ac voltage V of frequency f drives the convection instabilities [5,14,15]. In both systems normal *rolls,* characterized by a director field of the form

$$n_y = 0, \qquad n_z = A\cos(\mathbf{q}\cdot\mathbf{r})\cos(\pi z/d) \qquad (1)$$

to leading order, with a wave vector $\mathbf{q} = q_c\hat{\mathbf{x}}$ parallel to the anchoring direction, are preferred at onset. As usual, the magnetic field is scaled in units of a characteristic Fréedericksz field H_F [7,12]. In the regime $h = H/H_F \lesssim 3$, when $\epsilon = \Delta T/\Delta T_c(h) - 1$ (or $\epsilon = V^2/V_c^2 - 1$ in electroconvection, respectively) is slowly increased, the normal rolls bifurcate at $\epsilon = \epsilon_{ZZ}$ to oblique rolls, where $\mathbf{q}\cdot\hat{\mathbf{y}}$ becomes nonvanishing. At higher $\epsilon = \epsilon_{BV}$, a secondary oblique-roll mode of wave vector $\mathbf{k}$ becomes excited, leading to the bimodal varicose structure [13,15]. This sequence has been interpreted by an extended weakly

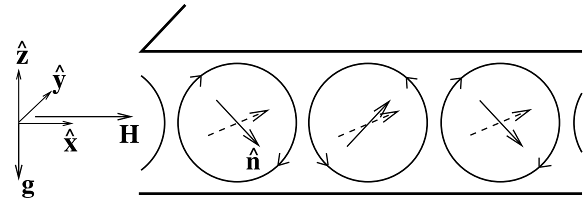

FIG. 1. Schematic convection cell with the periodic director field $\hat{\mathbf{n}}$ of a normal roll mode (1) (solid arrows). To avoid viscous torques created by the velocity field (circular stream lines), the director has a tendency to align along $\hat{\mathbf{y}}$, thus creating a homogeneous twist (2). The dashed arrows show the new homogeneous splay mode (3) superimposed on $\hat{\mathbf{n}}$. The director rotation towards $\hat{\mathbf{z}}$ presents a different option to avoid viscous torques in the roll-edges regions.

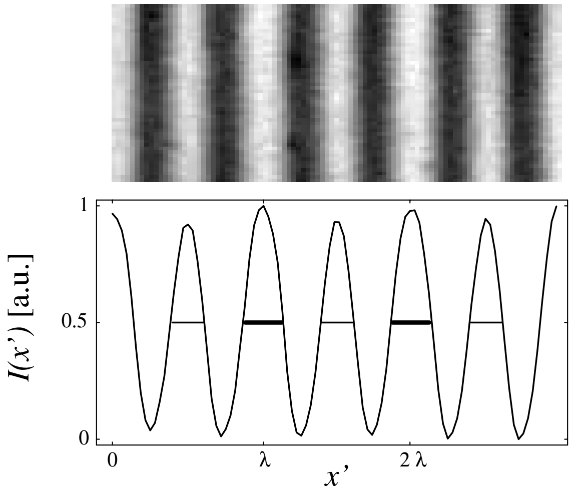

FIG. 2. Top: *Ordinary light* image of splay rolls in thermoconvection of the nematic 5CB in a cell of thickness 1.52 mm at $h = 3$, $\epsilon = 0.14$. Bottom: corresponding intensity line profiles $I(x')$ along the wave-vector direction (coordinate x'). The periodic roll-diameter modulation (marked by bars) cannot be explained by the theoretical expression for standard rolls deduced from (1), $I(x') \propto \overline{n_z^2}(x') = A^2 \cos^2 qx'$. The more general form $\overline{n_z^2}(x') = (A \cos qx' + \psi)^2$, corresponding to the superposition of rolls (1) and splay (3), leads instead to an accurate reproduction of the experimental profiles.

nonlinear analysis [14], which shows that the x-y homogeneous *twist mode*

$$n_y = \varphi \cos(\pi z/d), \qquad n_z = 0, \tag{2}$$

excited by nonlinear effects for $\epsilon > \epsilon_{ZZ}$, drives the zigzag and bimodal bifurcations.

Our experiments at larger h, however, reveal new patterns, with no change of periodicity of the rolls, but instead with an increasing asymmetry of the roll diameters. In ordinary light, where the intensity variations are proportional to the vertical average $\overline{n_z^2}$ of n_z^2 across the layer [16], every second roll becomes larger and brighter. From Fig. 2 it can be deduced that a homogeneous *splay mode*

$$n_y = 0, \qquad n_z = \psi \cos(\pi z/d), \tag{3}$$

is superimposed onto the periodic n_z variations (1), which leads to the notion of *splay rolls.* The excitation of homogeneous splay at large h is further confirmed by the observation of a new type of bimodal varicose structure (Fig. 3a). In contrast to the classical bimodal structure [13], the varicose pinchings are more pronounced at every second roll due to the enhanced splay. A quite accurate reconstruction of such a pattern is indeed obtained by using a combination $n_z = [A \cos(\mathbf{q} \cdot \mathbf{r}) + B \cos(\mathbf{k} \cdot \mathbf{r}) + \psi] \cos(\pi z/d)$ of the two corresponding roll modes (1) plus the splay mode (3). This new pattern is generic in nematic convection since it also appears in electroconvection in the presence of a planar magnetic field (Fig. 3b).

The various higher-order bifurcations and in particular a *subcritical* regime for $h > h_t \simeq 4$ [12] cannot be captured by weakly nonlinear methods. Thus a fully nonlinear analysis of the nematohydrodynamic equations based on Galerkin methods [17] has been developed. Our results for thermoconvection are summarized in the stability diagram Fig. 4. The diagram for electroconvection looks similar according to some first calculations. At $\epsilon = \epsilon_T$, we do find a bifurcation to *twist normal rolls,* which have been called "abnormal rolls" in electroconvection [10]. At larger h, above the codimension-2 point C_1, the bifurcation to *splay normal rolls* at ϵ_S is found. Between the codimension-2 points C_1 and C_4 splay rolls bifurcate to *splay-twist normal rolls* at ϵ_{ST} until the twist suppresses the splay at

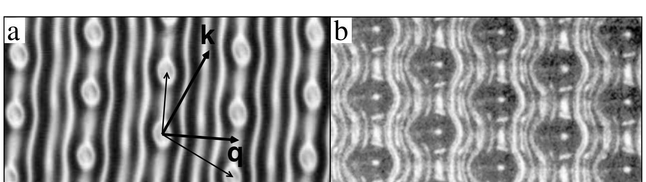

FIG. 3. (a): Picture of a splay-bimodal pattern in thermoconvection of the nematic MBBA in a cell of thickness 1.3 mm at $h = 3.3$, $\epsilon = 0.22$. With the use of *extraordinary light* the spatial variations of n_z are mapped into edge and center-lines caustics [16]. The thin (thick) arrows on the pattern indicate the direct (reciprocal) lattice base vectors. (b): Splay-bimodal in electroconvection of the nematic phase 5 in a cell of thickness 50 μm at $h = 5.1$, $f = 80$ Hz, $\epsilon = 0.58$. The double arrows at the bottom indicate on both pictures the modulation of the roll diameters.

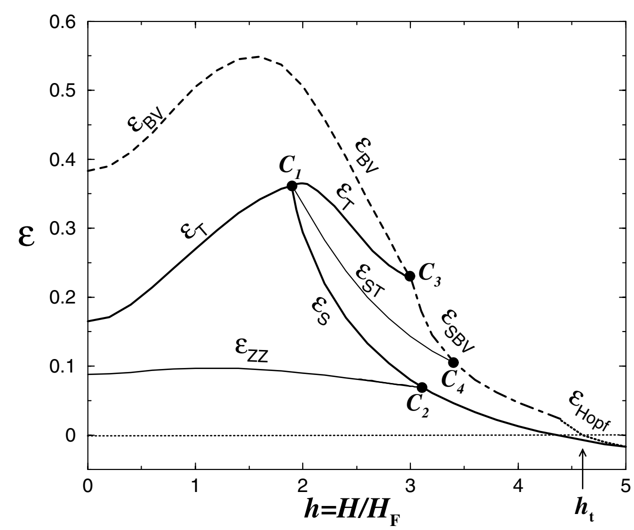

FIG. 4. Theoretical stability diagram for the thermoconvection of the nematic 5CB and rolls with a critical wave vector $\mathbf{q} = q_c(h)\hat{\mathbf{x}}$. The bifurcation lines above onset ($\epsilon = 0$) indicate zigzag (ZZ), twist (T), splay (S), and bimodal varicose (BV) bifurcations, together with their possible combinations, and a Hopf bifurcation close to the tricritical point h_t (see text). The codimension-2 points C_i where several bifurcation lines meet are marked.

ϵ_T between C_1 and C_3. At large ϵ bimodal instabilities are predicted, either leading to classical bimodal varicose at ϵ_{BV} or to splay bimodal varicose at ϵ_{SBV}. Note that between C_3 and C_4, the splay bimodal contains four active modes: the homogeneous splay mode and twist mode and the two periodic roll modes. At large h, we find stable subcritical splay roll solutions even slightly below the tricritical point h_t. The line ϵ_{SBV} merges with h_t, but the bifurcation becomes oscillatory.

Some basic features of the new splay rolls can be revealed by a generic description of the coupled periodic and homogeneous modes. Following the general scheme in [14] the resulting coupled-amplitude equations for the roll (A) and splay (ψ) amplitudes,

$$\tau \partial_t A = \epsilon A - g A^3 - \beta A \psi^2, \qquad \partial_t \psi = -\sigma_S \psi + \Gamma_S A^2 \psi, \tag{4}$$

are systematically derived from the nematohydrodynamic equations. The linear damping factor $\sigma_S = k_{11}/\gamma_1(\pi/d)^2(1 + h^2)$ does increase with h (k_{11} is the splay elastic constant, γ_1 a characteristic viscosity of the nematic), but can be compensated by the positive term $\propto \Gamma_S A^2$, leading to a continuous splay bifurcation at $\epsilon_S = g\sigma_S/\Gamma_S$, slightly lower than the Galerkin value. The positive sign of the nonlinear coefficient Γ_S is imposed by a combination of magnetic and viscous effects. The first one, which is associated with a term $+h^2 n_z^3$ in the n_z equation, originates from the magnetic quadrupolar nature of nematics, which aligned along $\hat{\mathbf{z}}$ would feel no magnetic torque. The second one is active for arbitrary h in thermally and electrically driven convection. It describes the general tendency of the director to avoid director-transverse velocity gradients: with the splay mode the director escapes vertically the velocity gradients due to the roll modulation Fig. 1.

From a quantitative point of view, there is a good agreement between theory and experiments in thermoconvection in the low-h regime and in particular for the bimodal bifurcations [18]. The measurements shown in Figs. 2 and 3a at larger h are also consistent with the theory. The roll-diameter modulation and the results for ϵ_{SBV} and the secondary wave vector $\mathbf{k}$ at the bifurcations to splay bimodals match within 15% the theoretical predictions. The Hopf bifurcation at large h, probably connected with the oscillating bimodal instability appearing at rather large ϵ in the limit $h \rightarrow 0$ [13,14], may explain the peculiar Nusselt number oscillations reported in [7] very close to onset in the subcritical regime [19].

In this work, new bifurcations have been identified in nematic convection and explained by the competition of homogeneous with periodic modes. Figure 4 proves that the balance achieved in the nonlinear regime between efficient convection, minimizing torques on the director and viscosity dissipation can be subtle. We point out that the orientational role of the magnetic field in the linear regime, which is well known in the context of the simple Fréedericksz transitions, cannot be extrapolated in the nonlinear regime: in our system the planar magnetic field does contribute to the excitation of a homogeneous splay. Of course, homogeneous splay can be already excited in the linear regime by a vertical magnetic field or a pretilt at the boundary; splay bimodals have in fact been observed in such cases [8] as well, though not explained at this time. To describe the slow spatial variations of homogeneous and periodic modes, Eqs. (4) have now to be generalized by including spatial derivatives. Thus contact will be made to the description of quasihomogeneous modes, which have recently attracted some interest [20]. Moreover, a theoretical description of walls between the equivalent $\pm\psi$ states observed in some experiments or of other defect structures will become possible [21].

In conclusion, our results show that the interaction between *multiple* homogeneous modes and periodic modes can lead to quite complicated scenarios in the nonlinear regime (Fig. 4). They should motivate further analyses of this competition in other extended dynamical systems.

We are grateful to L. Kramer and S. Kai for valuable discussions.

[1] See, e.g., M.C. Cross and P.C. Hohenberg, Rev. Mod. Phys. **65**, 851 (1994).
[2] G. Dewel, S. Métens, M'F. Hilali, P. Borckmans, and C.B. Price, Phys. Rev. Lett. **74**, 4647 (1995); H. Bachir, S. Métens, P. Borckmans, and G. Dewel, Europhys. Lett. **54**, 612 (2001).
[3] See, e.g., K.E. Daniels, B.B. Plapp, and E. Bodenschatz, Phys. Rev. Lett. **84**, 5320 (2000).
[4] P.G. de Gennes and J. Prost, *The Physics of Liquid Crystals* (Clarendon Press, Oxford, 1993).
[5] For a recent general presentation, see A. Buka and L. Kramer, *Pattern Formation in Liquid Crystals* (Springer-Verlag, New York, 1996).
[6] E. Dubois-Violette, G. Durand, E. Guyon, P. Manneville, and P. Pieranski, *Liquid Crystals,* edited by L. Liébert, Solid State Phys. Suppl. **14**, 147 (1978).
[7] See the article of G. Ahlers in [5] and L.I. Berge, G. Ahlers, and D.S. Cannell, Phys. Rev. E **48**, R3236 (1993).
[8] S. Kai and W. Zimmermann, Prog. Theor. Phys. Suppl. **99**, 458 (1989).
[9] J.T. Gleeson, Physica (Amsterdam) **239D**, 211 (1997); Nature (London) **385**, 511 (1997).
[10] E. Plaut, W. Decker, A.G. Rossberg, L. Kramer, W. Pesch, A. Belaidi, and R. Ribotta, Phys. Rev. Lett. **79**, 2367 (1997).
[11] S. Rudroff, H. Zhao, L. Kramer, and I. Rehberg, Phys. Rev. Lett. **81**, 4144 (1998).
[12] Q. Feng, W. Pesch, and L. Kramer, Phys. Rev. A **45**, 7242 (1992).
[13] E. Plaut, L. Pastur, and R. Ribotta, Eur. Phys. J. B **5**, 283 (1998).
[14] E. Plaut and W. Pesch, Phys. Rev. E **59**, 1747 (1999).

[15] R. Ribotta, A. Joets, and Lin Lei, Phys. Rev. Lett. **56**, 1595 (1986); R. Ribotta and A. Joets, J. Phys. (Paris) **47**, 739 (1986).

[16] E. Plaut, A. Joets, and R. Ribotta, J. Phys. III (France) **7**, 2459 (1997).

[17] Conceptionally we followed the basic idea of R. M. Clever and F. H. Busse [J. Fluid Mech. **65**, 625 (1974)] in expanding all fields into suitable test functions. Because of the complexity of nematohydrodynamics the generation of our code had to be automated by the extensive use of Mathematica and certain unix tools (lex and yacc). In the final code, the multiple solution branches and their bifurcations are followed individually by continuation and eigenvalue calculation methods.

[18] For rolls with the background wave vector $\mathbf{q}$ [$|\mathbf{q}| = 1.07 q_c$, $\angle(\mathbf{q}, \hat{\mathbf{x}}) = 8°$], in 5CB at $h = 0$ our theoretical results $\epsilon_{BV} = 0.32$, $|\mathbf{k}| = 0.87|\mathbf{q}|$, $\angle(\mathbf{k}, \hat{\mathbf{x}}) = -50°$ compare satisfactorily with the experimental values $\epsilon_{BV} \simeq 0.28$, $|\mathbf{k}| \simeq 0.9|\mathbf{q}|$, $\angle(\mathbf{k}, \hat{\mathbf{x}}) = -55 \pm 5°$.

[19] A clear identification of a hysteresis has failed up to now since it requires experiments very near to $\epsilon = 0$, where the dynamics of thermoconvection gets extremely slow.

[20] The competition of *one* quasihomogeneous mode with periodic modes has been shown to possibly lead to dramatic new effects in H. Riecke and G. D. Granzow, Phys. Rev. Lett. **81**, 333 (1998).

[21] Note that V. M. H. Ruutu *et al.,* [Nature (London) **382**, 334 (1996)] have shown that "pattern formation" in the early Universe can be mimicked by the dynamics of defects in intrinsically anisotropic fluids.