Soliton in Two-Band Superconductor

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There is a soliton in a superconductor having two bands (two-band superconductor), when the interband interaction is much smaller than the intraband interaction. This soliton is in a stable state. In the soliton the relative phase between two gap parameters rotates 0 to 2π (or $-\pi$ to π), where each gap resides in each band. A phase slip of the superconducting order parameter is accompanied with the soliton. The phase slip is not $n \times 2\pi$ where n is an integer. A soliton traps the flux inside the superconducting ring, of which magnitude is not integral multiples of the fluxoid quantum.

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Recently it was pointed out that multiple bands are essential to achieve high T_c in some superconductors having multiple bands (multiband superconductor) [1,2]. In a previous paper, I pointed out that a soliton trapped by a Josephson junction gave one of the clearest hallmarks of the multiband superconductor [3]. In that paper, I discussed the soliton of the ground state. In this Letter, I discuss that this soliton can be present without a Josephson junction as a stable state using a one-dimensional Ginzburg-Landau model.

Let us consider a superconductor having two bands (two-band superconductor) [4–17]. In this superconductor the pair in each band has its own phase and amplitude. I introduce a pseudo-order parameter ψ_1, ψ_2 defined as follows [18]:

$$\begin{aligned} \psi_1 &= \sqrt{N_1}\exp(i\theta_1), \\ \psi_2 &= \sqrt{N_2}\exp(i\theta_2). \end{aligned} \tag{1}$$

N_ν and θ_ν are the density of the pair and its phase on the band indexed by ν. This pseudoparameter can describe the superconducting state having four internal freedoms (N_1, θ_1, N_2, and θ_2). It is provided that $\sqrt{N_1}$ and $\sqrt{N_2}$ are real positive numbers.

Based on the Ginzburg-Landau model, I used Gibbs free energy density in the two-band superconductor for a one-dimensional case as follows [3,8,14,]:

$$\begin{aligned} g(x) = &\sum_{\nu=1,2} \alpha_\nu |\psi_\nu(x)|^2 + \sum_{\nu=1,2} \frac{\beta_\nu}{2} |\psi_\nu(x)|^4 \\ &+ \sum_\nu \frac{\hbar^2}{2m_\mu} \left| \frac{\partial \psi_\nu}{\partial x} \right|^2 + \gamma(\psi_1^\dagger \psi_2 + \psi_2^\dagger \psi_1), \end{aligned} \tag{2}$$

where x is a coordinate and m_ν is the mass of the pair. I assumed there was no external field. The interband interaction corresponds to $\gamma(\psi_1^\dagger \psi_2 + \psi_2^\dagger \psi_1)$. γ specifies the strength of the interband interaction. When it is positive, the relative phase of ψ_1 and ψ_2 is π, in other words, $\psi_1 \times \psi_2 < 0$. When it is negative, the relative phase of ψ_1 and ψ_2 is 0; in other words, $\psi_1 \times \psi_2 > 0$.

The Josephson junction has a chance to trap a soliton only when $\psi_1 \times \psi_2 < 0$ is satisfied. However, there is a soliton as a stable state in both cases when the thermal fluctuation and the quantum fluctuation are small.

When the intraband interaction is much larger than the interband interaction, $\sqrt{N_\nu}$ is specified with the intraband interaction. The role of the small interband interaction is for specifying the relative phase between two pseudo-order parameters. In this situation I take an approximation that $\sqrt{N_\nu}$ is independent of the coordinate and $\nabla\sqrt{N_\nu} \approx 0$. In this approximation, free energy can be described as follows:

$$\begin{aligned} g_\theta(x) &= \sum_{\nu=1,2} \frac{\hbar^2 N_\nu}{2m_\nu} |\nabla_x \theta_\nu|^2 \\ &\quad + 2\gamma\sqrt{N_1 N_2}\cos(\theta_1 - \theta_2), \\ g_{Ns}(x) &= \sum_{\nu=1,2} \alpha_\nu N_\nu + \sum_{\nu=1,2} \frac{\beta_\nu}{2} N_\nu^2, \\ g(x) &= g_\theta(x) + g_{Ns}(x). \end{aligned} \tag{3}$$

When no supercurrent flows anywhere in the real space ($Je = \sum_{\nu=1,2} \frac{e\hbar N_\nu}{m_\nu} \nabla_x \theta_\nu = 0$), I obtain $\theta_2 = -\frac{N_1 m_2}{N_2 m_1}\theta_1$ for $\gamma < 0$ and $\theta_2 = \pi - \frac{N_1 m_2}{N_2 m_1}\theta_1$ for $\gamma > 0$. By introducing $\varphi = \theta_1 - \theta_2$ for $\gamma < 0$ and $\varphi = \theta_1 - \theta_2 + \pi$ for $\gamma > 0$, I minimize the free energy with respect to φ ($\delta g = 0$). The equation obtained is sine-Gordon [3]:

$$\begin{aligned} \frac{1}{m_0} &= \frac{N_1 N_2}{m_1 N_2 + m_2 N_1}, \\ \frac{1}{L^2} &= 2\frac{|\gamma|}{\hbar^2}\sqrt{N_1 N_2}\, m_0, \\ \frac{\partial^2\varphi}{\partial x^2} &- \frac{1}{L^2}\sin\varphi = 0. \end{aligned} \tag{4}$$

A trivial solution is $\varphi = 0$ which is the ground state. The other local minimum gives a soliton, as shown in Fig. 1. The relative phase rotates from 0 to 2π for $\gamma < 0$ and $-\pi$ to π for $\gamma > 0$. The energy of one soliton (E_{soliton}) can be calculated by subtracting the energy of the system without a soliton from that with one soliton,

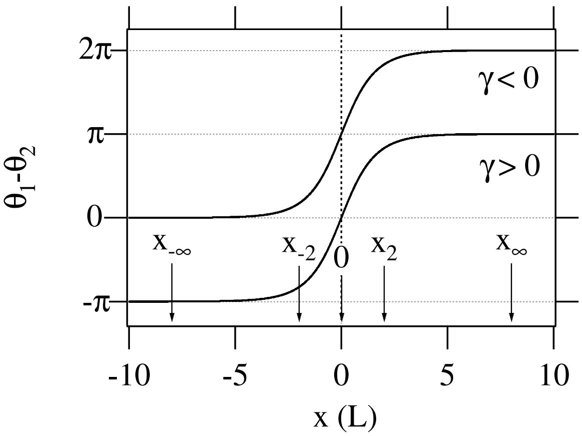

FIG. 1. Soliton in two-band superconductor. Relative phase between two pseudoparameters is depicted.

$$E_{\mathrm{soliton}} = 8\sqrt{2}\hbar\sqrt[4]{N_1N_2}\sqrt{\frac{|\gamma|}{m_0}}. \tag{5}$$

Figure 2 shows the schematic description of the soliton. I show ψ_1 and ψ_2 in the complex plane. As shown in Fig. 2, one can see the phase of the total order parameter slips at the soliton, and its magnitude is $\Theta_{\mathrm{soliton}} = \frac{\pm 2\pi}{1+\frac{m_2N_1}{m_1N_2}}$.

Leggett discussed the fluctuation of $\delta(\theta_1 - \theta_2)$ in 1966 [19–21]. According to his model, the fluctuation is limited within a small value. Therefore it is treated in terms of a harmonic oscillator. When this fluctuation grows to the nonlinear region and is stabilized, it becomes the soliton deduced in this Letter.

When I connect x_∞ to $x_{-\infty}$ in Fig. 1, I can make a ring. For a boundary condition, the phase slip due to the soliton should be compensated by the supercurrent. I replace θ_ν with $\theta_\nu - \frac{e}{\hbar}A$, where A is the vector potential [22]. In this case $Je = Je_A + Je_{\mathrm{soliton}}$, $Je_A = -e^2A(\frac{N_1}{m_1} + \frac{N_2}{m_2})$, and $Je_{\mathrm{soliton}} = \sum_{\nu=1,2}\frac{e\hbar N_\nu}{m_\nu}\nabla_x\theta_\nu$. I assume $Je_{\mathrm{soliton}} = 0$; in other words, there may be only a constant circulating supercurrent coming from the vector potential. (If $Je_{\mathrm{soliton}} \neq 0$, there would be a source and a sink of the supercurrent in the soliton. This situation is nonphysical.) Even if there is an offset due to the kinetic energy of this supercurrent, the soliton still gives a local minimum of the free energy [23]. The boundary condition is $\int\nabla_x\theta_\nu\,dx - \int\frac{e}{\hbar}A\,dx = 2n\pi$, where n is an integer. The induced flux inside the ring, Φ, by the supercurrent is $\int A\,dx = (\frac{-\Theta_{\mathrm{soliton}}}{2\pi} + n)\Phi_0$, where Φ_0 is fluxoid quantum. The amount of the self-induced flux inside the ring teaches us the number of solitons. (Strictly speaking, the number of antisolitons should be subtracted.) This fractional flux can be understood by the analogy of the half-flux quanta trapped by the grain boundary junction of the d-wave superconductor, where a crystallographic misalignment rotates the phase of the order parameter instead of the soliton [24–29].

The cuprate having more than two CuO_2 planes in a unit cell is one of the feasible examples of the multiband superconductor. We can tune the interband interaction by the doping level [30]. One of the candidates is $Cu_xBa_2Ca_3Cu_4O_y$ (Cu-1234) [31–34]. It has multiple bands and two crystallographically nonequivalent CuO_2 planes [35,36]. The doping level can be varied [37–39]. The specific heat and NMR studies suggest that the interband interaction is very weak [40–44]. By tuning the doping level, we can control the energy of the soliton through the interband interaction. When we quench the superconducting ring, thermally activated solitons may be quenched and survive at low temperatures against the thermal agitation. We can measure the self-induced flux inside the ring even with a time-consuming technique. By an instantaneous measurement of the total flux inside the ring, the steplike signal accompanied by the creation (or destruction) of the soliton can be observed when there is a fluctuation having a certain magnitude. By applying the magnetic field, we may generate a surviving soliton after releasing the field when a soliton is stable enough to survive against any fluctuation [45–48].

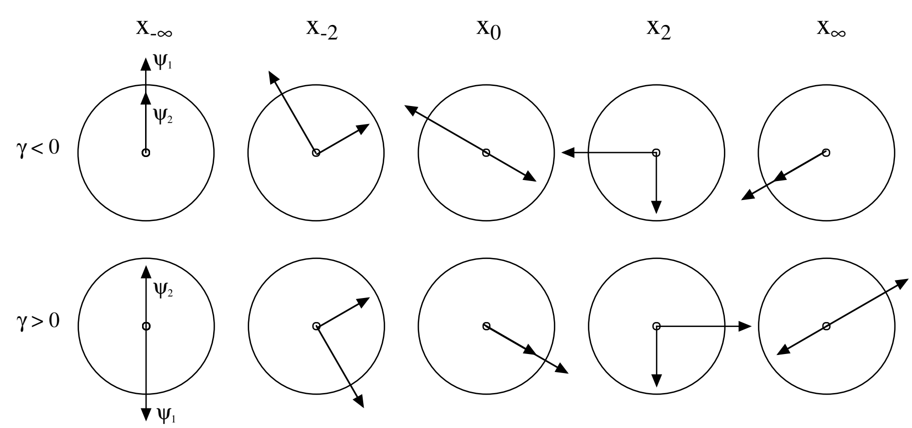

FIG. 2. Schematic diagram of the relative phase between two pseudo-order parameters in several slices at location specified x_i. x_i is labeled in Fig. 1.

In this Letter, I do not discuss the dynamics of the soliton in the mixed state of a type-II superconductor. That is a further problem.

In conclusion, there is a soliton in the two-band superconductor. It is a hallmark of the multiband superconductor.

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