Counting Statistics for Arbitrary Cycles in Quantum Pumps

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We study noise-related properties of current in adiabatic pumps. A symmetry of the problem allows us to relate the statistics of charge transport in the case of one-channel leads to the geometry of loops on a sphere (for many channels on a higher-dimensional manifold). This provides a unifying framework, which simplifies analysis of transport in various realizations of pumps. For each pumping cycle, the average current and its minimal variance are given by the areas enclosed by the corresponding loop on the sphere and on a minimal surface (soap film) spanned by this loop. We formulate conditions for quantization of the pumped charge.

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Adiabatic charge pumping has attracted considerable attention recently [1], largely motivated by the experiment of Switkes *et al.* [2]. A cyclic modulation of gate potentials can produce a dc current through an open system. In contrast to pumps in the Coulomb-blockade regime [3,4], pumping through open systems is governed by quantum effects. Analysis of the transport noise properties, which probe fundamental features of these Fermi systems, is important for potential applications.

In a system coupled to reservoirs, the excitation gap vanishes violating the true adiabaticity [5]. Still a compact description, in terms of a scattering matrix $S(t)$, is possible if the driving fields change weakly on the scattering time scale. Under this condition, the average current is a geometric property of the loop traversed by $S(t)$ in the unitary group [6]. Recently, mesoscopic fluctuations [7–10], the role of discrete symmetries [8,11], and charge quantization [7,8,12,13] were studied.

The full description of the noise requires the knowledge of the counting statistics. The probability $P(Q)$ of pumping the charge Q per cycle can be formally expressed via the determinant of an integral operator involving $S(t)$ [14]. While compact expressions were found for some pumping cycles [14–16], a general case requires further analysis.

Let us begin by sketching our main results (valid also in the presence of a voltage bias, which can be gauged away at the expense of a phase of the scattering matrix). Consider a system with two M-channel leads. We show that pumping cycles $S(t) \in U(2M)$ induce identical statistics $P(Q)$ if they differ only by separate rescattering of the left and right outgoing states. Thus, instead of loops in $U(2M)$, we can study loops $\mathbf{N}(t) = S^\dagger \sigma_3 S$ in a smaller, coset space $U(2M)/U(M) \times U(M)$. In the single-channel case, $\mathbf{N}(t) = \mathbf{n}\boldsymbol{\sigma}$ reduces to a loop $C = \{\mathbf{n}(t)\}$ on the unit sphere in 3D (see Fig. 1). We find that at low temperature the average pumped charge [17] is given by the area enclosed by C on the sphere (cf. Ref. [18]; we set the elementary charge $e = 1$),

$$\langle Q \rangle = A_{\text{sphere}}/4\pi \,, \tag{1}$$

which is defined modulo 1. A way of fixing the integer part of $\langle Q \rangle$ is discussed below.

If the loop $\mathbf{n}(t)$ [but not necessarily $S(t)$] is small, the pumped charge is quantized. Indeed, in Refs. [8,13] quantization was found under these conditions.

The current noise can also be expressed in geometric terms, namely, as an integral over the time disk defined as follows: For driving at frequency ω we replace the time axis by a unit circle C_t: $w = e^{i\omega t}$. The mapping $\mathbf{n}(w)$ from C_t onto the sphere has a unique harmonic ($\Delta \mathbf{n} = 0$) extension into the disk D_t. In these terms, the noise is given by

$$\langle\langle Q^2 \rangle\rangle = \frac{1}{8\pi} \int_{D_t} (\partial_i \mathbf{n})^2 \, d^2x, \qquad w = x_1 + ix_2 \,. \tag{2}$$

For a given contour C, the details of its traversal in time are determined by the shape of the driving pulse. Unlike the average charge (1), the dispersion (2) is sensitive to these details. Pulse shapes minimizing the noise were found in several cases [14,16]. Here we solve the problem of noise optimization for an arbitrary cycle. Specifically, we show that the minimal noise value [17] is the area of the minimal surface (soap film) spanned by C (see Fig. 1):

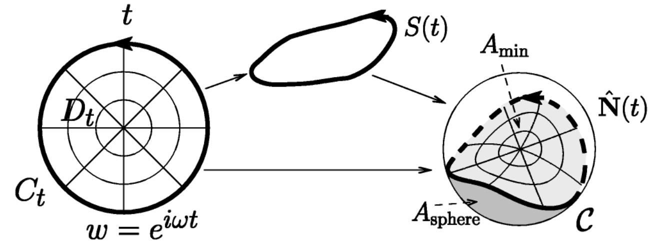

FIG. 1. A pumping cycle $S(t)$ defines a contour $\mathbf{N}(t)$ in the coset space, which determines the statistics. The areas enclosed by $\mathbf{N}(t)$ in the coset space, A_{sphere}, and on the minimal surface, $A_{\min}$, define the average current and the noise.

$$\langle\langle Q^2\rangle\rangle_{\min} = A_{\min}/4\pi\,. \qquad (3)$$

It is reached in a cycle for which the mapping $\mathbf{n}(w)$ is conformal, $(\partial_w \mathbf{n})^2 = 0$. Similar results hold in the many-channel case (see below).

Invariance.—We show that the statistics are invariant under a local symmetry group and are determined by the path in the corresponding coset space. Namely, the transformation

$$S(t) \to U(t)S(t), \qquad U(t) = \begin{pmatrix} U_L(t) & 0 \\ 0 & U_R(t) \end{pmatrix} \qquad (4)$$

just shifts the distribution $P(Q)$ by an integer, the relative winding number of the overall phases of U_L, U_R:

$$P(Q) \to P(Q - W), \qquad W = \frac{1}{8\pi i}\oint \mathrm{Tr}(dU\,U^\dagger \sigma_3)\,, \qquad (5)$$

where $\sigma_3 = \mathrm{diag}\{1_M, -1_M\}$. In particular, $P(Q)$ is invariant if the loop is trivial, $W = 0$.

Physically, multiplication of the scattering matrix (4) by $U(t)$ just redistributes the scattered particles between the left channels (U_L) and between the right channels (U_R), without affecting correlations at the scattering center. The outgoing states acquire an extra time-dependent phase which changes the time these particles need to reach the reservoirs. As a result, the extra charge $W_a \equiv \oint \mathrm{Tr}(dU_a^\dagger U_a)/4\pi i$ is transferred to the lead $a =$ L or R, and we get (5). For periodic $U(t)$ these numbers are integers. (Note that no net charge accumulation near the scatterer implies $W_L = -W_R$.)

Formally, the rule (5) for $\langle Q\rangle$ follows from Eq. (9) below [6]. For higher cumulants, we use the result [14,16] for the generating function $\chi(\lambda) = \sum_Q P(Q)e^{i\lambda Q}$,

$$\chi(\lambda) = \det\{1 + n_F(t',t)[S^\dagger_{-\lambda}(t)S_\lambda(t) - 1]\}. \qquad (6)$$

Here $S_\lambda(t) \equiv e^{-i\lambda\sigma_3/4}S(t)e^{i\lambda\sigma_3/4}$ and $n_F(t',t) = i/[2\pi(t' - t + i0)]$ is the Fourier transform of the Fermi distribution. In Ref. [16], by separating phases and amplitudes of $S(t)$, this result was presented in a form, implying the rule (5). Indeed, the determinant in Eq. (5) of Ref. [16] is invariant under (4) and the quantity $\hat{N}$ in the prefactor is shifted by W.

To express $\chi(\lambda)$ via $\mathbf{N}$, we notice that $S^\dagger_{-\lambda}S_\lambda = e^{i\lambda\sigma_3/4}S^\dagger e^{-i\lambda\sigma_3/2}Se^{i\lambda\sigma_3/4} = e^{i\lambda\sigma_3/4}e^{-i\lambda S^\dagger\sigma_3 S/2}e^{i\lambda\sigma_3/4}$. Using the identity $e^{-i\lambda\mathbf{N}/2} = \cos\frac{\lambda}{2} - i\sin\frac{\lambda}{2}\mathbf{N}$, we get

$$\chi(\lambda) = \det\{1 - \tfrac{1}{2}n_F(t',t)(e^{i\lambda\sigma_3} - 1)\sigma_3[\mathbf{N}(t) - \sigma_3]\}. \qquad (7)$$

At $T = 0$, multiplying by $1 + n_F(e^{-i\lambda\sigma_3/2} - 1)$, we obtain

$$\chi(\lambda) = \det[1 + n_F(t',t)(e^{-i\lambda\mathbf{N}(t)/2} - 1)]. \qquad (8)$$

Further, the result (8) is explicitly invariant under global rotations $\mathbf{N}(t) \to V^\dagger\mathbf{N}(t)V$ [corresponding to transformations [19] $S(t) \to S(t)V$].

Equations (6)–(8) involve $\mathbf{N}$, but not S, and, hence, can define $P(Q)$ only up to an integer offset. Indeed, the infinite product of the eigenvalues of these integral operators can be regularized in many ways [15]. Notice that, for the operator (8), due to strong degeneracy, one can choose eigenstates each spanning a narrow frequency range, of order ω. For those far above the Fermi level ($n_F = 0$) the eigenvalues are 1. Deep in the Fermi sea ($n_F = 1$), the eigenvalues appear in pairs $e^{\pm i\lambda/2}$ with product 1. Though regularization procedures can pair them in different ways, this can change $\chi(\lambda)$ only by an even power of $e^{\pm i\lambda/2}$, which gives an integer shift of Q.

Pumped charge.—The regularization of the expression for $\langle Q\rangle = \partial_\lambda\chi(\lambda = 0)$ requires the knowledge of the full $S(t)$ and gives an integral over the period $C = \{S(t)\}$ [6],

$$\langle Q\rangle = \frac{1}{4\pi i}\oint_C \mathrm{Tr}(\sigma_3\,dS\,S^\dagger)\,. \qquad (9)$$

The loop C can be shrunk to a point, uniquely up to continuous deformations. In the process it spans a surface D. (For a two-parametric pump [6] D can be constructed by shrinking the contour in the parameter plane.) Using Stokes' theorem, we rewrite (9) as a surface integral, which further reduces to the "area" of the corresponding surface $\mathcal{D}$ in the coset space,

$$\langle Q\rangle = \int_D \frac{\mathrm{Tr}(\sigma_3\,dS \wedge dS^\dagger)}{4\pi i} = \int_{\mathcal{D}} \frac{\mathrm{Tr}(\mathbf{N}\,d\mathbf{N} \wedge d\mathbf{N})}{16\pi i}\,. \qquad (10)$$

Note that the integrand is the curvature of the fiber bundle $S \to \mathbf{N} = S^\dagger\sigma_3 S$. In the single-channel case $\mathbf{N} = \mathbf{n}\boldsymbol{\sigma}$, and we obtain (1):

$$\langle Q\rangle = \frac{1}{8\pi}\int_{\mathcal{D}} \epsilon_{ijk}n_i\,dn_j \wedge dn_k\,. \qquad (11)$$

One can try to define the "integral part" Q_{int} of $\langle Q\rangle$ as follows: Let us parametrize scattering matrices as $S = US^0[\mathbf{N}]$ with a matrix S^0, defined for any $\mathbf{N}$, and a matrix U as in Eq. (4), and assign to each cycle $S(t)$ the winding number of the corresponding $U(t)$. This attempt fails, since there is no continuous global map $S^0[\mathbf{N}]$. In fact, any two loops in $U(2M)$ can be deformed into each other, i.e., any Q_{int} is discontinuous under certain contour deformations. However, continuous maps $S^0[\mathbf{N}]$ do exist for contractible regions, and one can introduce Q_{int} for contours $S(t)$, for which $\mathbf{N}(t)$ does not leave such a region. Examples are regions of matrices S without perfectly transmitting (or reflecting) channels. In particular, the integer $\hat{N}$ introduced in Ref. [16] changes abruptly when the pumping cycle contains a scattering matrix $S(t)$ with a perfectly transmitting channel.

At this point, we formulate sufficient conditions for the *quantization* of the pumped charge: The fractional part of $\langle Q\rangle$ vanishes for small contours $\mathbf{N}(t)$ in the coset space. An example: The minimal and maximal conductance, $g = 0$ and $g = M$, is achieved at the points $\mathbf{N} = \pm\sigma_3$. Hence, keeping g close to one of these values throughout the cycle guarantees the quantization. Evaluating the integral (10) for such cycles, we estimate the accuracy of quantization as $\delta Q \lesssim g$ for $g \approx 0$, and $\delta Q \lesssim M - g$ for $g \approx M$.

In the particular case of a single channel, $M = 1$, the scattering matrix can be parametrized by the conductance g and three phases:

$$S(g, \alpha, \beta, \varphi) = e^{i\varphi/2}\begin{pmatrix} \sqrt{1-g}\, e^{i\alpha} & i\sqrt{g}\, e^{i\beta} \\ i\sqrt{g}\, e^{-i\beta} & \sqrt{1-g}\, e^{-i\alpha} \end{pmatrix}. \tag{12}$$

The components of the unit vector $\mathbf{n}$ then are

$$n_z = 1 - 2g; \qquad n_x + in_y = -2i\sqrt{g(1-g)}\, e^{i(\alpha-\beta)}. \tag{13}$$

Using these expressions we can explain the charge quantization found, for instance, in Refs. [8,13]: For the pumping cycles studied, the system encircled the resonance point $g = 1$ in the parameter plane at a sufficient distance from it so that $g \approx 0$ throughout the cycle. The corresponding loop $\mathbf{n}(t)$ encircled the north pole ($g = 0$; Fig. 2). Since the interior of the loop in the parameter plane contained the resonance point, the surface $\mathcal{D}$ in (11) covered the *lower* part of the sphere, i.e., almost the whole sphere, $\langle Q \rangle \approx 1$.

Noise optimization.—For the noise, given by the λ^2 term in the Taylor series of $\ln\chi(\lambda)$, we obtain a double integral [14,16] over the unit circles $w = e^{i\omega t}$, $w' = e^{i\omega t'}$:

$$\langle\langle Q^2 \rangle\rangle = \frac{1}{32\pi^2} \oint\oint \frac{dw\, dw'}{(w - w')^2} \mathrm{Tr}\{[\mathbf{N}(t) - \mathbf{N}(t')]^2\}. \tag{14}$$

It can be compactly written in terms of the Fourier series $\mathbf{N}(t) = \sum_{k\geq 0} \mathbf{N}_k \exp(ik\omega t) + \mathrm{H.c.}$:

$$\langle\langle Q^2 \rangle\rangle = \frac{1}{4} \sum_{k>0} k\, \mathrm{Tr}[\mathbf{N}_k^\dagger \mathbf{N}_k]. \tag{15}$$

Furthermore, the mapping,

$$w \to \mathbf{N}(w) = \sum_{k\geq 0} \mathbf{N}_k w^k + \mathrm{H.c.}, \tag{16}$$

which is the (unique) harmonic extension of the mapping $t \to \mathbf{N}(t)$ from the circle into the disk $D_t : |w| < 1$, allows us to express the noise as the integral over D_t:

$$\langle\langle Q^2 \rangle\rangle = \frac{1}{16\pi} \int_{D_t} d^2x\, \mathrm{Tr}[(\partial_i \mathbf{N})^2], \qquad w = x_1 + ix_2, \tag{17}$$

which reduces to (2) in the single-channel case. Note that the integral (17) is well defined for any surface. Among the maps $\mathbf{N}(w)$ with the fixed value $\mathbf{N}(t)$ at the boundary the minimum is achieved for the harmonic surface (16).

Now we turn to optimization of pumping: The cyclic evolution of $S(t)$ is achieved by periodic changes in external parameters that control the scattering or the bias. For a fixed trajectory in the parameter space, varying the rate of motion does not affect the average pumped charge (9), (11) but does influence the noise. The noise (17) is bounded from below by the area of the surface $\mathbf{N}(w)$ [defined by the scalar product $\langle A, B \rangle \equiv \mathrm{Tr}(A^\dagger B)/2$]:

$$\langle\langle Q^2 \rangle\rangle \geq A[\mathbf{N}(w)]/4\pi\,. \tag{18}$$

Indeed, area spanned by $\partial_1 \mathbf{N}$ and $\partial_2 \mathbf{N}$ in the matrix space $\leq \frac{1}{2}(|\partial_1 \mathbf{N}|^2 + |\partial_2 \mathbf{N}|^2) = \frac{1}{4}\mathrm{Tr}[(\partial_i \mathbf{N})^2]$. The equality is achieved only if $\partial_i \mathbf{N}$ are orthogonal and have the same length, or, equivalently, $\mathrm{Tr}[(\partial_w \mathbf{N})^2] = 0$. This condition defines *conformal* mappings $\mathbf{N}(w)$.

The minimal noise value is given by the *minimal area* (3) of a surface spanned by the loop $\mathbf{N}(t)$, since both sides of Eq. (18) are minimized by a conformal harmonic surface [20]. Such a mapping also provides the optimal shape of the pumping pulse. Not surprisingly, the classes of harmonic and conformal maps are preserved by the $\mathrm{SL}_2(\mathbb{R})$ time-reparametrization symmetry [15], $w \to (w + a)/(1 + \bar{a}w)$. For N cycles $\mathbf{N}_N(w) \equiv \mathbf{N}_1(w^N)$ and $\mathbf{N}_1[\prod_{i=1}^N (w + a_i)/(1 + \bar{a}_i w)]$ give the same statistics.

Applications.—Our findings give a new perspective on the analysis of pumping cycles discussed in the literature. Consider the cycles

$$S_\beta(t) = e^{-i\phi(t)\sigma_3/2} S(0) e^{i\phi(t)\sigma_3/2}, \tag{19}$$

$$S_\alpha(t) = e^{i\phi(t)\sigma_3/2} S(0) e^{i\phi(t)\sigma_3/2}, \qquad \Delta\phi = 2\pi N\,. \tag{20}$$

The first of them, studied extensively by Levitov *et al.* [14,21] describes conductors under the voltage bias $-\hbar\dot{\phi}(t)/e$, as one can see by applying a gauge transformation. The cycle S_α was discussed in Ref. [16]. These cycles differ only by a transformation (4) with $U(t) = e^{i\phi(t)\sigma_3}$, hence, the statistics coincide up to a shift by the winding number: $P_\beta(Q - N) = P_\alpha(Q)$. Indeed, the same pulse shape $\phi(t) = \omega t$ [and others, generated by $\mathrm{SL}_2(\mathbb{R})$] was found optimal for both cycles. The statistics for this optimal cycle $S_\alpha(t)$ are related to the well-known binomial distribution for a conductor under a constant *positive* [22] bias by $P_\beta^{\mathrm{opt}}(N - Q) = P_\alpha^{\mathrm{opt}}(Q)$, in agreement with Ref. [16]. For a single channel, the vector $\mathbf{n}(t)$ follows a line of constant latitude for both S_β and S_α: $\beta(t) - \beta(0) = \phi(t)$ or $\alpha(0) - \alpha(t) = \phi(t)$. The pumped charge $\langle Q \rangle$ is given by the area above this line, g, for (19), and below this line, $1 - g$, for (20).

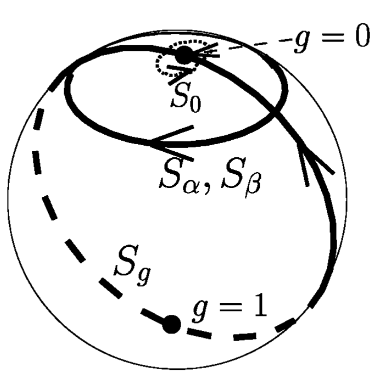

FIG. 2. Several pumping cycles $\mathbf{n}(t)$: S_β [21] and S_α [16] lead to the same counting statistics, which at $g = 1/2$ coincide with that for S_g [16]. The sketched contour S_0 [8,13] describes pumping of a unit charge per cycle.

The minimal noise value (3) is the area $g(1 - g)$ of the sphere's cross section.

The rotational invariance of $\chi(\lambda)$ implies that the counting statistics for any circle $\mathbf{n}(t)$ is the same as for biased conductors. In particular, the optimal pumping uniformly traverses the circle and gives rise to a binomial distribution. As an example, consider the cycle

$$S_g(t) = \begin{pmatrix} \cos\eta(t) & \sin\eta(t) \\ \sin\eta(t) & -\cos\eta(t) \end{pmatrix}, \qquad \Delta\eta = 2\pi, \quad (21)$$

during which the conductance $g = \sin^2\eta(t)$ oscillates. We find that $\langle Q \rangle = 0$ and $\mathbf{n}(t)$ traverses twice the meridian in Fig. 2. Thus, $P(Q)$ coincides with the distribution for the equator, S_β at $g = 1/2$, and the pulse $\eta(t) = \omega t$ from Ref. [16] is optimal. For this pulse $P(Q)$ is the shifted binomial distribution for $N = 2$ cycles, $P_g^{\mathrm{opt}}(Q) = P_\beta^{\mathrm{opt}}(Q + 1)|_{N=2}^{g=1/2} = \frac{1}{4}\binom{2}{Q+1}$, in agreement with Ref. [16].

Our geometric approach allows us to obtain relations between current and noise for broad classes of pumping cycles. For small loops $\mathbf{n}(t)$, the minimal surface lies on the sphere and the analysis simplifies. If the loop $S(t)$ is also small, the system is in the weak-pumping regime with $\langle Q \rangle, \langle\langle Q^2 \rangle\rangle \ll 1$, and for a general (possibly self-intersecting) loop we have $|\langle Q \rangle| = |A_+ - A_-|/4\pi \le \langle\langle Q^2 \rangle\rangle_{\min} = (A_+ + A_-)/4\pi$, where $A_\pm$ are the contributions to the enclosed area with positive (respectively, negative) orientations. The weak-pumping regime was studied very recently by Levitov [19]. He found that the transport is described by two uncorrelated Poisson processes, which transport charge to the right and to the left and in some cases reduce to a single process. Our inequality $\langle\langle Q^2 \rangle\rangle \ge |\langle Q \rangle|$ is in agreement with these findings, $A_\pm/4\pi$ being the rates of the two Poisson processes for an optimal cycle. The equality, the criterion for the reduction to a single Poisson process in the weak-pumping regime, is thus reached only for optimally traversed loops enclosing the area of a constant orientation (in particular, for non-self-intersecting loops). The example of such a cycle considered in Ref. [19] corresponds in our terms to $\mathbf{n}(t)$ traversing uniformly a small circle. Generally, for weak harmonic driving $\mathbf{n}(t)$ encircles an ellipse. The optimal pulse shape, given by the conformal map of D_t onto this ellipse, involves elliptic integrals [23]. Further, for a general small polygon the optimal pumping is given by the Schwarz-Christoffel formula, describing a map of D_t onto its interior (also reducing to elliptic integrals for a rectangle) [23].

The results for the weak-pumping regime can be generalized to the many-channel case. Using local complex coordinates in the coset space, we find that $\langle\langle Q^2 \rangle\rangle/|\langle Q \rangle| \ge 1$, the equality (corresponding to a single Poisson process) being reached only for optimal cycles with a complex analytic (or antianalytic) minimal surface $\mathbf{N}(w)$.

For the strong-pumping regime our description also gives new results. For instance, in the interesting case of a single channel and a contour $\mathbf{n}(t)$ without self-intersection, we find that $\langle\langle Q^2 \rangle\rangle_{\min} \le$ distance from $\langle Q \rangle$ to closest integer.

In conclusion, we have linked the counting statistics of charge pumping through an open (possibly voltage-biased) system to geometric properties of a loop on the sphere (or on its generalization in the many-channel case), induced by periodic variation of the scattering matrix. The average pumped charge and its minimal variance for an arbitrary pumping cycle are given by the area encircled by this loop on the sphere and the area of the minimal surface spanned by this loop, respectively. We have also found the shape of the driving pulse that optimizes the noise. Our results represent a unifying framework for analysis of transport statistics in various realizations of pumping.

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