Quantized Adiabatic Charge Transport in a Carbon Nanotube

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The coupling of a semimetallic carbon nanotube to a surface acoustic wave (SAW) is proposed as a vehicle to realize quantized adiabatic charge transport. We demonstrate that electron backscattering from a periodic SAW potential can be used to induce a miniband spectrum at energies near the Fermi level. Within the framework of Luttinger liquid theory, electron interaction is shown to enhance minigaps and thereby improve current quantization.

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The mechanism of quantized adiabatic transport, as first conceived by Thouless [1], involves a one-dimensional (1D) electron system subject to a periodic potential. If the potential varies slowly and periodically in time so that the Fermi level lies within a (perturbation induced) minigap of the instantaneous Hamiltonian, then an integer charge me is transported across the system during a single period. This results in a quantized current $j = mef$, where f is the frequency of the external field. If realized experimentally, such a device would present an important application as a current standard.

Electron properties of real 1D conductors, such as nanotubes or quantum wires, are dominated by electron interactions [2–6]. However, leaving aside general statements [7] about robustness of the quantization, the effect of interactions on quantized transport has not been explored. By investigating a theory of the interacting 1D system, the aim of this Letter is to establish the possibility to realize the regime of quantized adiabatic transport in metallic nanotubes, the purest 1D conductor [8–10] currently available.

Although the mechanism of quantized adiabatic transport is compelling in its simplicity, it has proven difficult to realize experimentally: its successful execution demands the fabrication of a host 1D system coupled to a sliding external perturbation to engineer a miniband spectrum with minigaps sufficiently large, so that disorder, thermal excitations, and finite size effects do not compromise the integrity of the quantization. Among existing 1D systems, one possibility is to use quantum wires coupled to a surface acoustic wave (SAW). The SAW field can be made strong enough to induce a gap, and the SAW wave number can be chosen to match $2p_F$ to pin electrons. However, since the densities for which adiabatic transport is most pronounced correspond to a few electrons per SAW spatial period (realistically, ca. a few microns), one would need wires with low electron 1D density of around 10^4 cm^{-1}. The densities currently available in such systems are at least an order of magnitude higher [11]. These difficulties have stimulated work on alternative mechanisms of current quantization [12] in GaAs split gate devices.

In this Letter we argue that a SAW coupled to a semimetallic carbon nanotube presents an ideal system in which quantized transport can be realized. The experimental arrangement is depicted in Fig. 1. A nanotube is placed between two metallic contacts on the surface of a piezoelectric crystal with a gate electrode nearby to allow adjustment of the Fermi level in the tube. In a piezoelectric substrate the SAW is accompanied by a wave of electrostatic potential that can have an amplitude of up to a few volts [13]. The potential decays both into the free space and substrate to a depth comparable with the wavelength λ_{SAW}. We assume that the tube is suspended at a height $\ll\lambda_{\mathrm{SAW}}$ above the substrate, so that there is no direct mechanical coupling and only the free space component of the SAW potential matters. When a SAW is launched from a transducer (such as an interdigitated electrode array) its electric field penetrates the tube and electron diffraction on the sliding SAW potential results in miniband formation. By positioning the Fermi level within the energy gap, the conditions for current quantization are fulfilled.

The high electron velocity ($v \approx 8 \times 10^7$ cm/s) in nanotubes makes it possible to obtain large minigaps. For a SAW-induced grating of period 200–300 nm, a crude estimate suggests minigap size $hv/\lambda_{\mathrm{SAW}} \sim 10$ meV.

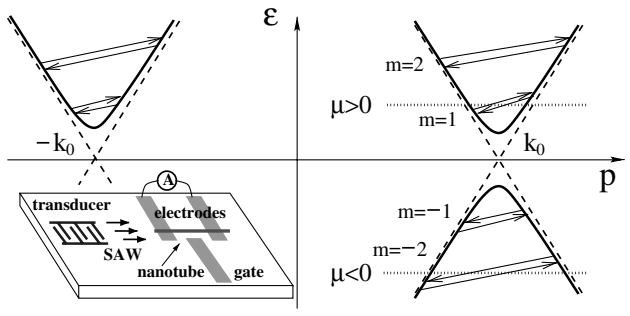

FIG. 1. The low energy spectrum of a metallic carbon nanotube (broken line) acquires a minigap (solid line) in the presence of a symmetry breaking perturbation. The backscattering transitions induced by the SAW potential are shown. Inset: proposed experimental arrangement consisting of a nanotube suspended between contacts, with a gate to the side, and a SAW source.

For comparison, the same periodic perturbation acting on a GaAs 1D channel with $v \approx 10^7$ cm/s will induce minigaps of an order of magnitude smaller. This figure can also be compared with a single-particle level spacing of $hv/L \sim 0.6$ meV expected for a tube of length $L = 3$ μm (and clearly resolved in experiment [8,9]). A further advantage of the nanotube over the GaAs system derives from the spectrum: the semimetallic nanotube system consists of two pairs of oppositely moving spin degenerate states that intersect exactly at the Fermi level (at half filling). Thus, despite the fact that the SAW wavelength is always much larger than the lattice constant a, minigaps will open close to the Fermi level (Fig. 1). Moreover, a minute doping or gating is sufficient to align the chemical potential with one of the minigaps.

Electron states in semimetallic nanotubes are described by a 1D Dirac equation rather than a Schrödinger equation. Below, it is shown that a selection rule protects the integrity of the Dirac band structure against backscattering due to a potential perturbation. Therefore, in the arrangement shown in Fig. 1, the SAW will not couple to electrons at all. In order to realize adiabatic charge transport, backscattering must be restored by applying an external perturbation that lowers the symmetry of the Dirac system (by mixing left and right states). This can be achieved by applying a magnetic field [14] along the nanotube axis. Also, in the majority of nominally metallic nanotubes such as the so-called "chiral" or "zigzag" nanotubes, a matrix element mixing left and right states appears [15] due to the curvature of the 2D carbon sheet rolled into a tube. Both effects open a minigap at the band center, as confirmed experimentally [16,17]. Below, we explore the influence of a SAW potential on the spectrum of the nanotube system within the framework of a free electron model [18] and, later, within the Luttinger liquid theory [2,3].

The long-range electron interaction in the spin- and valley-degenerate modes is symmetric with respect to the four "flavors." In the Luttinger liquid theory of nanotubes [2,3] this interaction is described by the forward scattering amplitude $V(q)$ with a form that depends on the electrostatic environment. In the absence of screening, $V_0(q) = e^2 \ln[(qd)^{-2} + 1]$, where d is the nanotube diameter. Accounting for the substrate dielectric constant ϵ, $V(q) = 2V_0(q)/(\epsilon + 1)$. Since the ratio $N = d/a$ is large (typically $N \sim 10$), backscattering and umklapp interactions are small scaling as $1/N$ [3]. Furthermore, the umklapp vertex also happens to be small numerically [19].

Therefore, taking into account the presence of a symmetry breaking perturbation Δ, and neglecting both backscattering and umklapp processes, the low energy states of the nanotube system (in the vicinity of the band crossing, $p = \pm k_0$) are described by the Dirac Hamiltonian

$$\mathcal{H} = \int dx \sum_{\alpha=1}^{4} \bar{\psi}_\alpha [-\hbar v \sigma_2 \partial_x + \Delta] \psi_\alpha + \frac{1}{2} \sum_q \hat{\rho}_q V(q) \hat{\rho}_{-q}, \quad (1)$$

where $\bar{\psi} = \psi^\dagger \sigma_1$. Here Pauli matrices $\sigma_{1,2}$ operate in the two-component Dirac operator space $\psi_\alpha = (\psi_r, \psi_l)_\alpha$, with pseudospin components corresponding to the right/left moving states, and $\hat{\rho}(x) = \sum_\alpha \psi_\alpha^\dagger(x) \psi_\alpha(x)$ represents the charge density operator. The second term in (1) describes the left/right mixing and yields a gap in the spectrum. Different mixing mechanisms lead to different values of Δ. For example, a parallel magnetic field [14] produces $\Delta = \hbar v \phi / R$, where R is the nanotube radius and $\phi = \Phi/\Phi_0$ is the magnetic flux through the nanotube cross section (measured in units of the flux quantum $\Phi_0 = hc/e$).

The harmonically varying electrostatic potential of the SAW decays exponentially in the direction normal to the surface: $Ae^{-kz} \sin k(x - ut)$, where u is the SAW velocity. Since the wavelength $\lambda_{SAW} = 2\pi/k$ is much larger than the tube diameter $2R$, one can ignore the potential variation e^{-kz} over the tube cross section. Indeed, for $\lambda_{SAW} = 1$ μm and $R = 1$ nm, the potential variation is less than 1%. The SAW velocity is small, $u \ll v$, and the 1D energy spectrum can therefore be obtained within a stationary approximation.

To simplify our analysis, let us first consider the noninteracting system. In the stationary approximation, the single-particle spectrum of each degenerate flavor can be obtained from the perturbed 1D Dirac system,

$$\epsilon \psi(x) = (-i\hbar v \partial_x \sigma_3 + \Delta \sigma_1 + A \sin kx) \psi(x). \quad (2)$$

Here the selection rule described earlier is manifest: for $\Delta = 0$, Eq. (2) separates into two independent equations for right and left moving particles. The SAW affects only the phase of the wave function. For $\Delta \neq 0$, the backscattering effect of the SAW potential is restored, and minigaps are induced in the spectrum. To explore the miniband structure it is convenient to implement a gauge transformation, $\psi(x) = e^{\frac{i}{2}\sigma_3 \lambda \cos kx} \psi'(x)$, where $\lambda = 2A/\hbar k v$, and

$$\epsilon \psi'(x) = (-i\hbar v \sigma_3 \partial_x + \Delta e^{-i\lambda \sigma_3 \cos kx} \sigma_1) \psi'(x). \quad (3)$$

The periodic system is characterized by Bloch states $\psi_p(x) = u_p(x) e^{ipx}$ with quasimomentum p taking values in the Brillouin zone defined by the SAW period, $-k/2 < p < k/2$. The corresponding energy spectrum can be easily obtained numerically (Fig. 2) by integrating the system of first-order differential equations (3) over the SAW spatial period, $0 < x < 2\pi/k$. The spectrum has an electron-hole symmetry, $\epsilon \to -\epsilon$, characteristic of a Dirac system. Equation (3) can also be solved analytically for $\Delta \ll \hbar k v$ by treating the second term as a perturbation [20]. Separated into Fourier components,

$$e^{-i\lambda \cos kx} = \sum_{m=-\infty}^{\infty} (-i)^m J_m(\lambda) e^{-imkx}, \quad (4)$$

where $J_m(\lambda)$ are Bessel functions, each harmonic of the perturbation (4) mixes right and left modes with $p - p' = mk$. When these states are in resonance (i.e., when $p = -p' = mk/2$, $\epsilon_m' = mkv/2$) , the spectrum can be found by standard two-wave matching. This gives energy gaps

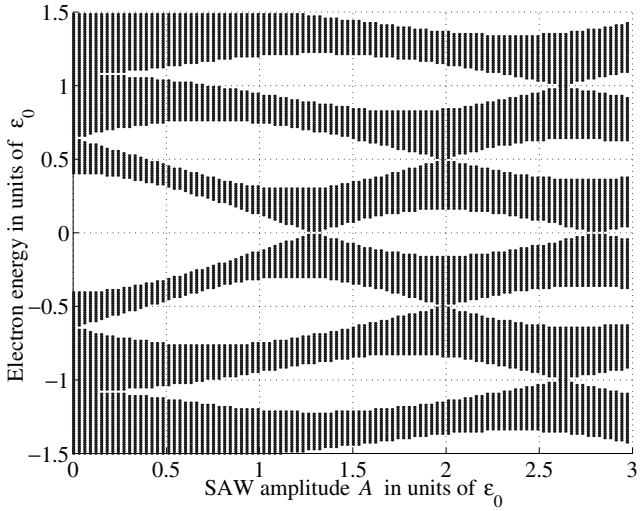

FIG. 2. Electron energy spectrum of Eq. (2) vs the SAW field strength A, scaled by $\epsilon_0 = \hbar k v$. The backscattering perturbation value $\Delta = 0.4\epsilon_0$ was used. Minigaps oscillate as a function of A, in agreement with the perturbation theory (5), vanishing at values close but generally not equal, to the roots of Bessel functions.

$$\Delta_m = 2\Delta |J_m(2A/\hbar k v)|, \tag{5}$$

which are oscillatory functions of the SAW amplitude A, with zeros at the nodes of Bessel functions. In particular, $\Delta_m \simeq 2\Delta(A/\hbar k v)^{|m|}/|m|!$ for $A \ll \hbar k v$.

Electrons in the half-filled (undoped) system represent a solid state analog of the Dirac vacuum: under the SAW perturbation, the many-body state carries neither charge nor current. For a weak SAW potential, this follows from adiabatic continuity: quantized transport takes place when the chemical potential μ falls in one of the minigaps. The value of the quantized current will remain the same [1] within a whole range of values of μ and A that stay within a gap. Since the spectral gap at the band center is adiabatically connected to the minigap at $A = 0$ (induced by the symmetry breaking perturbation Δ), it is evident that at half filling the current is zero. Similarly, for m fully occupied minibands, taking into account the fourfold valley and spin degeneracy, the electron density (counted from that at $\mu = 0$) is $\delta n = 4mk/2\pi$. This results in a current $j = eu\delta n$. Identifying $uk/2\pi$ with the SAW frequency f, we obtain the quantized current $j = 4mef$. The dependence of the energy gaps on A, shown in Fig. 2, describes the width of the plateaus of quantized current.

To complete our analysis it remains only to explore the integrity of the current quantization in the presence of electron interactions. To undertake this program it is convenient to first bosonize the Hamiltonian (1) setting $\psi_j(x) \propto \exp[i\sqrt{\pi}\,\phi_j(x)]$. Introducing the linear combination of bosonic fields

$$\begin{pmatrix} \Phi_0 \\ \Phi_1 \\ \Phi_2 \\ \Phi_3 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \end{pmatrix}\begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{pmatrix},$$

the part of the Hamiltonian (1) without the mass term $\Delta\bar{\psi}\psi$ is diagonalized. Setting $V_{\rm ext}(x) = V_{\rm g} + A\sin kx$, where $V_{\rm g}$ represents the external gate potential, and $K(q) = 1 + 4V(q)/\pi\hbar v$, the corresponding Lagrangian

$$\mathcal{L}_0 = \frac{1}{2}\sum_q [\partial_\tau \Phi_0(q)\partial_\tau \Phi_0(-q) + K(q)q^2\Phi_0(q)\Phi_0(-q)] + \int dx\left[\frac{1}{2}\sum_{a=1}^{3}(\partial_\mu \Phi_a)^2 + \frac{2}{\sqrt{\pi}\,\hbar v}V_{\rm ext}(x)\Phi_0(x)\right] \tag{6}$$

describes the dynamics of one charged and three neutral modes. Restoring the mass term perturbation, the total Lagrangian is given by $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_\Delta$, where

$$\mathcal{L}_\Delta = -2\Delta\int dx \sum_{\alpha=1}^{4}\cos(\sqrt{4\pi}\,\phi_\alpha). \tag{7}$$

Applied to $\mathcal{L}$, a conventional renormalization group approach demonstrates that the perturbation $\mathcal{L}_\Delta$ is relevant and grows. Depending on the density, controlled by $V_{\rm g}$, the resulting state can be gapped with a finite correlation length, or gapless.

Let us first focus on the influence of electron interactions on the energy gap at the band center considering the system at half filling and in the absence of the SAW (i.e, $V_{\rm ext} = 0$). Technically, this involves estimating the energy of a soliton field configuration ϕ_j (with any flavor j). A variational analysis which takes into account the renormalization due to the three neutral modes obtains

$$E_{\rm gap} \approx K^{1/2}E_0^{1/5}\Delta^{4/5}, \quad \text{where } E_0 = \frac{\hbar v}{d}, \tag{8}$$

substantially *larger* than the noninteracting result, Δ.

Similarly, the SAW-induced minigaps (5) are also enhanced by interaction. Considering the regime $\Delta \ll \hbar k v$, this enhancement is most straightforwardly demonstrated by mapping the SAW-induced gap onto the gap at the band center. This is achieved by a variable shift,

$$\phi_j \to \phi_j - (\sqrt{\pi}\,\hbar v)^{-1}\int_0^x \hat{K}^{-1}V_{\rm ext}(x')\,dx', \tag{9}$$

eliminating the term linear in Φ_0 from (6). [The operator in (9) is diagonal in Fourier representation, $\hat{K} = K(q)$.] At the same time, the mass term (7) is transformed as

$$\mathcal{L}_\Delta = -\Delta\int dx \sum_{f=1}^{4} e^{i(\sqrt{4\pi}\,\phi_i + \tilde{\lambda}\cos kx - 2\tilde{V}_{\rm g}x)} + \text{c.c.}, \tag{10}$$

where $\tilde{\lambda} = 2A/K\hbar k v$, $\tilde{V}_{\rm g} = V_{\rm g}/K\hbar v$, and $K = 1 + 8V(k)/hv$. [Indeed, the shift (9) is nothing but the bosonization representation of the gauge transformation used to solve the free fermion Dirac equation (2).] Now, by analogy with the treatment of Eq. (3) above, one can expand $\mathcal{L}_\Delta$ in Fourier components m. The density corresponding to m filled minibands can be chosen by setting

$\tilde{V}_g = mk/2$. In this case, all terms in the Fourier series (4) with $m \neq 2\tilde{V}_g/k$ give rise to expressions with oscillatory spatial dependence. Discarding these nonresonant terms one arrives at an expression of the form (7) with Δ replaced by $\Delta J_m(\tilde{\lambda})$. Being now formally equivalent to the problem at half filling considered above, we deduce that the interaction brings about a renormalization of the minigap such that

$$E_{\mathrm{gap}}^{(m)} \approx K^{1/2} E_0^{1/5} |\Delta J_m(2A/K\hbar k v)|^{4/5}. \qquad (11)$$

Several features of this result are worth noting: the general form of the gap dependence on the SAW amplitude, with nodes at the roots of Bessel functions, is *unaffected* by electron interaction. The magnitude of the minigap is *enhanced* by ca. $K^{1/2}(E_0/\Delta)^{1/5}$ as compared to the noninteracting case. The rescaling of the SAW amplitude A and of V_g by K, manifest in Eq. (10), describes the effect of screening due to the 1D electron system. For a substrate dielectric constant $\epsilon = 12$ (appropriate for GaAs) the screening factor is estimated to be $K \simeq 5$.

To complete our discussion, let us comment on the feasibility of the experiment (Fig. 1 inset). Maximal values of the SAW-induced minigaps in Fig. 2 are close to Δ, one-half of the value of the central gap. If a longitudinal magnetic field is used to open the central gap, then for a single-walled nanotube with a diameter 1.6 nm (such as that grown by Ref. [21]), and a field $B = 16$ T, one finds $\Delta \simeq 5$ meV. Applied to the spectra in Fig. 2 where $\Delta = 0.4\epsilon_0$ (i.e., $\epsilon_0 = 12$ meV) , this suggests a SAW wavelength of $\lambda_{\mathrm{SAW}} \simeq 0.25\ \mu$m. Taking a SAW velocity $u \approx 3 \times 10^5$ cm/s, this corresponds to a frequency $f = 12$ GHz resulting in a quantized current of around 8 nA. In order to reach a maximum value of the principal SAW-induced minigap shown in Fig. 2, the SAW potential should be around $A = 10$ meV. This value, obtained in the single electron approximation, should be corrected by the factor $K \simeq 5$ to account for screening. Thus a SAW potential of around a hundred meV may be required. These values do not present a problem even when a weak piezoelectric such as GaAs is used [12]. Moreover, for experiments with nanotubes, a SAW potential in the eV range can be made available by employing as a substrate a much stronger piezoelectric such as $LiNbO_3$. A strong piezoelectricity will also facilitate generation of the high frequency SAW required for the proposed experiment. (In $LiNbO_3$, SAW frequencies of ca. 17 GHz have been reported [22].) Alternatively, by using a chiral or zigzag metallic nanotube, where the central gap opens [15] due to the tube curvature, one can circumvent the need for a magnetic field. In this case, a gap is predicted [15] to be in a range up to 20 meV for a tube diameter of 1.6 nm allowing SAW-induced minigaps as large as 10 meV to be realized.

To summarize, we have considered a metallic carbon nanotube in the field of a slowly moving periodic potential. If the nanotube is subjected to a further perturbation that mixes right and left moving states, the coupling between the electrons and the SAW potential acts as a charge pump conveying electrons along the tube. An estimate of the miniband spectrum induced by electron diffraction on the sliding potential revealed that minigaps of ca. 10 meV are viable. We therefore conclude that the carbon nanotube combined with the SAW provides a promising system in which quantized adiabatic charge transport can be observed. As demonstrated above, the energy gaps that can be detected experimentally through quantization plateau widths are sensitive to the character of electron interactions. Thus, quantized transport in this strongly interacting system can be viewed as a novel probe of Luttinger liquid physics.

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