Sensitivity to Perturbations of a Time-Reversed Acoustic Wave in a Multiple Scattering Medium

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We present experimental results on the robustness of acoustic time-reversal focusing in a multiple scattering medium undergoing perturbations. Time reversal in such a medium can be viewed as a correlation technique, analogous to diffusive wave spectroscopy. Moreover, the recent introduction of telecommunication techniques based on time reversal in changeable media naturally raises the question of sensitivity to a perturbation of the medium. We consider the situation where the reversibility of an acoustic wave generated by a linelike source is gradually destroyed when a perturbation is added, either locally (by removing scatterers) or globally (by changing the temperature) to the multiple scattering medium.

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Time reversal of ultrasonic waves in an unperturbed medium was shown to be a very robust focusing process. Even in the presence of high-order multiple scattering [1,2], when a pulsed wave undergoes several hundred collisions as it traverses a sample, it is possible to make it travel back and focus onto its source, as long as the medium remains unchanged.

In this Letter, we consider the situation when the random multiple scattering medium is not stationary at the scale of a time-reversal experiment, which is of great practical importance. Precisely, we study the influence of two types of perturbation introduced in the medium before the propagation of the time-reversed wave: a local perturbation consisting of the introduction of a hole, and a global perturbation consisting of a temperature change.

The robustness of time-reversal experiments to such changes has become an important issue in several respects. First, based on time reversal or phase conjugation, new techniques of communication have recently been proposed [3]. But the media through which the information is transmitted (e.g., a city or the ocean) can evolve in time. In this perspective, a key problem is knowing whether communication is nonetheless possible in a changing environment. Second, real multiple scattering media such as some types of austenite steels, dense particle suspensions, or granular media are difficult to characterize with classical ultrasonic methods which are generally based on a single scattering assumption. In optics, correlation methods can be used to retrieve information about the medium: it is the field of the "diffusive wave spectroscopy" [4,5], where the intensity-intensity time correlations are exploited to characterize the dynamics of the medium. Recently, Cowan *et al.* developed analogous techniques in acoustics to characterize fluidized suspensions [6]. We show in this paper that a time-reversal experiment is a "natural correlator" which gives a direct measurement of the amplitude-amplitude time correlations. In other words, the breaking of time-reversal invariance due to a change in the medium is not only a problem to overcome, as is the case in the field of telecommunications, it can be taken advantage of to retrieve information about the dynamics of the medium.

The experimental setup is presented in Fig. 1. By essence, it is a 2D experiment. The source is a linelike wide-band transducer with a 3.2 MHz center frequency. The multiple scattering sample is a prototype made of 3000 steel rods immersed in water. The diameter of one rod is 0.8 mm, i.e., twice the average wavelength in water. Their concentration is $n = 0.1875\ \text{rods/mm}^2$. The elastic mean-free path was found to be 4 mm, whereas the sample thickness is $L = 40$ mm, which is characteristic of a strong multiple scattering regime. The diffusion constant was found to be $D = 3.2\ \text{mm}^2/\mu\text{s}$ [7].

The experiment can be described as follows: the source transmits a short pulse into the scattering medium (Fig. 1). The multiply scattered signals are recorded on an array of 128 transducers. The recorded signals last more than 250 μs. They are digitized over eight bits with a 20 MHz sample rate, time reversed and retransmitted into the medium; i.e., if $h_i(t)$ is the impulse response of the medium recorded on transducer No. i, $h_i(-t)$ is sent back into the medium. The finite spatial sampling (the array pitch is one wavelength) and the eight-bit quantization [8] themselves introduce small errors in the time-reversed wave field. However, this new wave field successfully travels back through the scattering sample and converges towards the initial source (Fig. 2).

Taking into account the reciprocity principle, the back and forth impulse responses are the same. The signal recreated at the source can thus be written as

$$s(t) = \sum_{i-1}^{128} h_i(t) \otimes h_i(-t) = \sum_{i=1}^{128} \int h_i(\tau) h_i(\tau + t)\, d\tau \,. \tag{1}$$

This quantity can be interpreted as an estimator of the autocorrelation function of the impulse response $\langle h(\tau)h(\tau + t)\rangle$ [8]. The maximum, reached at time $t = 0$, is the energy contained in the multiply scattered signal. It will serve as a reference (Fig. 2). If the medium

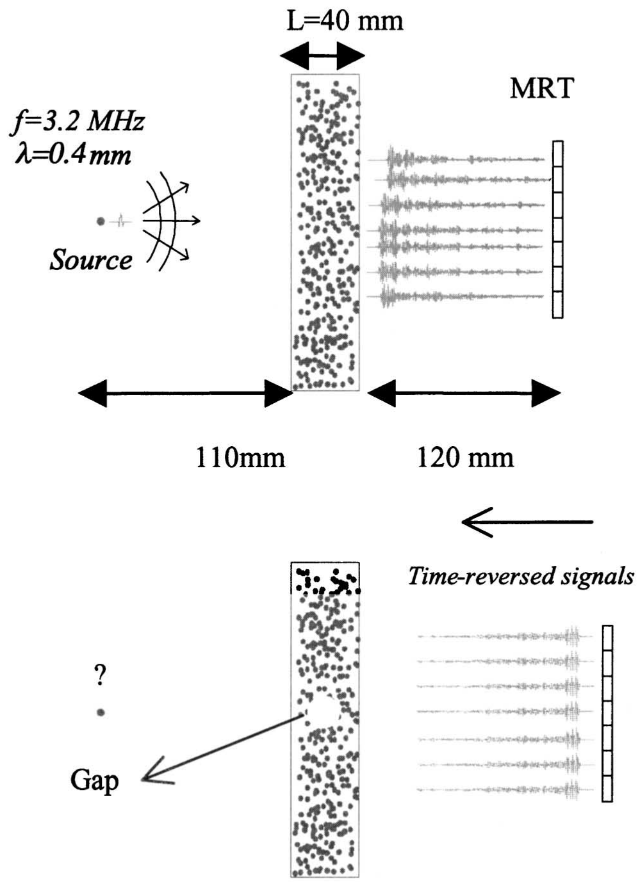

FIG. 1. Top: an ultrasonic source transmits a pulsed wave into a random medium. Bottom: the 128 transmitted signals are recorded on the transducer array and time reversed. At this step, a defect may be introduced in the medium or the temperature can be changed. Then, the time-reversed signals are retransmitted.

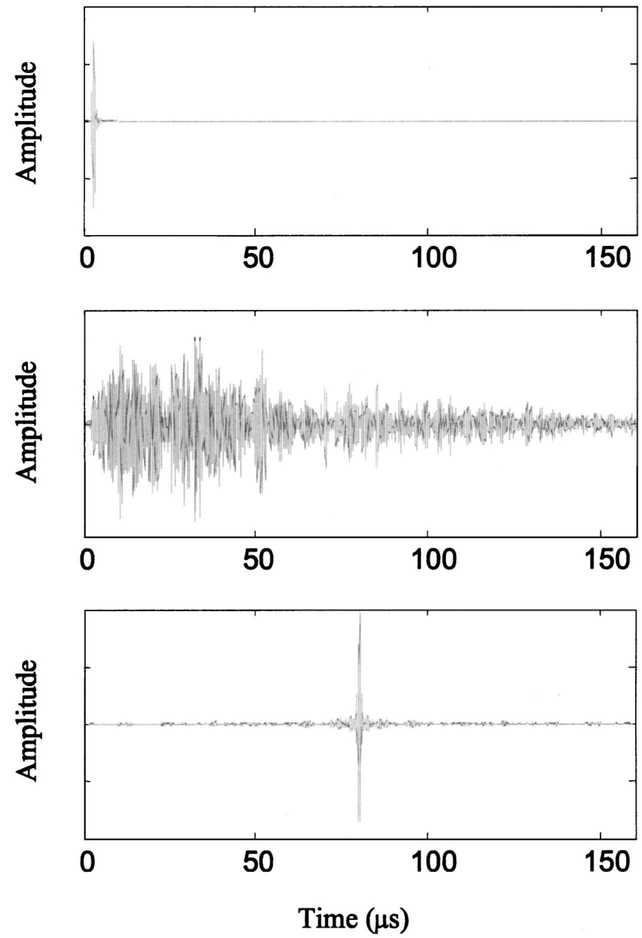

FIG. 2. Signal transmitted in water and received on transducer No. 64 (top). Signal transmitted through the multiple scattering sample and received on transducer No. 64 (middle). Signal $s(t)$ recorded at the source location after propagation of the time-reversed wave (bottom).

is perturbed before the time-reversed wave is sent back and if we denote by $g_i(t)$ the new impulse response of the medium, the signal recreated at the source is

$$s'(t) = \sum_{i=1}^{128} h_i(-t) \otimes g_i(t) = \sum_{i=1}^{128} \int h_i(\tau) g_i(\tau + t)\, d\tau\,, \tag{2}$$

which is an estimator of the intercorrelation function of the impulse responses before and after perturbation. Thus an acoustic time-reversal experiment in a changing environment is equivalent to the diffusive acoustic wave spectroscopy. The advantage of this method lies in the following: once the reference signal $h(t)$ has been acquired, the correlation is calculated *in situ* at a quick rate (typically each ms). Furthermore, as we use an array of 128 transducers as a time-reversal mirror, the estimator of the intercorrelation function is robust: we do not need the medium to move for averaging over realizations of disorder [8]. Averaging over disorder is replaced by averaging over the 128 contributions of the 128 transducers to the time-reversed signal.

The first perturbation that we studied consists of removing one or several rods (Fig. 1) before sending back the whole time-reversed field. This could illustrate a situation where a defect such as a hole or a crack progressively appears in a multiple scattering material under stress. The amplitude of the peak amplitude after time reversal is studied as a function of the number of removed rods (Fig. 3), i.e., as a function of the hole size. If the rods are removed randomly, the decrease in the peak is similar, at least if they are removed from the region illuminated by the ballistic beam, and if the transport mean-free path is not significantly affected by the change in density. We immediately notice in Fig. 3 that more than 60 rods are necessary to completely break the peak, which corresponds to about 2% of the total number of rods in the medium. We have to point out here that the perturbation brought to the medium when removing one rod is drastic. Indeed, the diameter of one rod is twice the wavelength. That is to say that all scattering paths that came through this location are no longer involved in the recompressed peak. In a ballistic regime, the decrease of the peak is expected to be roughly related to the size

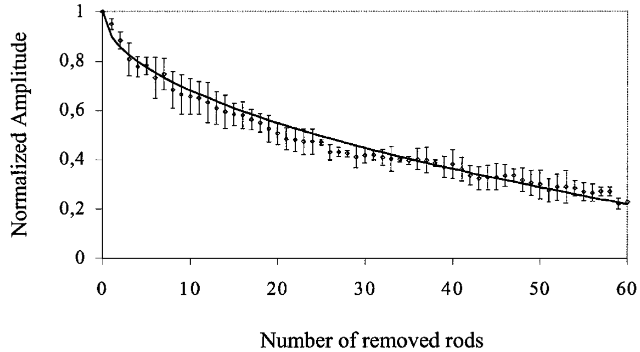

FIG. 3. Peak amplitude versus the number of removed rods. The experiment was carried out five times to obtain the averaged curve. The theoretical prediction based on Eq. (3) is in a continuous line.

of the affected zone compared to the transverse size B of the ballistic front at the same depth. In the multiple scattering regime, each point of the ballistic front acts as a diffusive source producing a diffuse halo whose transverse size is given by $\sqrt{DT}$, where T is the duration of the impulse response. Thus, the average estimation of the width W of the halo is $B + 2\sqrt{DT}$ at the depth where rods are removed. If we assume that on average multiply scattered waves recorded at time T after the ballistic front have seen the defect cT/L times, where c is the speed of sound in the medium, then the amplitude of the peak is roughly given by

$$1 - \frac{\text{defect size}}{\text{beam size}} \times \text{average number of round trips} =$$

$$1 - \frac{\sqrt{N/n}}{B + 2\sqrt{DT}} \times \frac{cT}{L}, \quad (3)$$

where N denotes the number of rods that have been removed and $T = 250\ \mu$s corresponds to the end of the time-reversal window, i.e., to the longest paths.

A fit of the experimental data in Fig. 3 gives $B = 140$ mm, a value compatible with the width of the first incoming wave front ("ballistic front") deduced from the classical relation $B = \lambda F/O$, where $F = 130$ mm is the distance between the source and the location where the rods are removed and $O = 0.39$ mm is the transverse size of the source.

We also perform "dynamic" time-reversal experiments: instead of time reversing the whole wave forms, we select short (5-μs-long) time windows in the multiply scattered signal. Then, we study the amplitude of the recreated peak versus the end time t_e of the selected time window (Fig. 4). The experimental results clearly show that the time-reversal process is much more sensitive when the window is selected in the later part of the signals, corresponding to the highest order of multiple scattering. The first time-reversal window in Fig. 4 contains the early arrivals (ballistic front), i.e., the lowest scattering order. In that case, more than 30 rods have to be taken out to break the

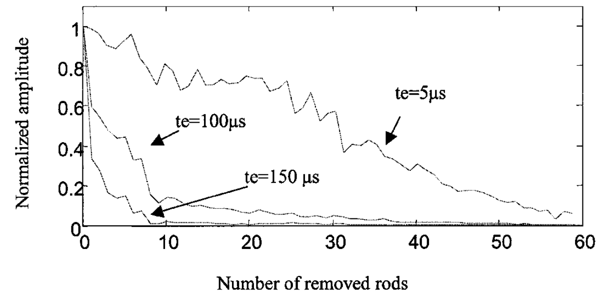

FIG. 4. Peak amplitude versus the number of removed rods for different positions t_e of the end of the time-reversal window.

peak by a factor of 2. But when the window is chosen far in the multiple scattering signal, the probability that the corresponding scattering paths crossed the perturbed region increases. It appears that, for $t_e = 150\ \mu$s, removing one rod is sufficient to decrease the peak by 50%. Following Ballentine and Zibin [9] and Snieder and Scales [10], we can define a time-reversal robustness parameter $\delta P_{1/2}$ as the perturbation necessary to diminish the peak amplitude by a factor of 2 (Fig. 5). A fit of the experimental data shows that $\delta P_{1/2}$ decreases as $1/t_e$. The enhanced sensitivity as the scattering order increases may be an interesting tool to detect the appearance of a very small defect in a multiple scattering sample.

The second perturbation that we have studied consists of varying continuously the temperature of the medium between 29 °C–25 °C. The evolution of the peak amplitude after time reversal as a function of temperature is presented in Fig. 6.

Obviously, the time-reversed peak is decreased when the temperature is changing. Indeed, the modification of the celerity in the medium introduces a phase shift on each scattering path, which implies a global decrease of the intercorrelation function. For moderate temperature changes in the studied range, the sound velocity increases linearly with temperature θ ($\alpha = \delta c/\delta\theta \approx 2.5\ \mu\text{m}\ \mu\text{s}^{-1}\ °\text{C}^{-1}$). Therefore, assuming that the scattered impulse response $h_i(t)$ spreads over a typical duration T, the duration of the perturbed impulse response $g_i(t)$ is changed by $\Delta T \sim \alpha T \Delta\theta/c$, where $\Delta\theta$ denotes the temperature change. If

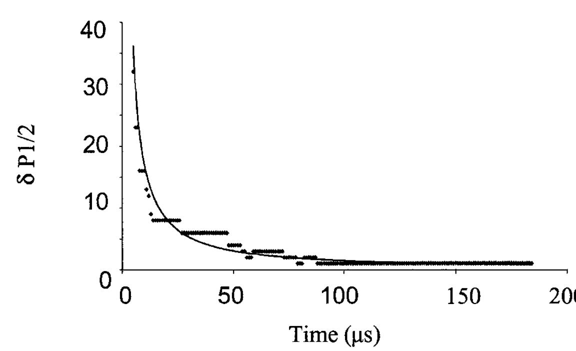

FIG. 5. Time-reversal robustness parameter versus the end time of the time-reversal window.

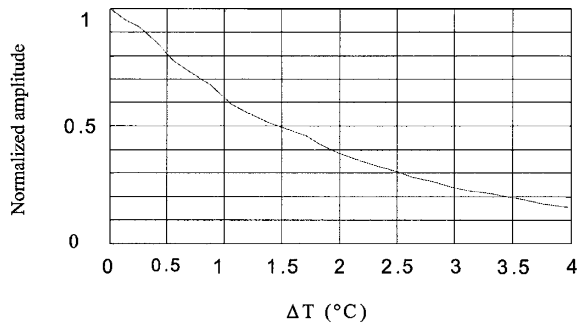

FIG. 6. Peak amplitude as function of a change in temperature.

we denote by t_m the median time, i.e., the time which divides the impulse response into two signals of equal energy, the intercorrelation will be significantly decreased when the dephasing between that time and the beginning of the impulse response corresponds to half a period. This yields a rough estimate of the robustness parameter $\delta P_{1/2} = cT_0/\alpha t_m \sim 1.7\ ^\circ\mathrm{C}$ with the period $T_0 = 0.3\ \mu\mathrm{s}$ and $t_m = 52\ \mu\mathrm{s}$, a value which is a little larger than the experimental one ($\sim$1.5 °C). Contrary to the previous experiment, the sensitivity to temperature change does not depend on time. A more precise model will be presented in a future paper.

We have highlighted in this paper the robustness of a time-reversal experiment in a multiple scattering medium submitted to a local or a global perturbation, which is a key problem in telecommunication techniques based on time reversal. Furthermore, we have shown that a time-reversal experiment performed in a perturbed random medium is a means to follow its time-dependent evolution. This method is quite analogous to the diffusive acoustic wave spectroscopy developed to study fluidized suspensions of particles [6]. However, it does not require a motion of the scatterers for averaging, and the correlation is "naturally" performed in the medium. Such a technique could be adopted in metallurgy to characterize a multiple scattering material under stress. In the physics of granular media, it could also be exploited to follow precisely the changes in the packing configuration produced by thermal expansion or by a high power acoustic wave propagating through it. Finally, it would be very interesting to perform these experiments when the propagation is nonlinear. In optics, Skipetrov and Maynard made very promising predictions concerning the instability of optical waves propagating in a nonlinear multiple scattering medium [11]. They show that the instability of the speckle manifests itself in the fluctuations of the intensity-intensity time correlation functions. The time-reversal technique may be very useful for an analogous study in acoustics.

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