

Unknowns after the SNO Charged-Current Measurement

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We perform a model-independent analysis of solar neutrino flux rates including the recent charged-current measurement at the Sudbury Neutrino Observatory (SNO). We derive a universal sum rule involving SNO and SuperKamiokande rates, and show that the SNO neutral-current measurement cannot fix the fraction of solar ν_e oscillating to sterile neutrinos. The large uncertainty in the standard solar model ^8B flux impedes a determination of the sterile neutrino fraction.

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The solar neutrino problem (SNP) is the discrepancy between the neutrino flux measured by solar neutrino experiments [1–6] and the predictions of the standard solar model (SSM) [7]. The SNP has defied non-particle-physics explanations [8]. The best-motivated solution is massive neutrinos with oscillations of solar electron neutrinos to mu and/or tau neutrinos. The SuperKamiokande (SK) experiment prefers solutions with large mixing between the mass eigenstates [9]. Very recently, the Sudbury Neutrino Observatory (SNO) Collaboration presented initial results of their charged-current (CC) measurement from about one year of operation, which again confirms the flux suppression [6]. The combination of SNO and SK data definitively establishes that the flux suppression of solar neutrinos is of particle physics origin, since it can be inferred that $\nu_{\mu,\tau}$ come from the Sun [6]. It is commonly believed that measurements of the neutral-current (NC) flux in the SNO experiment will decide whether oscillations to *sterile* neutrinos (that do not possess the standard model weak interaction) occur [10].

A motivating reason to postulate the existence of sterile neutrinos comes from the LSND accelerator experiment [11], which finds a $\nu_{\mu} \rightarrow \nu_e$ appearance probability of about 0.25%. To explain the solar and atmospheric [12] anomalies and the LSND data simultaneously, three distinct frequencies of oscillations are required. Since with the three known neutrinos there are only two independent oscillation frequencies, a fourth neutrino must be invoked. However, the invisible width of the Z boson places a constraint on the number of weakly interacting neutrinos to be very close to three [13]. The only way to evade this constraint is to require that the fourth neutrino be sterile. A recent combined analysis of solar and atmospheric data found that the active-sterile admixture can take any value between 0 and 1 at 99% C.L. for the preferred LMA (large mixing angle) solution to the SNP [14]. The SNO CC data are inconsistent with maximal mixing to sterile neutrinos at the 3.1σ level [6]. However, SNO did not address arbitrary active-sterile admixtures.

In this Letter, we perform a neutrino oscillation parameter-independent analysis of the solar neutrino rates in the ^{37}Cl [1], ^{71}Ga [2–4], and SK experiments, and

the recent CC measurement at SNO. The ^8B neutrinos represent a large fraction of the neutrinos incident at the SNO, SK, and ^{37}Cl experiments, as can be seen from Table I. Thus, the ^8B flux plays a crucial role in the interpretation of the results from these experiments. Unfortunately, the predicted value of the ^8B flux normalization is quite uncertain, mainly due to poorly known nuclear cross sections at low energies [15]. We find that if the fraction of solar ν_e that oscillate to sterile neutrinos is specified, the data determine the normalization of the ^8B solar neutrino flux. Alternatively, if the ^8B flux normalization is assumed to be that of the standard solar model (SSM), the range of the sterile neutrino fraction is determined. *However, the existing solar neutrino rate data and the forthcoming SNO NC measurement are not sufficient to determine the sterile neutrino content.* We discuss the additional measurements that are needed to determine, in a model-independent way, the oscillation probabilities and the fraction of solar ν_e that may be oscillating to sterile neutrinos.

Model-independent analysis.—Following the approach of Refs. [16] and [17] (in which we made the unique prediction $R_{\text{SNO}}^{\text{CC}} = 0.35_{-0.09}^{+0.16}$ for purely active oscillations with the SSM ^8B flux constraint), we divide the solar neutrino spectrum into three parts: high energy (consisting of ^8B and *hep* neutrinos), intermediate energy (^7Be , *pep*, ^{15}O , and ^{13}N), and low energy (*pp*). For each class of solar neutrino experiment the fractional contribution from each part of the unoscillated neutrino spectrum to the expected SSM rate can be calculated (see Table I). We define P_H , P_I , and P_L as the average oscillation probabilities for the high-, intermediate-, and low-energy solar neutrinos, respectively. We assume that the high-energy solar neutrino flux has absolute normalization β_H relative to the SSM calculation. If R is the measured rate divided by the SSM prediction for a given experiment, then with oscillations

$$R_{\text{Cl}} = 0.764\beta_H P_H + 0.236P_I, \quad (1)$$

$$R_{\text{Ga}} = 0.096\beta_H P_H + 0.359P_I + 0.545P_L, \quad (2)$$

$$R_{\text{SK}} = \beta_H P_H + r\beta_H \sin^2\alpha(1 - P_H), \quad (3)$$

TABLE I. Fractional contributions of the high-, intermediate-, and, low-energy neutrinos to the ^{37}Cl , ^{71}Ga , and SK signals without oscillations. The last column gives the 1σ normalization uncertainty for each part of the spectrum.

		^{37}Cl	^{71}Ga	SK SNO	Norm. Uncertainty
High	^8B , hep	0.764	0.096	1.000	18.0%
Inter.	^7Be , pep , ^{15}O , ^{13}N	0.236	0.359	0.000	11.6%
Low	pp	0.000	0.545	0.000	1.0%

$$R_{\text{SNO}}^{\text{CC}} = \beta_H P_H, \quad (4)$$

where $r \equiv \sigma_{\nu_\mu, \nu_\tau} / \sigma_{\nu_e} \approx 0.171$ is the ratio of the $\nu_{\mu, \tau}$ to ν_e elastic scattering cross sections on electrons. Here $\sin^2 \alpha$ is the fraction of ν_e that oscillates to active neutrinos, where α is a mixing angle in the four-neutrino mixing matrix that describes the linear combination of sterile and active neutrinos that participate in the solar neutrino oscillations. In the scheme of Eqs. (1)–(4), we are implicitly neglecting any small differences in the energy-dependent effects associated with the passage of active and sterile neutrinos through matter. We do not assign a normalization factor to the low-energy neutrinos because their flux uncertainty, which is constrained by the solar luminosity, is only 1% (see Table I). We also do not assign a normalization factor to the intermediate-energy neutrinos because it is likely that the uncertainties in this flux are well understood [15].

The solar neutrino data are summarized in Table II. We note that before the recent SNO CC result, the P_j were determined only if particular assumptions are made about the flux normalizations and sterile neutrino content [17]. With the addition of the SNO CC data, however, the quantities $\beta_H P_H$, P_I , and P_L can now be determined by R_{Cl} , R_{Ga} , and $R_{\text{SNO}}^{\text{CC}}$. We note that if the flux normalization β_H were known, the P_j would now be completely determined, regardless of the sterile content. This is because the ^{37}Cl , ^{71}Ga , and SNO CC measurements do not depend on whether the solar ν_e oscillate to active or sterile neutrinos. The SK data may be used to further constrain the parameters β_H , P_H , and $\sin^2 \alpha$, but without some assumption about either the ^8B flux normalization or sterile neutrino content there will still be one unconstrained degree of freedom. Thus there exists a family of solutions that fit the data exactly (with $\chi^2 = 0$), described by the relation

$$\sin^2 \alpha = (R_{\text{SK}} R_{\text{SNO}}^{\text{CC}}) / [r(\beta_H - R_{\text{SNO}}^{\text{CC}})], \quad (5)$$

TABLE II. Solar neutrino data expressed as the ratio $R = \text{data/SSM}$, including the experimental uncertainties. The ^{71}Ga number combines the results of the GALLEX, SAGE, and GNO experiments.

Experiment	Data/SSM
^{37}Cl	0.337 ± 0.030
^{71}Ga	0.584 ± 0.039
Super-K	0.459 ± 0.017
SNO CC	0.347 ± 0.028

shown in Fig. 1 by the solid curve. The amount of sterile content is not *a priori* known; in principle any value of $\sin^2 \alpha$ between zero and unity is still possible.

The 1σ and 2σ allowed regions from a fit to the rates in Table II with five parameters ($\sin^2 \alpha$, β_H , P_H , P_I , P_L) with the uncertainty in β_H determined by the fit, are also shown in Fig. 1. A pure sterile oscillation solution ($\sin^2 \alpha = 0$) is disfavored since experimentally $R_{\text{SK}} > R_{\text{SNO}}^{\text{CC}}$. However, for large enough β_H , $\sin^2 \alpha$ can be close to zero, although large flux normalizations are unlikely. For $\beta_H \leq 2$ (the 5σ bound from the SSM), we find that the pure sterile case ($\sin^2 \alpha = 0$) is not acceptable at the 2σ level. At the 1σ level and for $\beta_H \leq 2$, the values obtained from the above analysis are

$$\beta_H P_H = 0.35_{-0.07}^{+0.07}, \quad P_I = 0.31_{-0.31}^{+0.42}, \quad P_L = 0.81_{-0.33}^{+0.19},$$

$$0.6 \leq \beta_H \leq 2, \quad 0.14 \leq \sin^2 \alpha \leq 1. \quad (6)$$

These can be used to make statements about particular models of neutrino oscillations. For example, the LMA solution has the ordering $P_H \leq P_I \leq P_L$, while for the small mixing angle (SMA) solution P_I is significantly suppressed below both P_H and P_L . For $\beta_H \gtrsim 1.14$ the probability hierarchy of the LMA solution can be satisfied. Furthermore, the measured SNO spectrum appears to be undistorted compared to the SSM, which favors the LMA solution. The LOW solution has $P_H = P_I = P_L$, and thus is disfavored. At 2σ , the lowest allowed value of β_H is 0.47, which occurs for pure active mixing ($\sin^2 \alpha = 1$). The vacuum solution with $\delta m^2 \sim 5.5 \times 10^{-12} \text{ eV}^2$ and large mixing [18] is therefore barely acceptable at the 2σ level since the best-fit β_H for this solution is 0.47 (with very small uncertainties) [19].

In the near future SNO will also measure the NC reaction, which is related to the parameters by

$$R_{\text{SNO}}^{\text{NC}} = \beta_H P_H + \beta_H \sin^2 \alpha (1 - P_H). \quad (7)$$

Equations (3), (4), and (7) show that $R_{\text{SNO}}^{\text{NC}}$ does not provide independent information. There is in fact a universal sum rule:

$$R_{\text{SNO}}^{\text{NC}} = [R_{\text{SK}} - (1 - r)R_{\text{SNO}}^{\text{CC}}] / r$$

$$= 5.85 R_{\text{SK}} - 4.85 R_{\text{SNO}}^{\text{CC}}, \quad (8)$$

that holds for any value of $\sin^2 \alpha$ (this equation was known [17,19] for the case $\sin^2 \alpha = 1$). The SK and SNO data predict $R_{\text{SNO}}^{\text{NC}} = 1.00 \pm 0.24$. Although the SNO NC measurement will not provide a new constraint because the SK

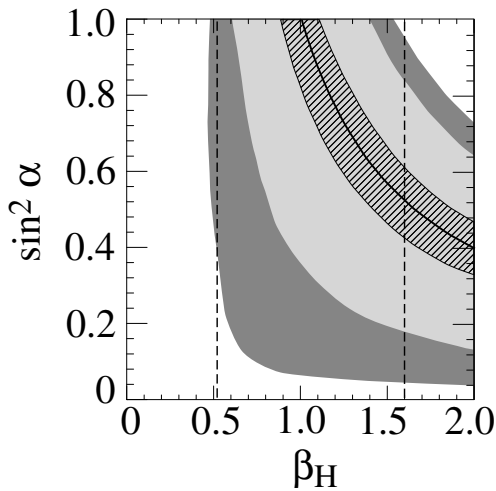


FIG. 1. Active neutrino fraction $\sin^2\alpha$ versus ${}^8\text{B}$ neutrino flux normalization β_H . The line represents solutions with $\chi^2 = 0$, and the 1σ and 2σ allowed regions are shaded. The hatched area is the 1σ allowed region if we replace the R_{SK} measurement by a hypothetical measurement $R_{\text{SNO}}^{\text{NC}} = 1 \pm 0.05$. The vertical dashes show the 3σ range allowed by the SSM.

data already supplies NC information, the SNO NC data could provide the more accurate measurement (since the $\nu_{\mu,\tau}$ NC cross sections are the same as that for ν_e , unlike in SK where they are much less). If we replace R_{SK} by $R_{\text{SNO}}^{\text{NC}} = 1 \pm 0.05$ (in anticipation of a measurement accurate to 5%–10% [20]), we find $\sin^2\alpha > 0.33$ at the 1σ level for $\beta_H \leq 2$. Another way to see why the SNO NC rate will not determine $\sin^2\alpha$ is to consider the ratio

$$R_{\text{SNO}}^{\text{NC}}/R_{\text{SNO}}^{\text{CC}} = 1 + \sin^2\alpha(1/P_H - 1). \quad (9)$$

Since P_H always appears in Eqs. (1)–(4) in the combination $\beta_H P_H$, $\sin^2\alpha$ cannot be extracted.

Can Borexino/KamLAND break the β_H , α degeneracy?—How then can the last degree of freedom (β_H or $\sin^2\alpha$) be eliminated? To make a *model-independent* determination of both β_H and $\sin^2\alpha$ (and hence also P_H), a different measurement that provides an independent constraint on the parameters must be used. For example, a measurement of the intermediate-energy solar neutrinos that involves a NC contribution such as in the Borexino [21] experiment or in the solar neutrino component of the KamLAND [22] experiment, would allow a separate determination of $\sin^2\alpha$ [23]. The resulting constraint would have the form $R_{B,K} = P_I + r \sin^2\alpha(1 - P_I)$, in analogy to Eq. (3), and would determine $\sin^2\alpha$. Values of $\sin^2\alpha$ and β_H [from Eqs. (1), (3), and (4)] are shown versus $R_{B,K}$ in Fig. 2. The value of β_H does not extend below about 1 because $\sin^2\alpha$ becomes greater than unity there. It is difficult for Borexino or KamLAND to determine $\sin^2\alpha$ and β_H because like SK, there is limited sensitivity to the NC component of the detected flux; the resulting uncertainty in $\sin^2\alpha$ would be $\delta \sin^2\alpha = \delta R_{B,K}/[r(1 - P_I)] \approx 8\delta R_{B,K}$.

Can adiabatic constraints break the β_H , α degeneracy?—If a particular model is assumed, then it can provide

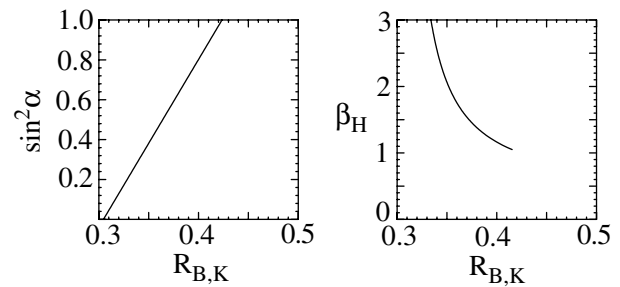


FIG. 2. Values of the active neutrino fraction $\sin^2\alpha$, and ${}^8\text{B}$ neutrino flux normalization β_H versus $R_{B,K}$.

the additional constraint to determine the parameters from current data. For example, if in the LMA solution all of the high-energy neutrinos and a fraction f of the intermediate-energy neutrinos are created above resonance, and a fraction $1 - f$ of the intermediate-energy neutrinos and all of the low-energy neutrinos are created below resonance, then since the neutrinos propagate adiabatically (i.e., the probability of jumping across the Landau-Zener-type level crossing from one adiabatic state to another is small) in the Sun we have approximately

$$P_H = \sin^2\theta, \quad P_L = 1 - \frac{1}{2} \sin^2 2\theta, \quad (10)$$

$$P_I = f \sin^2\theta + (1 - f)(1 - \frac{1}{2} \sin^2 2\theta), \quad (11)$$

where θ is the vacuum mixing angle and f can be directly related to the solar δm^2 (for a more detailed discussion, see Ref. [17]). Note that Eqs. (10) and (11) imply $P_H \leq P_I \leq P_L$ (for $\sin^2\theta \leq 1/2$). Now there are only four parameters (β_H , $\sin^2\alpha$, f , and θ), and all can be determined from the present data. Constraining $\beta_H \leq 2$, we find the best-fit point to be

$$\beta_H = 2.0, \sin^2\alpha = 0.42, \sin^2\theta = 0.17, f = 0.6, \quad (12)$$

with $\chi^2 = 0.51$. (There is a unique solution with zero χ^2 , but it has $\beta_H = 3.26$, which is unreasonably high [24].) The 1σ and 2σ allowed regions from a four-parameter fit are shown in Fig. 3. Note the similarity of the regions of Figs. 1 and 3; adiabatic constraints do not greatly help reduce the allowed region.

Including the SSM constraint on the ${}^8\text{B}$ flux.—To include the ${}^8\text{B}$ flux normalization as calculated in the SSM [7], we perform χ^2 analyses with $\beta_H = 1 \pm 0.18$. The results are shown in Fig. 4. The model-independent analysis yields a unique point with $\chi^2 = 0$ at $(\beta_H, \sin^2\alpha) = (1.0, 1.0)$. The left panel of Fig. 4 shows that the $\sin^2\alpha$ range is not improved. However, as shown in the right panel of Fig. 4, imposition of the ${}^8\text{B}$ flux constraint in addition to adiabatic constraints, does lead to a smaller $\sin^2\alpha$ range. In this case, the best-fit parameters are

$$\beta_H = 1.1, \sin^2\alpha = 1.0, \sin^2 2\theta = 0.83, f = 0.15, \quad (13)$$

with $\chi^2 = 3.1$. Thus, for this solution mainly high-energy neutrinos are created above resonance and the critical energy [17] lies close to the *pep* line at 1.44 MeV, which translates to $\delta m^2 = 4.8 \times 10^{-5} \text{ eV}^2$.

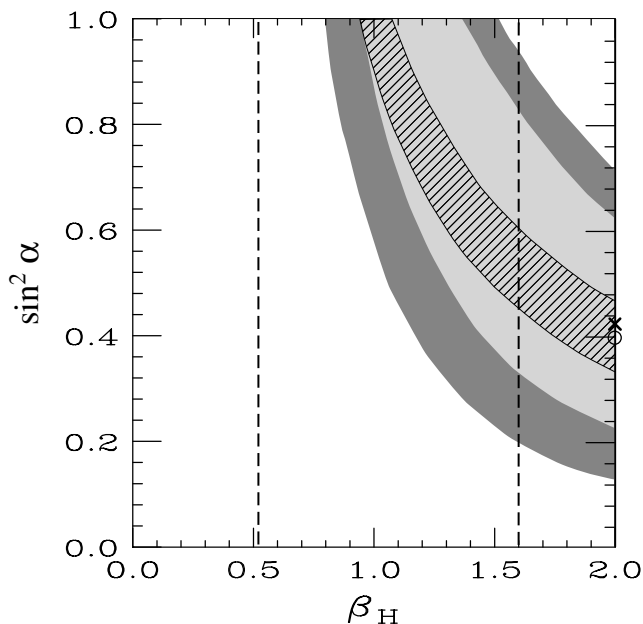


FIG. 3. $\sin^2\alpha$ versus β_H with adiabatic constraints imposed. The cross marks the best-fit point $(\beta_H, \sin^2\alpha) = (2.0, 0.42)$ for $\beta_H \leq 2$; the 1σ and 2σ allowed regions are shaded. With R_{SK} replaced by $R_{SNO}^{NC} = 1 \pm 0.05$, the best-fit shifts slightly to the circle and the hatched region is the 1σ allowed region. The dashed lines show the 3σ range allowed by the SSM.

Summary.— After including the recent SNO CC results in a model-independent analysis of solar neutrino flux rate data, there remains one free parameter. The locus of solutions may be represented by a curve in the plane of the active neutrino fraction, $\sin^2\alpha$, and the ^8B neutrino flux normalization β_H . We have shown that the forthcoming SNO NC data will not fully constrain the last degree of freedom; in fact, there is a universal sum rule involving R_{SNO}^{NC} , R_{SNO}^{CC} , and R_{SK} that must be satisfied, independent of the sterile neutrino content of the solar neutrino flux. The adiabatic constraint for the LMA does not appreciably

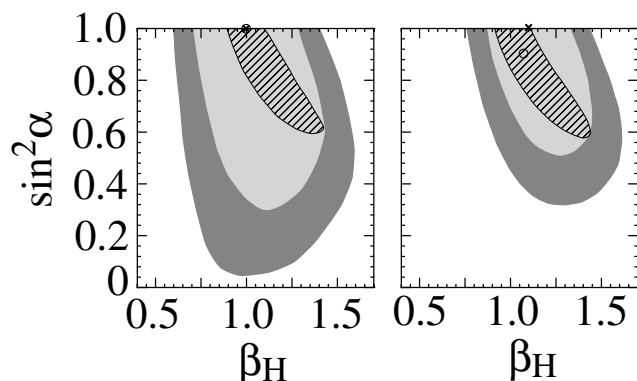


FIG. 4. $\sin^2\alpha$ versus β_H with the ^8B flux constraint imposed. The left panel shows the result of a model-independent analysis, and the right panel has adiabatic constraints in addition. The crosses mark the best-fit points; the 1σ and 2σ allowed regions are shaded. The hatched areas are the 1σ allowed regions if we replace R_{SK} by $R_{SNO}^{NC} = 1 \pm 0.05$; the circles mark the corresponding best-fit points.

reduce the allowed region in $\sin^2\alpha$. Even when we impose the SSM ^8B flux constraint, the sterile neutrino fraction is not determined.

In principle, measurements of ν_e scattering for the intermediate-energy neutrinos in Borexino/KamLAND could break the degeneracy of allowed solutions, but because the NC sensitivity of these experiments is relatively weak, a very precise measurement would be required to determine $\sin^2\alpha$ and β_H . What is needed is a measurement of neutrino-nucleon NC scattering for the intermediate-energy neutrinos.

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- [1] B. Cleveland *et al.*, *Astropart. Phys.* **496**, 505 (1998).
- [2] J. Abdurashitov *et al.*, *Phys. Rev. C* **60**, 055801 (1999).
- [3] W. Hampel *et al.*, *Phys. Lett. B* **447**, 127 (1999).
- [4] M. Altmann *et al.*, *Phys. Lett. B* **490**, 16 (2000).
- [5] S. Fukuda *et al.*, *Phys. Rev. Lett.* **86**, 5651 (2001).
- [6] Q. Ahmad *et al.*, *Phys. Rev. Lett.* **87**, 071301 (2001).
- [7] J. Bahcall, M. Pinsonneault, and S. Basu, *Astrophys. J.* **555**, 990 (2001).
- [8] J. Bahcall and H. Bethe, *Phys. Rev. D* **47**, 1298 (1993); N. Hata, S. Bludman, and P. Langacker, *Phys. Rev. D* **49**, 3622 (1994); K. Heeger and R. Robertson, *Phys. Rev. Lett.* **77**, 3720 (1996).
- [9] S. Fukuda *et al.*, *Phys. Rev. Lett.* **86**, 5656 (2001).
- [10] J. Bahcall, P. Krastev, and A. Yu. Smirnov, *Phys. Rev. D* **63**, 053012 (2001); M. Gonzalez-Garcia and C. Pena-Garay, *ibid.* **63**, 073013 (2001); J. Bahcall, P. Krastev, and A. Yu. Smirnov, *ibid.* **62**, 093004 (2000); M. Maris and S. Petcov, *ibid.* **62**, 093006 (2000); S. Bilenkii and C. Giunti, *Z. Phys. C* **68**, 495 (1995).
- [11] A. Aguilar *et al.*, hep-ex/0104049.
- [12] Y. Fukuda *et al.*, *Phys. Lett. B* **467**, 185 (1999); W. Allison *et al.*, *ibid.* **449**, 137 (1999); M. Ambrosio *et al.*, *ibid.* **517**, 59 (2001).
- [13] D. Groom *et al.*, *Eur. Phys. J.* **C15**, 1 (2000).
- [14] M. Gonzalez-Garcia, M. Maltoni, and C. Pena-Garay, *Phys. Rev. D* **64**, 093001 (2001).
- [15] J. Bahcall, *Neutrino Astrophysics* (Cambridge University Press, Cambridge, England, 1989).
- [16] V. Barger, R. Phillips, and K. Whisnant, *Phys. Rev. D* **43**, 1110 (1991).
- [17] V. Barger, D. Marfatia, and K. Whisnant, *Phys. Lett. B* **509**, 19 (2001).
- [18] R. Raghavan, *Science* **267**, 45 (1995); P. Krastev and S. Petcov, *Phys. Rev. D* **53**, 1665 (1996).
- [19] J. Bahcall, P. Krastev, and A. Yu. Smirnov, *J. High Energy Phys.* **0105**, 015 (2001).
- [20] A. McDonald and S. Oser (private communication).
- [21] C. Arpesella *et al.*, edited by G. Bellini *et al.* (University of Milano Press, Milano, 1992), Vols. 1 and 2.
- [22] KamLAND proposal, Stanford-HEP-98-03.
- [23] A. Gouvea and H. Murayama, *J. High Energy Phys.* **0008**, 025 (2000); *Phys. Rev. Lett.* **82**, 3392 (1999).
- [24] P. Krastev, S. Petcov, and L. Qiuyu, *Phys. Rev. D* **54**, 7057 (1996).