Observation of Islands of Stability in Soft Wall Atom-Optics Billiards

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We report on numerical and experimental observations of islands of stability induced in a Bunimovich stadium atom-optics billiard by a soft wall repulsive potential. A deviation from exponential decay of the survival probability of atoms in an open billiard is observed, and explained by the presence of these stable islands and a sticky region surrounding them. We also investigate islands in dispersing billiards with soft walls, and predict a new mechanism for their formation.

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The billiard is one of the most widely studied conservative Hamiltonian systems, since it is a very simple system yet it demonstrates many classical and quantum mechanical properties of more complex dynamical systems. In some billiards the dynamics is integrable (e.g., in circular, rectangular, or elliptical billiards), whereas in other cases (e.g., the Sinai billiard, the Bunimovich stadium) the motion is chaotic and ergodic [1,2]. But what will happen to such a chaotic and ergodic billiard when the potential of the boundary is changed into a smooth function? This question is especially interesting in the context of understanding the origin of statistical mechanics, for which the billiard problem is a widely used paradigm [3]. For example, the Sinai billiard [1] is mathematically analogous to the motion of two disks on a two-dimensional torus, a crude approximation for gas molecules in a chamber, and already this simple system is proved to be ergodic. Recently, however, it was proven that if the potential between the two disks is smooth, as is the actual potential between gas molecules, there exist elliptic periodic orbits, hence the system is not ergodic [4]. Smooth potentials that cause the appearance of elliptic islands inside a chaotic phase-space have also been recently explored in general scattering billiards [5,6].

Investigations of the quantum properties of billiards reveal Poison level statistics for a smooth potential wall, and a sharp change into Wigner-Dyson statistics below some critical "softness parameter." This transition is accompanied by a dynamical localization-delocalization transition of the eigenstates [7,8].

From a practical point of view, physically realizable potentials are inherently soft, and the softness of the potential may result in a mixed phase-space with a hierarchical structure of islands. This structure greatly affects the transport properties of the system (e.g., induces nonexponential decay of correlations), since trajectories from the chaotic part of phase-space are trapped for long times near the boundary between regular and chaotic motion [3]. As an important example, these considerations were studied in the context of ballistic nanostructures, and found to be the origin of the fractal nature of magnetoconductance fluctuations in quantum dots [9–13]. However, in these systems the wall softness is often accompanied by nonideal effects (such as scattering from impurities) and hence its role is still controversial.

In this Letter we investigate the influence of a soft potential wall on an experimental system—the recently realized atom-optics billiard [14,15]. We show that, by changing the wall softness, it is possible to induce islands of stability in the phase-space of an otherwise chaotic billiard. The island position is determined experimentally by mapping a projection of the phase-space of the system. This ability to precisely control the phase-space structure of an experimental system is unique for the atom-optics billiards.

The billiard that we study is a tilted Bunimovich stadium (see insets of Fig. 1 below), which is chaotic and ergodic. It is composed of two semicircles of different radii (64 and 31 μm), connected by two nonparallel straight lines (192 μm long). When the potential of the wall becomes softer, a stability region appears around the singular trajectory which connects the points where the big semicircle

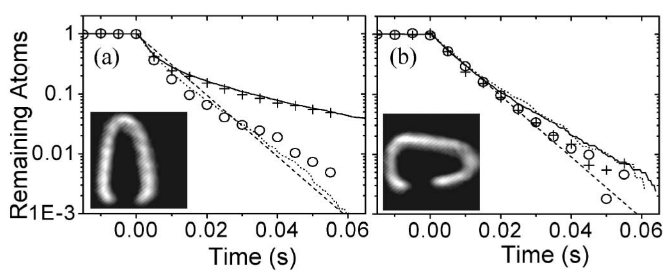

FIG. 1. Experimental results for the decay of cold atoms from a tilted-stadium-shaped atom-optics billiard, with two different values for the softness parameter: $w_0 = 14.5$ μm ($\circ$), and $w_0 = 24$ μm (+), and for two different hole positions. (a) The hole is located inside the big semicircle. The smoothing of the potential wall causes a growth in stability and a slowing down in the decay curve. (b) The hole includes the singular point; no effect for the change in w_0 is seen. Also shown are the results of full numerical simulations, with the experimental parameters (see text) and no fitting parameters. The dashed line is e^{-t/τ_c}, the decay curve for an ideal (hard wall) billiard. The insets show measured cross sections of the (averaged) intensity of the laser creating the soft wall billiards, in the beam's focal plane. The size of the images is 300×300 μm.

joins the straight lines (see Fig. 2 below). We first present our experimental results for the decay rate of cold atoms from billiards with different wall softness, through a small hole located at different places along the boundary. Subsequently, we describe numerical simulations for the dynamics of atoms in the billiard, and show that the structure of phase-space varies with the wall softness in a way which explains the experimental results.

As described in detail in [15], the billiard is realized by the use of a laser beam which is rapidly (100 kHz) scanned using two perpendicular acousto-optic scanners (AOSs). The laser is tuned 0.5 nm above the atomic resonance (D_2 line of ^{85}Rb), hence applying a repulsive force on the atoms. By controlling the AOSs, we create the required billiard shapes which confine the atoms in the transverse direction. The instantaneous potential is given by the dipole potential of the laser beam: $U(x,y,t) = U_0 \exp\{-2[(x - x_0(t))^2 + (y - y_0(t))^2]/w_0^2\}$, where the curve $[x_0(t), y_0(t)]$ is the shape of the ideal billiard, along which the center of the Gaussian laser beam scans. w_0 is the laser beam waist and is used as the softness control parameter. U_0 is the potential height, which is kept constant for different values of w_0 by changing the laser power. Fast enough scanning of the beam results in an effective time-averaged potential wall [16]. We experimentally control the softness of the billiard's walls using a telescope with a variable magnification, which is located prior to the AOS's such that w_0 can be changed without affecting the billiard's size and shape. It was found [16] that, during the experiment, an atom spontaneously scatters less than one photon on average. Furthermore, in the range of atomic densities realized in the billiard, the mean collision time between atoms is longer than the experiment time [16], hence the motion of the atoms between reflections from the walls can be regarded as strictly ballistic.

The loading scheme of cold atoms into the billiard differs from that described in [15]. It is intended to achieve a more monoenergetic ensemble of atoms, with a mean kinetic energy of about half the billiard potential height, and a narrow spread around that mean value. Laser cooled ^{85}Rb atoms are loaded from a magneto-optical trap into a 0.5 mK deep red-detuned one-dimensional optical lattice with a beam waist of 240 μm. The atoms are then transferred from the lattice into the billiard, which is displaced by $\Delta = 250$ μm, by pushing them with a pulse of a strong on-resonance beam which is perpendicular to the billiard beams. Simultaneously with the pushing, the lattice beams are switched off in a time constant of $\sim$400 μs, which is adiabatic in the longitudinal direction, resulting in further cooling in that direction [17]. In this way an atomic cloud is formed with an rms velocity spread of $\sim 6.3 v_{\mathrm{rec}}$ in the radial direction, $< 2.1 v_{\mathrm{rec}}$ in the longitudinal direction, and moving with an average velocity of $v = 18 v_{\mathrm{rec}}$ towards the billiard (v_{rec} is the atom's recoil velocity, $\sim$6 mm/s for ^{85}Rb). Further reduction of the radial velocity spread, and especially a decrease in the number of very slow atoms, is achieved by capturing only a central velocity group of atoms in the billiard, which is turned on at the proper time ($t = \Delta/v = 2.3$ ms). Typically, 3×10^5 atoms are loaded into the billiard. The resulting quasi-monoenergetic ensemble of atoms has a well-defined decay time through the hole, given by $\tau_c = \pi A / vL$, where A is the billiard's area and L is the length of the hole [18]. The cooling in the longitudinal direction ensures that the system can be approximated as a two-dimensional system [19]. We allow additional 65 ms of collisions with the billiard's walls, such that the direction of the transverse velocity is randomized, and then open a hole in the billiard's boundary, through which atoms can escape. The hole is produced by switching off one of the AOSs for $\sim$1 μs every scan cycle, synchronously with the scan. The number of atoms remaining in the trap is measured using fluorescence detection [16]. The ratio of the number of trapped atoms with and without the hole, as a function of time, is the main data of our experiments. Note that for systems with mixed phase-space the decay depends on the position of the hole. If the hole does not include trajectories which belong to a stable island and its vicinity, these trajectories are trapped for long times, and a strong deviation from exponential decay is observed. On the other hand, if the hole contains such trajectories, they will decay fast and the rest of the atoms (the majority) will decay exponentially.

In Fig. 1, experimental results for the decay from a tilted stadium with two different values of the softness parameter ($w_0 = 14.5$ μm and $w_0 = 24$ μm) are presented. It can be seen that, when the hole is located entirely inside the big semicircle [Fig. 1(a)], the soft wall causes an increased stability and a slowing down in the decay curve. When the hole includes the singular point where the semicircle meets the straight line [Fig. 1(b)], no effect for the change in w_0 is seen. We show below that these results can be explained by the formation of a stable island around the singular trajectory, and a sticky region around it. Figure 1 also includes the results of numerical simulations, which include the three-dimensional atomic and laser beam distributions, atomic velocity spread, laser beam scanning and gravity, and no fitting parameters. As can be seen, there is very good agreement between the simulated and measured decay curves. Similar decay measurements and simulations for a circular atom-optics billiard showed no dependence on w_0 in the range 14.5–24 μm, and no dependence on the hole position.

To understand these observations, it is useful to look at how the phase-space of the system changes with changing w_0. In Fig. 2, results of numerical simulations for classical trajectories of Rb atoms inside the tilted-stadium billiard are shown. For clarity, we assume a monoenergetic ensemble (with $v = 20 v_{\mathrm{rec}}$), a two-dimensional system, and no gravity. The dimensions of the billiard are equal to the experimental ones. Phase-space information is presented using a Poincaré surface of section, showing v_x versus x at every trajectory intersection with the billiard's symmetry axis ($y = 0$), provided that $v_y > 0$.

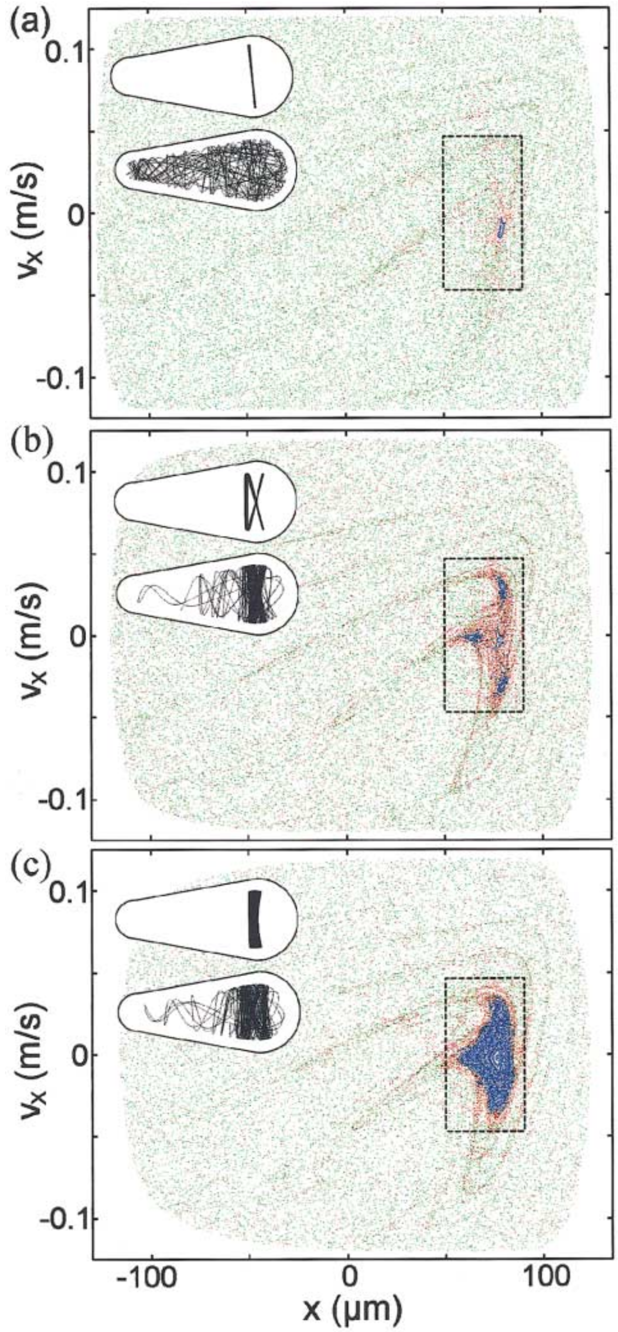

FIG. 2 (color). Poincaré surface of section for monoenergetic atoms confined in a tilted-stadium atom-optics billiard with parameters specified in text, and three different values of the softness parameter w_0. Trajectories inside an island are marked in blue, sticky trajectories [21] are marked in red, and chaotic trajectories are marked in green. (a) $w_0 = 18$ μm. A small elliptic island appears close to the trajectory which connects the two singular points. Upper inset: a trajectory in the island. Lower inset: a typical chaotic trajectory in this billiard. (b) $w_0 = 27$ μm. Three additional islands appear around the central one, and correspond to the periodic trajectory shown in the upper inset. Around these islands there is a large area of stickiness, where the trajectories spend a long time (see trajectory in lower inset). (c) $w_0 = 30$ μm. The three previous islands merge into one large elliptic island (see trajectory in upper inset), with some stickiness around it (lower inset).

In Fig. 2(a), the $w_0 = 18$ μm case is shown. A small elliptic island (marked in blue) appears around the trajectory which connects the two singular points, as can also be seen from the upper inset, which shows a trajectory in the island [20]. In the lower inset, a typical chaotic trajectory is shown. For $w_0 < 12$ μm, no islands with area larger than 10^{-4} of the total phase-space (the resolution of the simulations) were observed. In general, the island size increases with the increase of w_0, as can be seen from Figs. 2(b) and 2(c) which correspond to $w_0 = 27$ and 30 μm, respectively. For $w_0 = 27$ μm, three additional islands (blue) appear around the central one, and correspond to the periodic trajectory shown in the upper inset. Around these islands there is a large area of "stickiness" (marked in red [21]), where the trajectories spend a long time. Such a "sticky" trajectory is presented in the lower inset of Fig. 2(b). The exact structure of the island and its vicinity depends on the softness parameter w_0 in a sensitive way, as can be seen from the $w_0 = 30$ μm case [Fig. 2(c)], where the three previous islands merged with the central one into one big elliptic island, with some stickiness around it.

We also performed similar numerical simulations for totally scattering billiards. As predicted in [6], an island is formed around a singular orbit, periodic in the ideal billiard and tangent to (or parallel to the tangent of) one of the billiard's arcs. A second type of island, for which we are not aware of a theoretical treatment, is trajectories which go near one (or more) of the corners of the billiard. In all three cases the island appears around a singular periodic orbit in a limiting ideal billiard, as follows from [6].

For the tilted stadium, a simple model can explain the formation of the island. For the singular periodic trajectory, the curvature of the ideal billiard at the reflection points is not well defined: it is R^{-1} (the radius of the big semicircle) when approaching the singular point from the semicircle side, and 0 when approaching from the side of the straight line. When the walls are softened, the billiard can be approximated as the potential contour line which has a height equal to the kinetic energy of the particles. The curvature in the vicinity of the singular points will now vary smoothly from 0 to R^{-1}. As a result, the sum of the radius of curvature in the two reflection points will be larger than the distance between them ($2R$), a condition which ensures linear stability of the periodic orbit [22]. However, this is only a partial explanation, since in a soft wall billiard the reflection point depends on the angle of reflection, and hence is not always from the same contour line. In a numerical simulation for a hard wall stadium with a shape of the above contour, the decay was indeed slower than from the ideal billiard, but substantially faster than from the soft wall billiard, indicating a combined effect of the stable resonator at the reflection points caused by smoothing the singularity, and of the soft wall itself.

In Fig. 3, the measured fraction of remaining atoms at $5\tau_c$ ($= 42.5$ ms) after a hole is opened in the big semicircle is plotted as a function of the softness parameter, w_0,

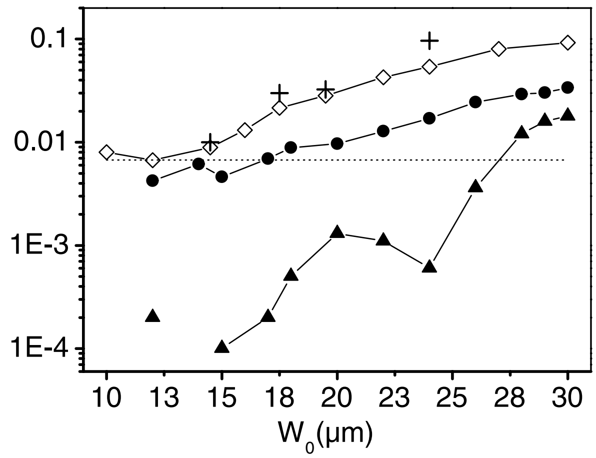

FIG. 3. Fraction of remaining atoms at $5\tau_c$ after the hole is open, as a function of the softness parameter (w_0), for a hole in the big semicircle. (+): experimental results, (◇): numerical simulation. The dashed line is e^{-5}, the expected value for small w_0 . Also shown are values for the island (▲) and island + stickiness (•) sizes as a fraction of the phase-space area, calculated from the two-dimensional phase-space simulations. Lines connecting the symbols are added to guide the eye.

together with the results of numerical simulations. A very good agreement exists between the decay simulations and the measured data, and both converge to the expected value of e^{-5} for small w_0. To intuitively understand the origin of the increased stability, the results of the two-dimensional phase-space simulations for the island and island + stickiness size (as a fraction of phase-space) [21] are also presented in the figure. These simulations reveal that the size of the island + stickiness grows monotonically with w_0, in a similar way to the decay results. The island size itself is much smaller, and has a nonmonotonic dependence on w_0. These facts suggest that the remaining atoms in the experiment, at $5\tau_c$, are mainly due to stickiness and demonstrate the important influence of the stickiness on the dynamics of the billiard.

In conclusion, numerical and experimental observations of the formation of islands of stability in a Bunimovich stadium with soft walls were presented. The island position was located experimentally by mapping a projection of the phase-space of the system. The ability to control the phase-space structure can be used, for example, to construct experimentally a "Maxwell's demon" as suggested in [23], by carefully designing the phase-space structure of two contacting billiards. The experimental control of phase-space structure will enable one to also explore mesoscopic or quantum effects in chaotic and mixed phase-space systems, for example, by placing a Bose-Einstein condensate inside the optical billiard.

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[19] During the 120 ms of the experiment, an atom travels on average a distance of 1.5 mm in the longitudinal direction, less than half the trap's length, ~4 mm.
[20] A 1% difference in the lengths of the two straight lines of the stadium used in both simulations and experiment, causes a slight tilt of the island. However, similar island location and hierarchy were also observed in the simulations for a perfectly symmetric billiard, although with different island sizes.
[21] A sticky trajectory is defined as one which spends inside a box surrounding the island (shown in Fig. 2) a time which is more than 3 times longer than the time expected for a random trajectory.
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