New Determination of the Electron's Mass

Thomas Beier,[1] Hartmut Häffner,[1,2] Nikolaus Hermanspahn,[2] Savely G. Karshenboim,[3,4] H.-Jürgen Kluge,[1] Wolfgang Quint,[1] Stefan Stahl,[2] José Verdú,[1,2] and Günther Werth[2]
[1]*Gesellschaft für Schwerionenforschung, 64291 Darmstadt, Germany*
[2]*Institut für Physik, Universität Mainz, 55099 Mainz, Germany*
[3]*D.I. Mendeleev Institute for Metrology (VNIIM), 198005 St. Petersburg, Russia*
[4]*Max-Planck-Institut für Quantenoptik, 85748 Garching, Germany*
(Received 29 August 2001; published 19 December 2001)

A new independent value for the electron's mass in units of the atomic mass unit is presented, $m_e = 0.000\,548\,579\,909\,2(4)$ u. The value is obtained from our recent measurement of the g factor of the electron in $^{12}C^{5+}$ in combination with the most recent quantum electrodynamical (QED) predictions. In the QED corrections, terms of order α^2 were included by a perturbation expansion in $Z\alpha$. Our total precision is three times better than that of the accepted value for the electron's mass.

DOI: 10.1103/PhysRevLett.88.011603 PACS numbers: 14.60.Cd, 06.20.Jr, 31.30.Jv, 32.10.Fn

The mass of the electron is a fundamental physical constant, currently known to a relative precision of 2.1×10^{-9} [1,2]. It is also connected to other fundamental constants, in particular to those which describe the properties of atoms. To the Rydberg constant R_∞ it is related via

$$R_\infty = \alpha^2 \frac{m_e c}{2h}, \tag{1}$$

where the fine-structure constant α and the Planck constant h also enter the expression. Those are currently known to precisions of 3.7×10^{-9} and 3.9×10^{-8}, respectively [1]. The numerical value of c is fixed by definition, and the Rydberg constant is known to an accuracy of at least 0.008 ppb [1,3,4] and thought to become even more precise in the near future [5]. Therefore a precise value of the electron's mass also allows us to obtain more precise values for the other constants present in Eq. (1).

Recent measurements of the atomic mass of the electron were carried out determining either the proton-electron mass ratio m_p/m_e [6–11], the antiproton-electron mass ratio [10], or $m_e/m(^{12}C^{6+})$ [2,12]. Most of these experiments employed suitable Penning-trap techniques, where electrons and ions were loaded alternately into the same trap. By determining the cyclotron frequency

$$\omega_c = \frac{q}{m} B \tag{2}$$

(m is the mass and q the charge) for each type of particle and assuming the same magnetic field strength B, the desired mass ratio was obtained. Farnham *et al.* [2] carried out the most precise of these measurements by subsequently observing a single $^{12}C^{6+}$ ion and clouds of 5–13 electrons. They obtained

$$m_e = 0.000\,548\,579\,911\,1(12)\ \text{u}. \tag{3}$$

The recent compilation of the recommended values of fundamental physical constants [1] is mainly based on this value.

An indirect measurement was performed by Wineland *et al.* [13] who measured the ground-state g factor for $^9Be^+$ in a Penning trap by a laser-fluorescence technique. Comparing their experimental result with the value then predicted by theory [14], it was possible to derive the mass ratio $m(^9Be^+)/m_e$ and thus also m_p/m_e to a precision of 3.4×10^{-7}.

We have successfully developed and tested a setup for investigating g factors of highly charged ions [15,16]. Microwave irradiation induces spin-flip transitions within the system. For zero nuclear spin, the transition frequency is equal to the electronic Larmor precession frequency ω_L, given by

$$\omega_L = g \frac{e}{2m_e} B, \tag{4}$$

where e is the positive elementary charge unit. Calibrating the magnetic field by the cyclotron frequency ω_c [Eq. (2)] of an ion, the mass of the electron is obtained as

$$m_e = \frac{g}{2} \frac{e}{q} \frac{\omega_c}{\omega_L} m_{\text{ion}}, \tag{5}$$

where m_{ion} denotes the mass of the ion. At any level of precision under investigation here, the charge ratio e/q can be considered as a ratio of integers. Thus only the g factor and the frequency ratio ω_L/ω_c have to be known to determine the electron's mass in units of the ion mass. For hydrogenlike systems, the g factor can be accurately calculated ([17–20], for low-Z systems also [21–23], and references therein). The frequency ratio ω_L/ω_c for $^{12}C^{5+}$ was recently measured by us [15]. We will discuss our experiment first and outline the theoretical considerations afterwards.

A single $^{12}C^{5+}$ ion is stored in the magnetic field (3.8 T) of a cryogenic Penning trap (Fig. 1) which is described in detail in [24,25]. The trapping potentials are generated by a stack of 13 cylindrical electrodes of 0.7 cm inner diameter. Two potential minima can be created along the axis, which are separated by about 2 cm. Single highly

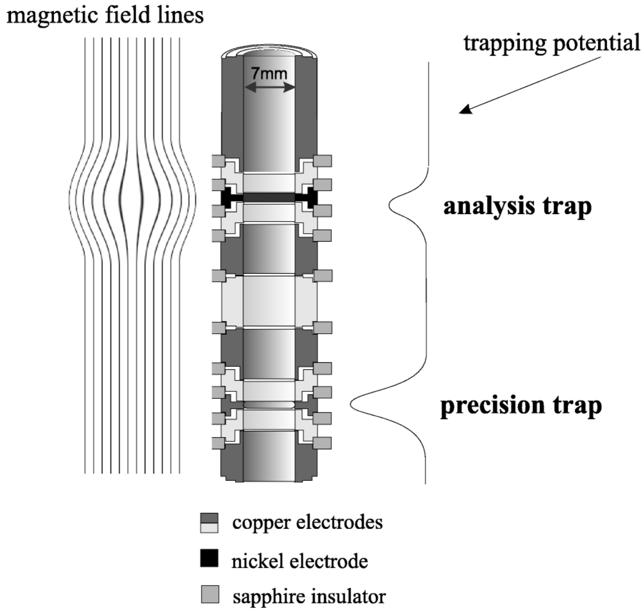

FIG. 1. Sketch of the double Penning trap.

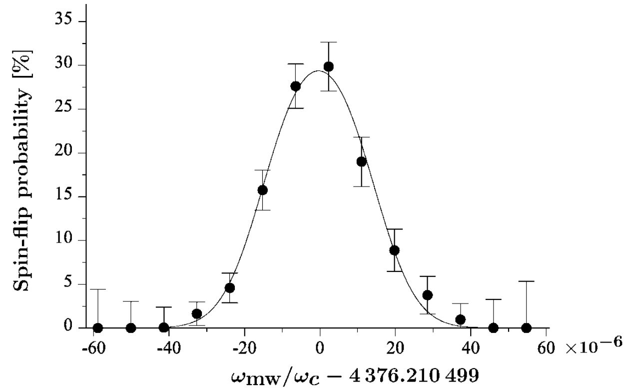

FIG. 2. Normalized Larmor resonance for $^{12}C^{5+}$ measured in the precision trap. We plot the spin-flip probability versus the frequency ratio $\omega_{\rm mw}/\omega_c$, corrected for finite cyclotron energy. The solid line is a fit of a Gaussian. The error margins are calculated by assuming a binomial distribution of the spin-flip probability.

charged ions have been stored for months, and neither vacuum ($p < 10^{-16}$ mbar) nor transport between the two traps restrict the storage time of an ion. To determine the Larmor frequency, the spin-flip rate is recorded as a function of the frequency $\omega_{\rm mw}$ of an applied microwave field and of the cyclotron frequency ω_c of the ion [15,25].

To investigate spin flips, we first analyze the direction of the spin in the so-called analysis trap, where the magnetic field has a considerable quadratic component, $B = B_0 + B_2 z^2 + \ldots$, $B_2 = 10$ mT/mm^2 [16]. Because of this inhomogeneity, the axial frequency of the ion slightly differs for both spin orientations. The ion is transferred to the precision trap, where the magnetic field is much more homogeneous ($B_2 = 8\ \mu$T/mm^2). Microwave irradiation takes place, and simultaneously the cyclotron frequency is measured by an image-current technique. To determine the final spin state, the ion is moved back to the analysis trap. This double-trap technique circumvents the limitations imposed by the magnetic inhomogeneity which is required for the detection of the spin direction [26] and which limited us to a precision of 10^{-6} in an earlier experiment [16].

Employing a Gaussian least-squares fit to the spin-flip resonance in Fig. 2, the center of the curve can be determined within 5% of the relative line width of 7×10^{-9}. We modeled the small asymmetry of the resonance according to Brown [27] and investigated it experimentally by increasing the axial energy. Its influence on the ratio ω_L/ω_c was found to be less than 2×10^{-10}. Corrections are performed for finite ion-oscillation amplitudes by extrapolating to vanishing energies. Finally, the frequency ratio ω_L/ω_c can be extracted to a relative precision of 5×10^{-10}:

$$\frac{\omega_L}{\omega_c(^{12}\mathrm{C}^{5+})} = 4\,376.210\,498\,9(19)\,(13)\,. \qquad (6)$$

Here, the first uncertainty is statistical [15], whereas the second one is obtained from the estimation of possible systematical shifts [15,25]. Employing the published value for the electron's mass [1], we obtain $g(^{12}\mathrm{C}^{5+}) = 2.001\,041\,596\,3(10)\,(44)$, where the first error results from quadratically combining the uncertainties of (6) and the second error is due to the uncertainty in the known value of the electron's mass. We report all our uncertainties as one-sigma margins, i.e., standard deviations.

The theoretical value of $g(^{12}\mathrm{C}^{5+})$ is given by the Dirac value for the g factor in the ground state of a hydrogenlike ion with nuclear charge Z, $g_{1s} = (2/3)\,[1 + 2\sqrt{1 - (Z\alpha)^2}\,]$, plus additional corrections for finite nuclear size and mass, and for QED effects. The QED effects are calculated up to order $(\alpha/\pi)^4$ for the free electron (cf. [28], and references therein). Additional bound-state QED corrections are known nonperturbatively in $Z\alpha$ up to order (α/π) [19,20]. For the current theoretical value we adopt

$$g(^{12}\mathrm{C}^{5+}) = 2.001\,041\,589\,9(10)\,. \qquad (7)$$

The individual contributions to the theoretical value are presented in Table I. The value in (7) and also Table I deviate from that quoted in [15] in several respects. First, the updated value [1] for the fine-structure constant α was employed, $\alpha = 1/137.035\,999\,76(50)$. All calculations presented in [19] and cited in [15] are based on the value $\alpha = 1/137.035\,989\,6$. This change does not affect any of the bound-state QED calculations at the precision given here. However, the leading Dirac-theory term and the order-(α/π) term for the free-QED contributions are sensitive to it on the 10^{-10} level. Employing the old value increases the total theoretical number by 1×10^{-10}. The recent uncertainty of α causes an uncertainty of 2×10^{-11} in the prediction for g. Since the value for α from [1] is mainly based on the $g - 2$ measurement for

TABLE I. Theoretical contributions to $g(^{12}C^{5+})$. Where no error margin is given, it is less than one unit of the last digit. The uncertainty for the bound-state QED of order (α/π) is purely numerical, that for bound-state QED, order $(\alpha/\pi)^2$, results from employing a perturbation series in $Z\alpha$.

Dirac theory (including binding)	1.998 721 354 4	[37]
Finite-size correction	+0.000 000 000 4	[38]
Recoil	+0.000 000 087 6	[35]
QED, free, order (α/π)	+0.002 322 819 5	[39]
QED, bound, order (α/π)	+0.000 000 844 2(9)	[19,23]
QED, free, orders $(\alpha/\pi)^2$ to $(\alpha/\pi)^4$	−0.000 003 515 1	[28]
QED, bound $(\alpha/\pi)^2$, $(Z\alpha)^2$ term	−0.000 000 001 1(4)	[31]
Total theoretical value	2.001 041 589 9(10)	

the free electron [29], we have employed, in addition, the non-QED value from [30], $\alpha = 1/137\,036\,003\,7(33)$. It leads to the same theoretical prediction for g but in that case the uncertainty resulting from α is 1×10^{-10}.

A second improvement in Table I results from the new estimates for bound-state QED corrections of order (α/π). The so-called "magnetic-loop" vacuum polarization correction, also termed "potential correction," up to now had an error margin of 3×10^{-10} assigned to it [19,20]. Recent estimations [23] show this term to be of the order $(\alpha/\pi)(Z\alpha)^7$ and for carbon therefore less than 1×10^{-11} in absolute magnitude. The numerical calculations for the leading vacuum polarization contributions (the "wave-function correction" in [19,20]) were recently confirmed by an analytic calculation [22,23] to better than 10^{-10}. In total, this leaves an uncertainty of 9×10^{-10} from the self-energy contributions to the bound-state QED corrections of order (α/π), without changing the numerical value of the corresponding entry.

The third change in the table results from taking into account the existing $Z\alpha$ expansion for QED corrections of the order $(\alpha/\pi)^2$. In [15], a rather large error margin was employed instead. For that expansion, only the leading term is known which is of purely kinematic origin [31,32]. It is given by

$$g_{(\alpha/\pi)^2,(Z\alpha)^2} = 2\left(\frac{\alpha}{\pi}\right)^2 \frac{(Z\alpha)^2}{6} \times (-0.328\ldots), \quad (8)$$

where the last number is the coefficient of the $(\alpha/\pi)^2$ term in the expansion for the g factor for the free electron, cf. formula (B6) in [1]. An estimate for the error margin resulting from the $Z\alpha$ expansion is obtained by subtracting the leading $(Z\alpha)^2$ term from the numerical value of the order-(α/π) term and multiplying the result by (α/π) [23,33]. To be conservative the obtained estimate is multiplied by a factor of 2 by which also bound-state terms of order $(\alpha/\pi)^3$ and higher are thought to be taken into account.

The fourth improvement in Table I is due to recent evaluations for the recoil effect that accounts for the finite mass of the nucleus. Shabaev and Yerokhin [34,35] have presented results to all orders in $Z\alpha$ for the order (m_e/M_N), where M_N is the mass of the nucleus. In addition Yelkhovsky independently calculated the $(Z\alpha)^4$ term for the order (m_e/M_N) [36]. These new evaluations do not affect the value in Table I but only its uncertainty which now can be estimated to be less than 0.5×10^{-10}.

Experiment and theory for the g factor of $^{12}C^{5+}$ agree within 1.5 standard deviations. Our experiment forms one of the most stringent tests of QED in any highly charged system up to now. The relative uncertainties of both values are of the order 10^{-9}, and the largest uncertainty results from the knowledge of the electron's mass. Therefore it is reasonable to turn around our arguments and instead derive a new value for the atomic mass of the electron. Hereby we rely on the validity of quantum electrodynamics to our level of precision.

The mass of $^{12}C^{5+}$ in terms of the mass of neutral carbon, $m(^{12}C) \equiv 12$ u, is given by

$$m(^{12}C^{5+}) = m(^{12}C) - 5m_e + E_B/c^2. \quad (9)$$

where $E_B = 579.835(1) \times 10^{-9}$ u c^2 is the cumulative binding energy for all 5 electrons [1]. By employing Eqs. (5) and (9), the experimental value of the frequency ratio ω_L/ω_c (6), and the theoretical prediction for the g factor (7), we obtain for the atomic mass of the electron,

$$m_e = 0.000\,548\,579\,909\,24(29)\,(27)\ \text{u}. \quad (10)$$

The first error margin results from the experimental uncertainty of the ratio ω_L/ω_c and the second results from the error margin of the theoretical prediction. The uncertainty of the binding energies affects the result for the electron's mass only on the relative level of 10^{-13}. In total, our uncertainty amounts to 4×10^{-13} u which corresponds to a relative precision of 7.3×10^{-10}. Our new value for the electron's mass is independent from the other high-precision measurement [2].

We stress that the numerical value of the mass of the electron also enters into our theoretical predictions. The dominant influence of m_e in $g(^{12}C^{5+})$ is given by the recoil correction. A change of the electron's mass at the level of 10^{-7} would lead to a change of Δg_{recoil} of the order 1×10^{-14}. This is the dominant effect, and, although the mass of the electron is also present in all other bound-state calculations via the nuclear size and the numerical renormalization procedures, the effects are orders of magnitude smaller. Our theoretical number does not change when inserting the new value.

Comparing our result (10) with the one from [2], Eq. (3), we find only a small difference of about 1.5 standard deviations. Our error margins are three times smaller. For the proton-electron mass ratio, we obtain

$$\frac{m_p}{m_e} = 1836.152\,673\,3(14), \quad (11)$$

where the Committee on Data for Science and Technology value for the atomic mass of the proton was employed, $m_p = 1.007\,276\,466\,88(13)$ u. The uncertainties were added quadratically.

In conclusion, we have obtained a three times more accurate value for the atomic mass of the electron than that recommended in [1]. We consider our value as an important consistency check. In addition, our experimental method differs from the one used by Farnham *et al.* [2]. We perform measurements on a single particle. This rules out the possibility of different spatial positions of electrons and ions used for the comparison. Furthermore, we measure the Larmor and the cyclotron frequency simultaneously, thus avoiding systematic uncertainties due to temporal fluctuations of the magnetic field. Our experimental setup is applicable to any hydrogenlike system without major changes. In $^4He^+$ the bound-state QED effects, which at present cause the largest theoretical uncertainty, are an order of magnitude smaller than in $^{12}C^{5+}$. The mass of this ion is known to a precision of 2.5×10^{-10} [1] and thus this system is also suitable for a determination of the electron's mass in our setup.

We are grateful to R. N. Faustov, P. J. Mohr, G. Soff, B. N. Taylor, and V. A. Yerokhin for many enlightening and stimulating discussions and, in particular, to V. M. Shabaev and A. S. Yelkhovsky for making their results accessible to us prior to publication. Financial support was obtained from the European Union under Contract No. ERB FMRX CT 97-0144 (EUROTRAPS network).

[1] P. J. Mohr and B. N. Taylor, Rev. Mod. Phys. **72**, 351 (2000).
[2] D. L. Farnham, R. S. Van Dyck, Jr., and P. B. Schwinberg, Phys. Rev. Lett. **75**, 3598 (1995).
[3] T. Udem *et al.,* Phys. Rev. Lett. **79**, 2646 (1997).
[4] C. Schwob *et al.,* Phys. Rev. Lett. **82**, 4960 (1999).
[5] F. Biraben *et al.,* in *The Hydrogen Atom. Precision Physics of Simple Atomic Systems,* edited by S. G. Karshenboim *et al.* (Springer, Berlin, 2001), p. 17.
[6] G. Gärtner and E. Klempt, Z. Phys. A **287**, 1 (1978).
[7] G. Gräff, H. Kalinowsky, and J. Traut, Z. Phys. A **297**, 35 (1980).
[8] R. S. Van Dyck, Jr. and P. B. Schwinberg, Phys. Rev. Lett. **47**, 395 (1981).
[9] R. S. Van Dyck, Jr., F. L. Moore, D. L. Farnham, and P. B. Schwinberg, Int. J. Mass Spectrom. Process. **66**, 327 (1985).
[10] G. Gabrielse *et al.,* Phys. Rev. Lett. **65**, 1317 (1990).
[11] B. de Beauvoir *et al.,* Phys. Rev. Lett. **78**, 440 (1997).
[12] R. S. Van Dyck, Jr., D. L. Farnham, and P. B. Schwinberg, Phys. Scr. **T59**, 134 (1995).
[13] D. J. Wineland, J. J. Bollinger, and W. M. Itano, Phys. Rev. Lett. **50**, 628 (1983); **50**, 1333 (1983).
[14] L. Veseth, J. Phys. B **16**, 2891 (1983).
[15] H. Häffner *et al.,* Phys. Rev. Lett. **85**, 5308 (2000).
[16] N. Hermanspahn *et al.,* Phys. Rev. Lett. **84**, 427 (2000).
[17] S. A. Blundell, K. T. Cheng, and J. Sapirstein, Phys. Rev. A **55**, 1857 (1997).
[18] H. Persson, S. Salomonson, P. Sunnergren, and I. Lindgren, Phys. Rev. A **56**, R2499 (1997).
[19] T. Beier *et al.,* Phys. Rev. A **62**, 032510 (2000).
[20] T. Beier, Physics Rep. **339**, 79 (2000).
[21] H. Grotch and R. A. Hegstrom, Phys. Rev. A **4**, 59 (1971).
[22] S. G. Karshenboim, V. G. Ivanov, and V. M. Shabaev, Can. J. Phys. **79**, 81 (2001).
[23] S. G. Karshenboim, V. G. Ivanov, and V. M. Shabaev, Zh. Eksp. Teor. Fiz. **120**, 546 (2001) [JETP **93**, 477 (2001)].
[24] M. Diederich *et al.,* Hyperfine Interact. **115**, 185 (1998).
[25] H. Häffner *et al.* (to be published).
[26] R. S. Van Dyck, Jr., P. B. Schwinberg, and H. G. Dehmelt, Phys. Rev. D **34**, 722 (1986).
[27] L. S. Brown, Ann. Phys. (N.Y.) **159**, 62 (1985).
[28] V. W. Hughes and T. Kinoshita, Rev. Mod. Phys. **71**, 133 (1999).
[29] R. S. Van Dyck, Jr., P. B. Schwinberg, and H. G. Dehmelt, Phys. Rev. Lett. **59**, 26 (1987).
[30] A. Jeffrey *et al.,* Metrologia **35**, 83 (1998).
[31] H. Grotch, Phys. Rev. Lett. **24**, 39 (1970).
[32] S. G. Karshenboim, Phys. Lett. A **266**, 380 (2000).
[33] S. G. Karshenboim in *The Hydrogen Atom. Precision Physics of Simple Atomic Systems,* edited by S. G. Karshenboim *et al.* (Springer, Berlin, 2001), p. 651; hep-ph/0008227.
[34] V. M. Shabaev, Phys. Rev. A **64**, 052104 (2001).
[35] V. M. Shabaev and V. A. Yerokhin, physics/0110056, 2001 (http://xxx.lanl.gov) .
[36] A. Yelkhovsky, hep-ph/0108091, 2001 (http://xxx.lanl.gov).
[37] G. Breit, Nature (London) **122**, 649 (1928).
[38] D. A. Glazov and V. M. Shabaev, physics/0108046, 2001 (http://xxx.lanl.gov).
[39] J. Schwinger, Phys. Rev. **73**, 416 (1948).