

A Complexity View of Rainfall

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We show that rain events are analogous to a variety of nonequilibrium relaxation processes in Nature such as earthquakes and avalanches. Analysis of high-resolution rain data reveals that power laws describe the number of rain events versus size and number of droughts versus duration. In addition, the accumulated water column displays scale-less fluctuations. These statistical properties are the fingerprints of a self-organized critical process and may serve as a benchmark for models of precipitation and atmospheric processes.

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Rainfall and rainfall-related quantities have been recorded for centuries [1,2]. All these measurements, however, have the disadvantage of low temporal resolution and low sensitivity. The rain measurements are based on the simple idea of collecting rain in a container and measuring the amount of water after a certain time. The time intervals between readings are typically hours or days. Even with the most sophisticated of these conventional methods, the fine details of rain events cannot be captured at all and very light rain might not be recorded due to evaporation or insufficient sensitivity of the instrument, making it impossible to address questions regarding single rain events.

Recently, high-resolution data have been collected with a compact vertically pointing Doppler radar MRR-2, developed by METEK [3]. The instrument is operated by the Max-Planck-Institute for Meteorology, Hamburg, Germany, at the Baltic coast Zingst (54°43'N 12°67'E) under the Precipitation and Evaporation Project (PEP) in BALTEX [4]. Rain rate, liquid water content, and drop size distribution were obtained from the radar Doppler spectra, based on a method described by Atlas [5–7]. At vertical incidence, the Doppler shift can be identified with the droplet fall velocity. As, in the atmosphere, larger drops fall faster than smaller drops, spectral bins can be attributed to corresponding drop sizes. For a given size, the scattering cross section of the droplets can be calculated by Mie theory [8]. This yields the number density of drops which is proportional to the spectral power divided by the corresponding cross section. The rain rate $q(t) = \sum_i n_i V_i v_i$, where n_i is the number density of drops of volume V_i falling with velocity v_i . The detection threshold for rain rates under the pertinent operation parameters was $q_{\min} = 0.005$ mm/h. Below this threshold, $q(t) = 0$ by definition.

Precipitation profiles up to some thousand meters altitude can be observed. At present, the quantitative retrieval is restricted to rain. Snow and hail can be identified from the form of the Doppler spectra but have been excluded from the quantitative analysis. The analyzed data refer to

250 m above sea level and have been collected from January to July 1999 with 1-min resolution.

The processes that make a cloud release its water content are only very little understood. However, with the high temporal resolution of 1 min, single rain events can be identified and characterized. Previous work focused on the rainfall during a fixed period of time [9–11]. What makes the present analysis fundamentally new is the identification of a rain event as the basic entity. We define an event as a sequence of successive nonzero-rain rates. Sequences of zero-rain rates in between rain events are called drought periods. The event size is defined as the released water column in mm, $M = \sum_t q(t)\Delta t$, where $\Delta t = 1$ min, that is, the time integral of the rain rate over an event. In Fig. 1, the number density of rain events per year $N(M)$ versus

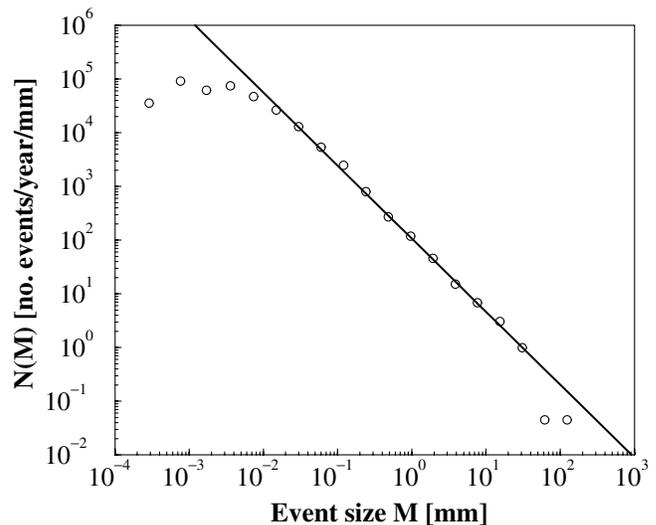


FIG. 1. The number density of rain events per year $N(M)$ versus event size M (open circles) on a double logarithmic scale. A rain event is defined as a sequence of consecutive nonzero-rain rates (averaged over 1 min). This implies that a rain event terminates when it stops raining for a period of at least 1 min. The size M of a rain event is the water column (volume per area) released. Over at least 3 decades, the data are consistent with a power law $N(M) \propto M^{-1.36}$, shown as a solid line.

event size M is displayed on a double-logarithmic plot. In a certain scaling regime, extending over at least 3 decades, the number density of rain events obeys a simple power law

$$N(M) \propto M^{-1.36}, \quad (1)$$

represented by the solid line in Fig. 1.

Figure 2 displays the number density of interoccurrence times (drought durations) $N(D)$ between successive rain events. The drought duration is power-law distributed

$$N(D) \propto D^{-1.42}, \quad (2)$$

implying there is no typical duration of droughts. We were not able to detect a lower or an upper cutoff in this relation. Both the lower end (1 min) and the upper end (2 weeks) still lie within the scaling region of the power law. The observed deviation around a period of 1 day is related to the daily meteorological cycle.

It is compelling that the distributions of sizes of rain events and drought periods are simple power laws. This result could prove very useful in relation to drought hazard assessment or flooding hazard assessment. In order to calculate the expected number $\bar{N}(T)$ of droughts with period $D > d$ in a given time period T , we would have to integrate $N(D)$. Assuming, for simplicity, that the upper cutoff diverges, $N(D) = \text{const} \times D^{-1.42}$, we find $\bar{N}(T) = T \times N(D > d) = T \times \frac{\text{const}}{0.42} \times d^{-0.42}$.

The question of having a reliable water supply is of utmost importance. Hurst [1,2] posed the following problem: How can one design a reservoir so that it never overflows or empties? He considered an incoming signal $q(t)$ over a

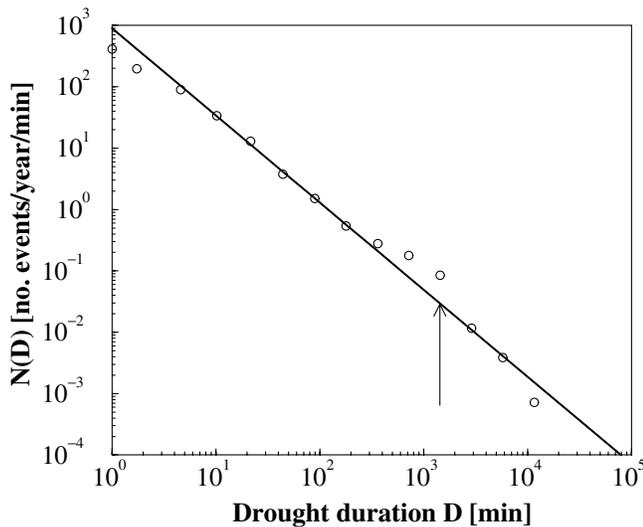


FIG. 2. The number density of droughts per year $N(D)$ versus drought duration D (open circles) on a double logarithmic plot. The drought duration is the time, measured in minutes, between two successive rain events. The displayed solid line is a power law $N(D) \propto D^{-1.42}$. The arrow indicates a time interval of one day. The data deviate from the power-law behavior at time intervals corresponding to about a day, reflecting the daily meteorological cycle.

time period τ . In our case, $q(t)$ is the rain rate. The actual level of water in a reservoir (or a river) is determined by

$$X(t, \tau) = \sum_{u=1}^t [q(u) - \langle q \rangle_\tau] \Delta t, \quad (3)$$

where $\Delta t = 1$ min and

$$\langle q \rangle_\tau = \frac{1}{\tau} \sum_{t=1}^{\tau} q(t) \Delta t \quad (4)$$

denotes the average influx in the considered time period τ . The water level needed for the reservoir never to empty is given by the range

$$R(\tau) = \max_{1 \leq t \leq \tau} X(t, \tau) - \min_{1 \leq t \leq \tau} X(t, \tau). \quad (5)$$

One can now determine the dimensionless ratio $R(\tau)/S(\tau)$ as a function of τ , where $S(\tau)$ is the standard deviation of the influx $q(t)$ in the period τ . For uncorrelated random events, this ratio increases as

$$R(\tau)/S(\tau) \propto \tau^H, \quad (6)$$

where the Hurst exponent $H = 1/2$. However, Hurst [1,2] discovered that for water level fluctuation in the Nile, $H \approx 0.77$. Figure 3 displays the water level $X(t, \tau)$ in a virtual reservoir for the rain data with $\tau = 266\,611$ min.

Figure 4 demonstrates that $R(\tau)/S(\tau) \propto \tau^H$, $H \approx 0.76$ is obeyed over more than 4 decades with $\tau \in [10 \text{ min}, 266\,611 \text{ min} \approx 6 \text{ months}]$.

It is important to notice that these fluctuations are a result of the fluctuating rain rate alone and imply a correlation between rain events over the whole temporal range studied in this Letter. This extends Hurst's result [2],

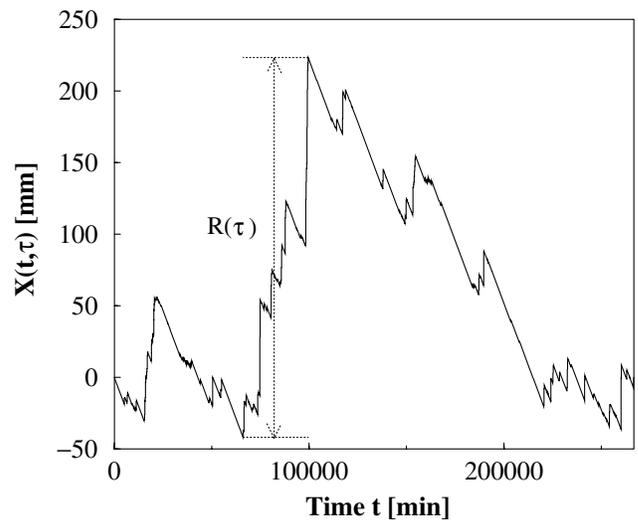


FIG. 3. Reservoir level $X(t, \tau)$ in mm for the entire record of duration $\tau = 266\,611$ min. The parts of the curve with negative slope correspond to dry periods (droughts) where there is no rain, only the mean outflux. The parts of the graph with positive slope are periods with rain events. The steepness of the line measures the difference between the influxes and the average outflux. The range $R(\tau)$ is indicated with a dashed line.

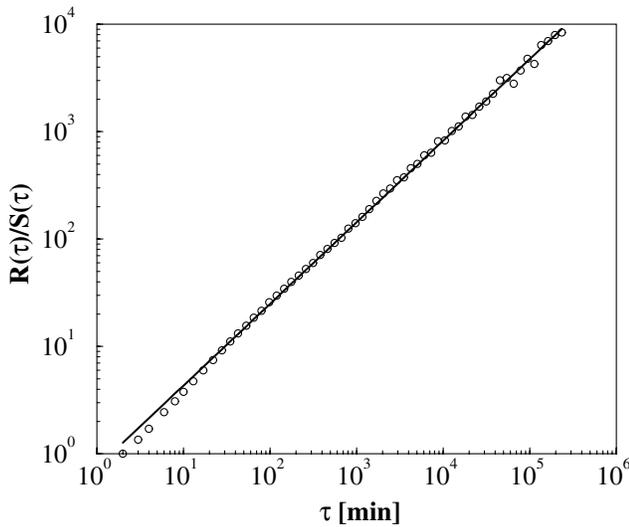


FIG. 4. Log-log plot of values of $R(\tau)/S(\tau)$ against the corresponding period of length τ (open circles). The solid line is a power law $R(\tau)/S(\tau) \propto \tau^H$, $H = 0.76$. The data are consistent with a power law over at least 4 decades. There is a lower cutoff around $\tau \approx 10$ min but no upper cutoff is apparent.

which he found was valid in the temporal range $\tau \in [1 \text{ yr}, 1080 \text{ yr}]$.

It can be illustrated directly that the fluctuations of the reservoir are statistically invariant under a transformation that changes the time scale by a factor b and the level by a factor b^H [12]. In Fig. 5, the x axis of Fig. 3 has been rescaled with a factor b and the y axis with a factor b^H , and the similarity is indeed striking.

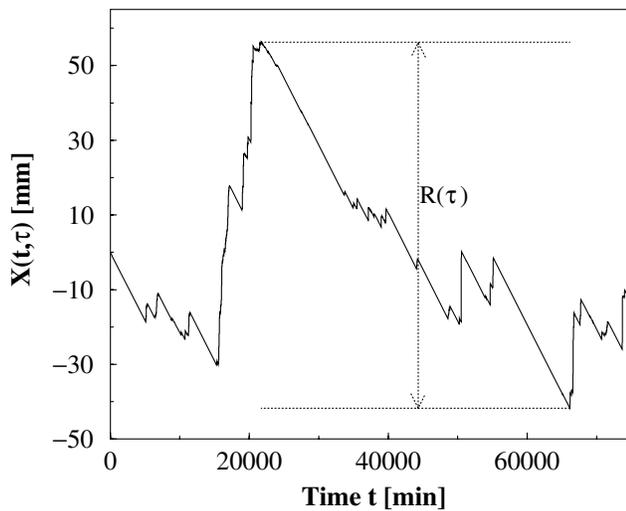


FIG. 5. Reservoir level $X(t, \tau)$ in mm for the initial part of the record with a duration of $\tau = 74\,743$ min. The corresponding range $R(\tau)$ is indicated with a dashed line. Compared with Fig. 3, the x axis has been rescaled with a factor $b = 266\,611/74\,743 \approx 3.57$ while the y axis has been rescaled with a factor $b^H \approx 2.63 \approx 300/115$ demonstrating that the reservoir level is a self-affine fractal.

The power-law number density of rain events is consistent with a self-organized critical process. The concept of self-organized criticality [13–15] refers to the tendency of nonequilibrium systems driven by slow constant energy input to organize themselves into critical states where all scales are relevant. The characteristic feature of self-organized critical systems, even if their dynamics are incomprehensibly complex, is that the intermediately stored energy is eventually released in sudden bursts with no typical size.

A well-known example of such a system is the Earth’s crust. Currents in the liquid core of the Earth drive the crust slowly and fairly constantly. The energy deposited by these currents is intermediately stored in tension building up between the tectonic plates and then suddenly released in earthquakes. The number of earthquakes per year with a seismic moment S exceeding s is given by the Gutenberg-Richter law [16]

$$N(S > s) \propto s^{-B}; \tag{7}$$

that is, there is no typical size for earthquakes. This suggests that all the earthquakes have the same physical origin and that the Earth’s crust is poised in a critical state.

Avalanches in a pile of grains might also display self-organized criticality: when grains are dropped onto a pile, one by one, the pile ultimately reaches a stationary critical state in which its slope fluctuates about a constant angle of repose, with each new grain being capable of inducing an avalanche on any of the relevant size scales [17].

From the perspective of self-organized criticality, rain events do not look very different from earthquakes or avalanches. If a rain shower, regardless of its duration or intensity, is defined as an event, the correspondence to avalanches in granular media and avalanches in the crust of the Earth is striking. The atmosphere is the system under investigation and corresponds to the Earth’s crust or the granular pile. It is driven by a slow and constant energy input from the Sun. In particular, water is evaporated from the oceans. The energy is stored in the form of water vapor in the atmosphere. It is then suddenly released in bursts when the vapor condenses to water drops. The power-law distribution of the number density of rain events versus size is equivalent to the Gutenberg-Richter law for earthquakes and the power-law distribution of avalanche size. There is no constant drizzle accounting for the constant evaporation but rain events of a wide range of sizes. One could imagine having a classification of rain events according to their size just as earthquakes are classified according to their position on the Richter scale.

In summary, we found that simple power laws describe the number density of occurrence of rain events of a given size and drought periods. Moreover, Hurst’s analysis from the 1950s on water level fluctuations was extended by more than 4 decades, from a year down to minutes. This insight will inevitably inspire new research into the modeling of precipitation and atmospheric processes as well as serve as

a benchmark for existing models and might be useful in, e.g., drought and flooding hazard assessment. On a more general level, our analyses show that new insights can be obtained from taking the very general point of view of complexity and self-organization theory. It may serve as an example of how to use this approach in situations that seem too complex to be accessible to quantitative analysis.

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[1] H. E. Hurst, *Nature* (London) **180**, 494 (1957).

[2] H. E. Hurst, *Long-Term Storage: An Experimental Study* (Constable & Co. Ltd., London, 1965).

[3] MRR-2, *Physical Basis*, p. 21 (1998). Available from METEK GmbH, Fritz-Straßmann-Straße 4, D-25337 Elmshorn, Germany.

[4] Information on the BALTEx project is available from the Web site <http://w3.gkss.de/baltex/>.

[5] D. Atlas, R. C. Srivastava, and R. S. Sekhon, *Rev. Geophys. Space Phys.* **11**, 1 (1973).

[6] D. Klugmann, K. Heinsohn, and H. J. Kirtzel, *Contrib. Atmos. Phys.* **69**, 247 (1996).

[7] M. Löffler-Mang and M. Kunz, *J. Atmos. Ocean. Technol.* **16**, 379 (1999).

[8] G. Mie, *Ann. Phys.* **25**, 377 (1908).

[9] S. Lovejoy, in *Proceedings of the 20th Conference on Radar Meteorology* (AMS, Boston, 1981), p. 476.

[10] S. Lovejoy and B. B. Mandelbrot, *Tellus* **37A**, 209 (1985).

[11] S. Lovejoy and D. Schertzer, *J. Appl. Meteorol.* **29**, 1167 (1990).

[12] J. Feder, *Fractals* (Plenum Press, New York, 1988).

[13] P. Bak, C. Tang, and K. Wiesenfeld, *Phys. Rev. Lett.* **59**, 381 (1987).

[14] P. Bak, *How Nature Works: The Science of Self-Organized Criticality* (Springer-Verlag, New York, 1996).

[15] H. J. Jensen, *Self-Organized Criticality: Emergent Complex Behavior in Physical and Biological Systems* (Cambridge University Press, Cambridge, U.K., 1998).

[16] B. Gutenberg and C. F. Richter, *Bull. Seismol. Soc. Am.* **34**, 185 (1944).

[17] V. Frette, K. Christensen, A. Malthé-Sørensen, J. Feder, T. Jøssang, and P. Meakin, *Nature* (London) **379**, 49 (1996).