Instability, Chaos, and "Memory" in Acoustic-Wave–Crack Interaction

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A new class of nonlinear acoustic phenomena has been observed for acoustic wave interactions with cracked defects in solids. Parametric modulation of crack stiffness results in fractional acoustic subharmonics, wave instability, and generation of chaotic noiselike acoustic excitations. Acoustic-wave impact on a crack is shown to exhibit amplitude hysteresis and storage for parametric and nonlinear acoustic effects. The measured storage time amounts to several hours and is believed to be due to a long-term relaxation of thermally induced microstrain within a crack area.

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Nonlinear acoustic wave propagation in both ideal and imperfect solids is known to be accompanied by classical effects of higher harmonic generation, waveform distortion, resonance frequency shift, etc. [1,2]. However, a general family of nonlinear *oscillation* phenomena in strongly nonlinear and parametric systems is much broader and includes subharmonic bifurcations, dynamic instability, and chaos, well known in many branches of physics [3].

This Letter demonstrates that stochastic scenario can also be inherent in *propagating acoustic wave* phenomena as a result of their nonlinear interaction with cracked defects. Such a defect (crack) is considered to be a strongly nonlinear oscillator with an asymmetric stiffness characteristic [4]. Parametrically driven nonbonded contact of a crack can, therefore, display a nondeterministic behavior and accommodate a greater diversity of nonlinear oscillation effects to propagating acoustic waves in solids.

An inverse problem includes the monitoring of crack mechanical parameters using the data of nonlinear acoustic wave probing. Since acoustic instability effects have a resonance nature one can expect them to be an extremely sensitive indicator of a crack mechanical state. In particular, we were able to observe that acoustic wave impact on a crack was followed by a long-term relaxation of its nonlinear elastic properties, whereas linear elasticity was found to be unchanged.

Acoustic wave-crack interaction was studied for surface acoustic waves (SAWs) in YZ-LiNbO$_3$ substrate ($10 \times 60 \times 2.5$ mm) with a crack of irregular shape (≈ 2 mm deep and ≈ 8 mm long) across the sample length. Two identical sets of three interdigital transducers each (fundamental frequencies 15, 30, and 45 MHz, correspondingly) were used to generate and receive SAWs. By combining the input and output transducers the higher harmonics as well as fractional SAW subharmonics could be detected. A pulse (1–100 μsec) or cw voltage (amplitude up to 10 V) can be used for SAW generation. A monofrequency operation revealed the SAW transmission through and reflection from the crack to exist steadily over the 15–45 MHz range so that its faces were found partly closed by the internal stress.

As the amplitude of the acoustic wave incident on the crack increased, a number of superharmonic (2ω and 3ω) and subharmonic ($\omega/2$, $3\omega/2$, $\omega/3$, $2\omega/3$) components were observed in the reflected acoustic field (nonlinear reflection mode). Figure 1 shows the amplitude of $3\omega/2$ subharmonic as a function of the input voltage at ω frequency. One can clearly see steplike thresholds [$(V_{\mathrm{IN}})_1 \geq 3$ V; $(V_{\mathrm{IN}})_2 \geq 4$ V] followed by more stable "plateaus." The sharp amplitude increase at the threshold indicates a transition into the instability region where avalanchelike development of oscillations takes place. Multiple thresholds observed in our experiments (two thresholds in Fig. 1), apparently, correspond to parametric resonances within different parts of the crack aperture.

By the end of the avalanchelike buildup the instability manifests the amplitude self-modulation of the SAW subharmonics (Fig. 2). The modulation frequency changes from hundreds of kHz to units of MHz for the input voltages corresponding to the "plateau" area in Fig. 1. Physically, this is an indication of sideband spectral components generation within a parametric frequency band around the subharmonic line instead of a single-frequency mode exactly at the threshold. Note that a similar self-modulation

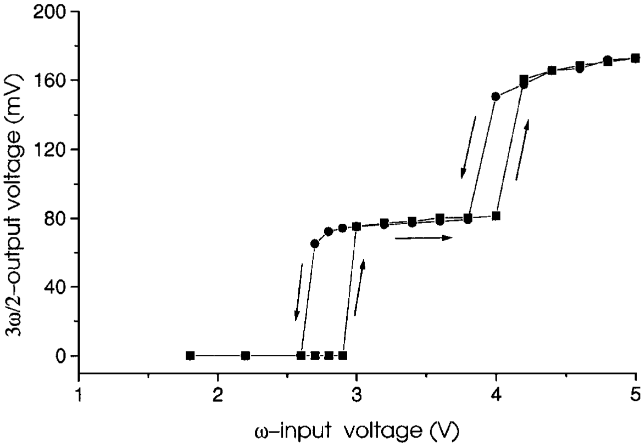

FIG. 1. Amplitude of $3\omega/2$ subharmonic as a function of ω-frequency input voltage.

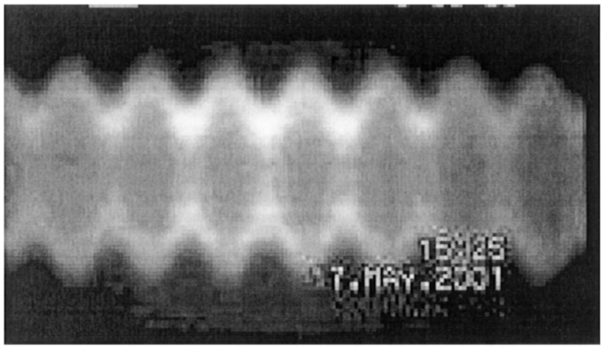

FIG. 2. “Self-modulation” effect for $3\omega/2$ subharmonic.

instability was reported to accompany fractional subharmonic oscillations in coupled quartz resonators [5].

Similar behavior was observed for the higher acoustic harmonics nonlinearly reflected from the crack area (Fig. 3). At low input ($V \leq 1.3$ V) the harmonic amplitudes increase uniformly “purely” due to crack nonlinearity without any signs of parametric contribution. However, subsequent steplike behavior indicates the initiation of the impact of even parametric resonances (of $2m\omega/2$ frequencies) accompanied by self-modulation and further instability development for the higher harmonics driven parametrically at higher input.

Away from the thresholds, the amplitude modulation turns into chaotic beats until finally (for $V_{\mathrm{IN}} \geq 4.5$ V in Figs. 1 and 3) a temporal instability is fully developed: the amplitudes of nonlinear spectral components fluctuate randomly and both sub- and superharmonics change into noiselike excitations (Fig. 4). It is worthwhile to note that neither transmitted nor reflected linear acoustic waves (of ω frequency) showed signs of the threshold or unstable behavior.

The subharmonic and instability effects were found to depend dramatically on the ambient conditions (temperature, humidity, pressure, etc.). For instance, we were able to control remotely the threshold transition to instability and back with a sound wave radiated in air by a loudspeaker. Such an extreme sensitivity can be used to measure minor dc variations in the crack mechanical state caused by acoustic wave in a solid.

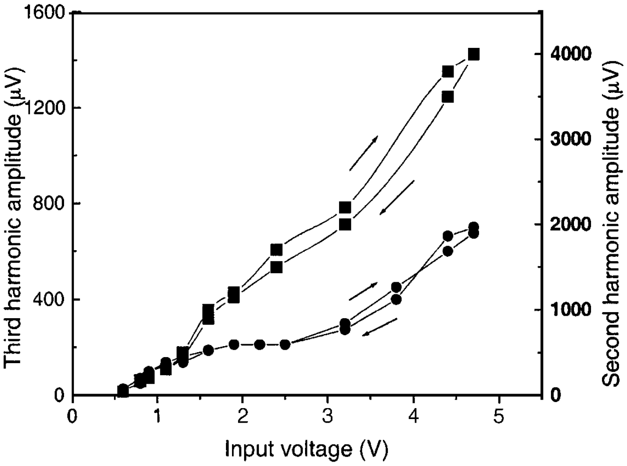

FIG. 3. Dynamic characteristics of the second (■) and third (●) harmonics.

These effects are illustrated in Figs. 1 and 3 by an evident hysteresis in the amplitudes behavior of the sub- and superharmonics. The hysteresis might be associated with an inherently hysteretic “retreat” of the oscillations from the instability area [6]. It also may be an indication that the state of the crack is modified by an incident acoustic pulse so that the defect does not fully recover during subsequent measurements. The measurements showed that the threshold levels and the area of hysteresis loops in Fig. 1 are proportional to the duration of acoustic wave impact. Longer impacts shift the parametric threshold to the lower input, spread the hysteresis loops, and smooth them over the parametric resonance area. The sensitivity to the acoustic pulse length implies that the hysteretic behavior of acoustic nonlinearity is caused by changes in the crack mechanical state. These changes are proportional to acoustic energy delivered to the crack and are stored until it recovers from the acoustic impact.

The results of experimental study of crack nonlinear “memory” are shown in Fig. 5 where a 15 MHz pump (reading-in) acoustic wave (length $\Delta\tau$) was used to strike the crack. A read-out 30 MHz acoustic pulse probes the crack after the pump wave irradiation and a 45 MHz reflected $3\omega/2$ acoustic subharmonic is observed. The probing wave amplitude is adjusted below the threshold level so that no subharmonic could be observed without the pump wave. Acoustic impact changes the state of the crack and increases greatly the subharmonic amplitude, which then recovers to the original zero level within time interval T (“memory” time). From Fig. 5, 25 s-long acoustic impact is accompanied by $\approx$5 min storage of the subharmonic signal, thus providing the value of memory factor $T/\Delta\tau \approx 12$. Very close values of $T/\Delta\tau \approx 10$–15 obtained for a different impact duration in similar experiments with pulse 15 MHz pump waves suggest that this is

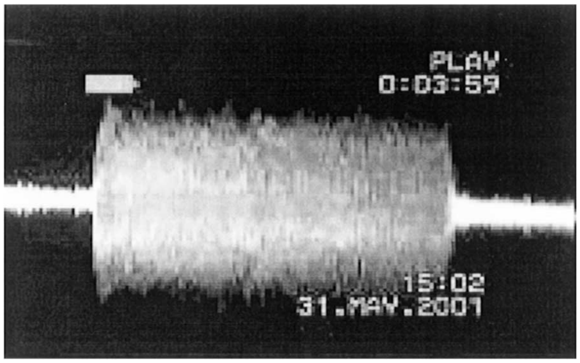

FIG. 4. Chaotic behavior of $3\omega/2$ subharmonic wave above the threshold.

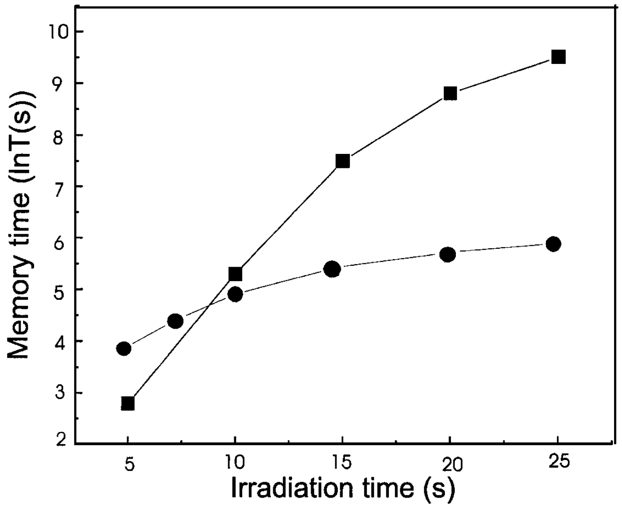

FIG. 5. "Memory" time for parametric (●) and nonlinear (■) thresholds as a function of pump wave exposure.

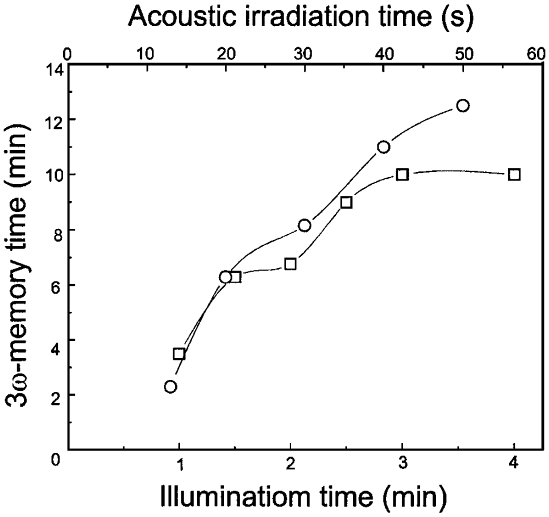

FIG. 6. 3ω storage time as a function of optical (□) and acoustic (○) irradiation time.

a characteristic parameter of the crack recovery for subharmonic (i.e., parametric) threshold.

According to Fig. 3, at low input (below the parametric threshold $V_{\mathrm{IN}} \approx 1.5$ V) the crack was nonlinearly driven and the higher harmonic generation was entirely due to the crack nonlinearity. To study storage characteristics in this case, the crack was irradiated with a 30 MHz wave (CW-pump mode). To read out the nonlinear changes induced in the crack, a 15 MHz probing pulse was used and the third harmonic (45 MHz) generation observed in the reflection from the crack. Similar to the above experiments, the probing wave amplitude was adjusted below the nonlinear threshold level ($V_{\mathrm{IN}} \leq 0.5$ V, Fig. 3) and no higher harmonics could be observed without the pump wave. The pump wave acoustic impact changes the crack parameters and brings the higher harmonic amplitudes beyond the nonlinear threshold. Extremely long storage (T up to 4 h) was observed for acoustic impacts ($\Delta\tau$) as short as 25 s. According to Fig. 5, the memory factor $T/\Delta\tau$ for the nonlinear threshold ranges from 100 to 500, i.e., exceeds considerably its parametric counterpart.

A physical origin for nonlinear and parametric "memory" of the crack is assumed to be associated with relaxation of its mechanical parameters after being modified by an acoustic impact. The latter may be caused by a direct mechanical acoustic wave impact (radiation pressure, elastic dc effects) or indirectly by heating the crack area followed by a slow strain relaxation. To examine the storage mechanism, the pump acoustic wave in the previous experiment was replaced by a focused light beam to produce a direct heating of the crack area. A comparison of the crack nonlinear recovery for the two types of impacts is given in Fig. 6. The striking similarity between the results of measurements implies that the main reason for the crack nonlinear memory is concerned with slow relaxation of thermally induced microstrain within the fractured defect.

Unlike slow dynamics of linear elastic properties reported recently for rocks [7], the crack memory manifests itself exclusively for nonlinear acoustic effects: for the input strain amplitudes below $\approx 10^{-5}$, neither transmitted nor reflected linear acoustic waves (of ω frequency) demonstrated any evidence of instability or storage. This proves acoustic nonlinearity to be a sensitive forerunner of minor variations in mechanical properties of cracked defects.

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