Algebraic Fermi Liquid from Phase Fluctuations: "Topological" Fermions, Vortex "Berryons," and QED$_3$ Theory of Cuprate Superconductors

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Within the phase fluctuation model for the pseudogap state of cuprate superconductors we identify a novel statistical "Berry phase" interaction between the nodal quasiparticles and fluctuating vortex-antivortex excitations. The effective action describing this model assumes the form of an anisotropic Euclidean quantum electrodynamics in (2 + 1) dimensions (QED$_3$) and naturally generates non-Fermi liquid behavior for its fermionic excitations. The doping axis in the x-T phase diagram emerges as a *quantum critical line* which regulates the low energy fermiology.

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Perhaps the most intriguing property of high temperature superconductors is the anomalous character of their normal state [1]. This "strange metal" stands in stark contrast to the relatively benign features of the superconducting phase which can be understood rather accurately within the framework of a d-wave BCS-like phenomenology with well-defined quasiparticle excitations [2].

In this Letter we propose a theory of the pseudogap phase in cuprate superconductors based on the following premise: a successful phenomenology of the strange metal should be built by starting from a comprehensive understanding of the adjacent superconducting state and its excitations. The spirit of our approach is the traditional one [1,3] but turned upside down. Usually, the strategy is to first understand the normal state before we can understand the superconductor. In the cuprates, however, it is the superconducting state that appears "conventional" and its quasiparticles "less correlated" and better defined. Having adopted this "inverted" paradigm, we proceed to study the interactions of the quasiparticles with the collective modes of the system, i.e., fluctuating (anti) vortices (our strategy here is similar to that of Ref. [4]). We show that in d-wave superconductors these interactions take a form of a gauge theory which shares considerable similarity with the quantum electrodynamics in (2 + 1)-dimensions (QED$_3$). In the superconducting state, where vortices are *bound,* the gauge fields of the theory are *massive* and the low energy quasiparticles remain well-defined excitations. This is the mundane Fermi liquid state in our inverted paradigm. In the normal state, however, as vortices *unbind*, our QED$_3$-like theory enters its *massless* phase and it abandons this "inverted Fermi liquid" protectorate in favor of a weakly destabilized Fermi liquid characterized by a power law singularity in the fermion propagator which we call *algebraic Fermi liquid.* We compute the spectral properties of fermions in our theory and find that they capture some key qualitative aspects of the available experimental data.

We concentrate on the portion of the pseudogap phase above the shaded region and below T^* in Fig. 1. We assume that Cooper pairs are formed at or somewhat below T^* but the long-range phase coherence sets in only at the superconducting transition temperature $T_{SC} \ll T^*$ [5]. Between T_{SC} and T^* the phase order is destroyed by unbound vortex-antivortex excitations of the Cooper pair field [6–8]. In this pseudogap regime the d-wave superconducting gap is still relatively intact [4,5] and the dominant interactions are those of nodal quasiparticles with fluctuating vortices. There are *two* components of this interaction: First, vortex fluctuations produce variations in superfluid velocity which cause Doppler shift in quasiparticle energies [9]. This effect is classical and already much studied [10,11]. Second, there is a purely quantum "statistical" interaction, tied to a geometric "Berry phase" effect that winds the phase of a quasiparticle as it encircles a vortex [12,13]. It is this quantum mechanical interaction that ultimately causes the destruction of the Fermi liquid in the pseudogap phase.

Our starting point is the partition function

$$Z = \int \mathcal{D}\Psi^\dagger(\mathbf{r},\tau) \int \mathcal{D}\Psi(\mathbf{r},\tau) \int \mathcal{D}\varphi(\mathbf{r},\tau)\, \exp[-S], \tag{1}$$
$$S = \int d\tau \int d^2r \{\Psi^\dagger \partial_\tau \Psi + \Psi^\dagger \mathcal{H} \Psi + (1/g)\Delta^*\Delta\},$$

where τ is the imaginary time, $\mathbf{r} = (x, y)$, g is an effective coupling constant, and $\Psi^\dagger = (\bar\psi_\uparrow, \psi_\downarrow)$ are the standard Grassmann variables. The Hamiltonian $\mathcal{H}$ is given by

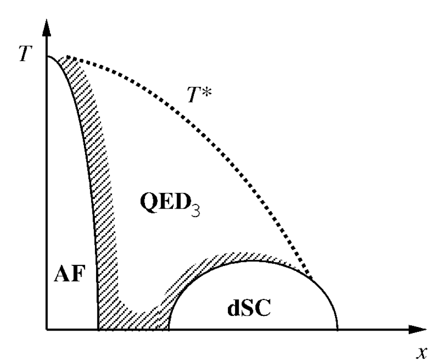

FIG. 1. Phase diagram of a cuprate superconductor.

$$\mathcal{H} = \begin{pmatrix} \hat{\mathcal{H}}_e & \hat{\Delta} \\ \hat{\Delta}^* & -\hat{\mathcal{H}}_e^* \end{pmatrix}, \tag{2}$$

with $\hat{\mathcal{H}}_e = \frac{1}{2m}(\hat{\mathrm{p}} - \frac{e}{c}\mathbf{A})^2 - \epsilon_F$, $\mathbf{p} = -i\nabla$ (we take $\hbar = 1$), and $\hat{\Delta}$ is the d-wave pairing operator [12], $\hat{\Delta} = (1/k_F^2)\{\hat{p}_x,\{\hat{p}_y,\Delta\}\} - (i/4k_F^2)\Delta(\partial_x\partial_y\varphi)$, where $\Delta(\mathbf{r},\tau) = |\Delta|\exp[i\varphi(\mathbf{r},\tau)]$ is the center-of-mass gap function. $\int \mathcal{D}\varphi(\mathbf{r},\tau)$ denotes an integral over smooth ("spin wave") and singular (vortex) phase fluctuations. Amplitude fluctuations are suppressed below T^*.

It is convenient to eliminate the phase $\varphi(\mathbf{r},\tau)$ from the pairing term (2) in favor of $\partial_\mu\varphi$ terms $[\mu = (x,y,\tau)]$ in the fermionic action. In order to avoid dealing with nonsingle-valued wave functions we employ the singular gauge transformation devised in Ref. [12]:

$$\bar{\psi}_\uparrow \to \exp(i\varphi_A)\bar{\psi}_\uparrow, \qquad \bar{\psi}_\downarrow \to \exp(i\varphi_B)\bar{\psi}_\downarrow, \tag{3}$$

where $\varphi_A + \varphi_B = \varphi$. Here $\varphi_{A(B)}$ is the singular part of the phase due to $A(B)$ vortex defects: $\nabla \times \nabla\varphi_{A(B)} = 2\pi\hat{z}\sum_i q_i\delta(\mathbf{r} - \mathbf{r}_i^{A(B)})$, with $q_i = \pm 1$ denoting the topological charge of the ith vortex and $\mathbf{r}_i^{A(B)}(\tau)$ its position. The labels A and B represent some convenient but otherwise arbitrary division of vortex defects [loops or lines in $\varphi(\mathbf{r},\tau)$] into two sets. As discussed in [12] this transformation guarantees that the fermionic wave functions remain single valued and the effect of branch cuts is incorporated directly into the fermionic part of the action:

$$\mathcal{L}' = \bar{\psi}_\uparrow[\partial_\tau + i(\partial_\tau\varphi_A)]\psi_\uparrow + \bar{\psi}_\downarrow[\partial_\tau + i(\partial_\tau\varphi_B)]\psi_\downarrow + \Psi^\dagger\mathcal{H}'\Psi,$$

where the transformed Hamiltonian $\mathcal{H}'$ is

$$\begin{pmatrix} \frac{1}{2m}(\hat{\pi} + \mathbf{v})^2 - \epsilon_F & \hat{D} \\ \hat{D} & -\frac{1}{2m}(\hat{\pi} - \mathbf{v})^2 + \epsilon_F \end{pmatrix},$$

with $\hat{D} = (\Delta_0/2k_F^2)(\hat{\pi}_x\hat{\pi}_y + \hat{\pi}_y\hat{\pi}_x)$ and $\hat{\pi} = \hat{\mathrm{p}} + \mathbf{a}$.

The transformation (4) generates a "Berry gauge potential" $a_\mu = \frac{1}{2}(\partial_\mu\varphi_A - \partial_\mu\varphi_B)$ which describes half-flux Aharonov-Bohm scattering of quasiparticles on vortices and mimics the effect of branch cuts in quasiparticle-vortex dynamics [12,13]. This is in addition to the "Doppler" gauge field $v_\mu = \frac{1}{2}(\partial_\mu\varphi_A + \partial_\mu\varphi_B)$ which denotes the classical part of the quasiparticle-vortex interaction. All choices of the sets A and B are *equivalent*—different choices represent different singular gauges, and v_μ is invariant under such transformations. To symmetrize the partition function with respect to this singular gauge we define a generalized transformation (3) as the sum over all possible choices of A and B, i.e., over the entire family of singular gauge transformations. This is an Ising sum with 2^{N_l} members, where N_l is the total number of vortex defects in $\varphi(\mathbf{r},\tau)$. This symmetrization leads to the new partition function $Z \to \tilde{Z} = \int\mathcal{D}\tilde{\Psi}^\dagger\int\mathcal{D}\tilde{\Psi}\int\mathcal{D}v_\mu\int\mathcal{D}a_\mu\exp[-\int d\tau\int d^2r\,\tilde{\mathcal{L}}]$ in which the half-flux-to-minus-half-flux (Z_2) symmetry of the singular gauge transformation (3) is manifest:

$$\tilde{\mathcal{L}} = \tilde{\Psi}^\dagger[(\partial_\tau + ia_\tau)\sigma_0 + iv_\tau\sigma_3]\tilde{\Psi} + \tilde{\Psi}^\dagger\tilde{\mathcal{H}}\tilde{\Psi} + \mathcal{L}_0[v_\mu, a_\mu], \tag{4}$$

where $\mathcal{L}_0$ is the "Jacobian" of the transformation given by

$$e^{-\int d\tau\int d^2r\,\mathcal{L}_0} = 2^{-N_l}\sum_{A,B}\int\mathcal{D}\varphi(\mathbf{r},\tau) \times \delta[v_\mu - \tfrac{1}{2}(\partial_\mu\varphi_A + \partial_\mu\varphi_B)] \times \delta[a_\mu - \tfrac{1}{2}(\partial_\mu\varphi_A - \partial_\mu\varphi_B)].$$

Here σ_μ are the Pauli matrices and $\tilde{\mathcal{H}} = \mathcal{H}'$. We call the quasiparticles $\tilde{\Psi}^\dagger = (\bar{\tilde{\psi}}_\uparrow, \tilde{\psi}_\downarrow)$ appearing in (4) "topological fermions" (TF's). TF's are the natural fermionic excitations of the pseudogapped normal state. They are electrically neutral and are related to the original quasiparticles by the inversion of transformation (3).

To proceed we must extract the low energy, long-distance properties of the Jacobian (4). This is done by focusing on the fluctuations of two gauge fields v_μ and a_μ in the fluid of vortex excitations. We use the saddle-point approximation to compute the leading (quadratic) terms in $\mathcal{L}_0$ for two cases of interest: (i) the thermal vortex-antivortex fluctuations in 2D layers and (ii) the space-time vortex loop excitations relevant for low temperatures ($T \ll T^*$) in the underdoped regime (but still above the shaded region in Fig. 1). The computation is straightforward but the algebra is laborious and will be presented elsewhere [14]. Here we quote only the final results whose form is ultimately dictated by the symmetries of the problem. For the case (i),

$$\mathcal{L}_0 \to \frac{T}{2\pi^2 n_l}[(\nabla\times\mathbf{v})^2 + (\nabla\times\mathbf{a})^2], \tag{5}$$

where n_l is the average density of free vortex defects. Both $\mathbf{v}$ and $\mathbf{a}$ have a Maxwellian *bare* stiffness and are *massless* in the normal state. As one approaches $T_{\rm SC}$, $n_l \sim \xi_{\rm SC}^{-2} \to 0$, where $\xi_{\rm SC}(x,T)$ is the superconducting correlation length, and $\mathbf{v}$ and $\mathbf{a}$ become *massive*. Similarly, for the case (ii), the quantum fluctuations of *unbound* vortex loops result in [14]

$$\mathcal{L}_0 \to \frac{1}{2\pi^2}\left[K_\tau(\partial\times a)_\tau^2 + \sum_i K_i(\partial\times a)_i^2\right], \tag{6}$$

where K_τ and K_i ($i = x,y$) are functions of x and T: $K_i \sim \xi_{\rm SC}$ and $K_\tau \sim \xi_{\rm SC}^z$, with z being the dynamical exponent. The Maxwell form of $\mathcal{L}_0$ is dictated by symmetry: the *bare* propagators for v_μ and a_μ, $\mathcal{D}_v^0(\mathbf{q},i\omega)$ and $\mathcal{D}_a^0(\mathbf{q},i\omega)$, are massless in the normal state and massive within a superconductor. Note that we dropped v_μ from (6)—the reason for this is made apparent below.

The physical picture advanced in this Letter rests on the following observations: v_μ couples to the TF "charge" in the same way as the real electromagnetic gauge field. Consequently, if we integrate out TF's in Eq. (4) to obtain the renormalized (or dressed) gauge field propagators $\mathcal{D}_v(\mathbf{q},i\omega)$ and $\mathcal{D}_a(\mathbf{q},i\omega)$, we find that $\mathcal{D}_v^{-1}(\mathbf{q} \to$

$0, i\omega = 0) \rightarrow$ const, i.e., the Doppler gauge field v_μ is *massive.* This is a consequence of the Meissner response of TF's. Physically, this means that the integration over the quasiparticles leads to the familiar long-range interactions between vortices. In contrast, the "Berry" gauge field a_μ couples to the TF *spin*. This implies that any contribution of TF's to the stiffness of a_μ must be *massless*: a singlet superconductor retains the global SU(2) spin symmetry ensuring that $\mathcal{D}_a^{-1}(\mathbf{q} = 0, i\omega = 0) = 0$.

When we combine this with the bare propagators implied by Eqs. (5) and (6) the following physical picture emerges. In the superconducting state, both v_μ and a_μ are massive by virtue of vortex excitations being bound in finite loops. The massive character of v_μ and a_μ protects the coherent TF excitations from being smeared by vortex fluctuations. The coupling of TF's to the gauge fields v_μ and a_μ is *irrelevant*. This is our inverted Fermi liquid phase.

In the normal (pseudogap) state, the situation changes dramatically. The bare propagators for v_μ and a_μ are now massless but the renormalization by the medium of TF's *screens* these bare propagators and still keeps v_μ massive. Thus, TF coupling to the Doppler shift and "spin waves" remains *irrelevant* even in the normal state. The Berry gauge field a_μ, however, is now truly *massless* since the spin polarization in the medium of TF's *cannot* fully screen the massless bare propagator. Instead, by computing the TF polarization, we find $\mathcal{D}_a^{-1} \propto \frac{1}{8}\sqrt{\omega^2 + q^2}$ for $(\mathbf{q}, i\omega) \rightarrow 0$; stiffer than the Maxwellian form [Eqs. (5) and (6)], but still massless. The massless gauge field a_μ produces strong scattering at low energies and affects qualitatively the spectral properties of TF's.

The low energy quasiparticles are located at the four nodal points of the d_{xy} gap function: $(\pm k_F, 0)$ and $(0, \pm k_F)$, hereafter denoted as $(1, \bar{1})$ and $(2, \bar{2})$, respectively. Linearizing the fermionic spectrum in the proximity of these nodes leads to the effective Lagrangian,

$$\mathcal{L}_D = \sum_{\alpha=1,\bar{1}} \Psi_\alpha^\dagger [D_\tau - i v_F D_x \sigma_3 - i v_\Delta D_y \sigma_1] \Psi_\alpha + \sum_{\alpha=2,\bar{2}} \Psi_\alpha^\dagger [D_\tau - i v_F D_y \sigma_3 - i v_\Delta D_x \sigma_1] \Psi_\alpha + \mathcal{L}_0[a_\mu], \qquad (7)$$

where $\Psi_\alpha^\dagger$ is a two-component nodal spinor, α is a node index, $D_\mu = \partial_\mu + i a_\mu$, and $\mathcal{L}_0$ is given by (6). We have dropped the Doppler gauge field v_μ since it is massive both below and above T_{SC} and irrelevant for our purposes.

The Lagrangian (7) and the physics it embodies are our main results. In the normal state, a_μ becomes massless and the problem of quasiparticle interactions with vortex fluctuations takes the form of topological fermions interacting with massless "berryons," i.e., quanta of the Berry gauge field a_μ. We recognize the above theory as equivalent (apart from the intrinsic anisotropy) to the Euclidean quantum electrodynamics of massless Dirac fermions in (2 + 1) dimensions. The remarkable feature of QED$_3$ is that it naturally generates a non-Fermi liquid phenomenology for its fermionic excitations. This property of QED$_3$ has led to previous suggestions that it should be, in some form, relevant to cuprate superconductivity [15,16]. However, the *physical content* of QED$_3$ as an effective low energy theory in this Letter is entirely different from those earlier works.

We now discuss the low energy phenomenology governed by the TF propagator: $G_\alpha^{-1}(\mathbf{k}, \omega) = G_{\alpha 0}^{-1}(\mathbf{k}, \omega) - \Sigma_\alpha(\mathbf{k}, \omega)$, where $G_{\alpha 0}^{-1}$ is a free Dirac propagator at node α. We first consider the $T = 0$ case in the isotropic limit ($v_F = v_\Delta$) where explicit results are readily obtained. In this case $G_{\alpha 0}^{-1} = \omega - v_F k_x \sigma_3 - v_\Delta k_y \sigma_1$ and we find

$$\Sigma_\alpha = \frac{8}{3\pi^2 N}(-\omega + v_F k_x \sigma_3 + v_\Delta k_y \sigma_1) \ln(\Lambda/p), \qquad (8)$$

with $p = (-\omega^2 + v_F^2 k_x^2 + v_\Delta^2 k_y^2)^{1/2}$, Λ is a high energy cutoff, and $N = 2$ is the number of pairs of nodes.

The essential feature of the TF propagator is the singular behavior of the self-energy $\Sigma_\alpha(\mathbf{k}, \omega)$ which arises from the massless nature of the dressed berryon propagator $\mathcal{D}_a(\mathbf{q}, \omega)$ and is logarithmic in the leading order. This result can be formalized as the leading term in a large N expansion. Ultimately, the resummation of such an expansion [15] yields a power law singularity $G_\alpha \propto p^{\eta - 1}$, with a small exponent $\eta = -8/3\pi^2 N$. For our purposes, having to deal with both the anisotropy and finite T, the leading order form (8) is more convenient since it allows for explicit computation of various quantities. Once we move beyond the leading order, the vertex corrections to Σ_α are necessary and the algebra becomes impenetrable. Furthermore, the available experiments are unlikely to distinguish between $\eta = 0^+$ and a small finite exponent.

The singularity in Σ_α heralds the breakdown of the Fermi liquid behavior in the normal state. To see this, consider Eq. (8) for $E_\mathbf{k} \equiv (v_F^2 k_x^2 + v_\Delta^2 k_y^2)^{1/2} \ll |\omega|$. We find $\Sigma_\alpha \propto -(8/3\pi^2 N)\omega \ln(\Lambda/\sqrt{-\omega^2})$. The residue of the fermion pole vanishes as $\omega \rightarrow 0$, $Z(\omega) \sim 1/\ln|\omega|$, while its width goes as $\Sigma_\alpha'' = -(4/3\pi N)|\omega|$. This behavior is reminiscent of the marginal Fermi liquid (MFL) expression for the self-energy, assumed on phenomenological grounds by Varma *et al.* [17]. Note, however, that our QED$_3$ TF propagator implied by Eq. (8) remains *qualitatively different* from the MFL *ansatz* [17], both by the fact that $\ln(\Lambda/p)$ is replaced by a weak power law (thus *algebraic* Fermi liquid) and by the momentum dependence of $\Sigma_\alpha(\mathbf{k}, \omega)$. As shown below it is this combined momentum-frequency dependence that provides a natural explanation for some of the remarkable features of the fermionic spectral function in cuprates observed in the angle-resolved photoemission spectroscopy (ARPES) experiments [18]. Also, we emphasize that our results apply to the pseudogap phase below T^*. The physics of the normal state at higher temperatures is beyond the scope of our present theory.

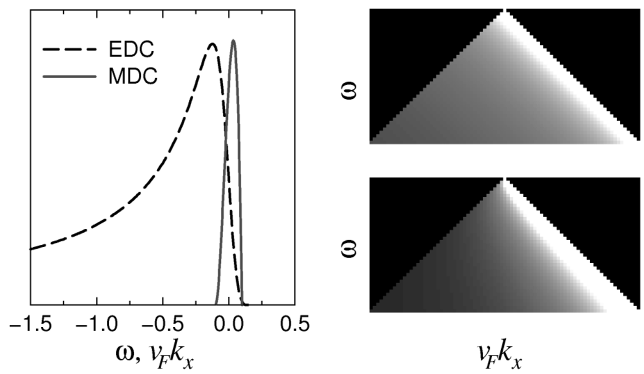

FIG. 2. Energy versus momentum distribution curves of $A(\mathbf{k}, \omega)$. Left: EDC cut taken for $\mathbf{k} = 0$ (coincident with a nodal point), and MDC cut taken for $\omega = 0^-$ and $k_y = 0$. Both curves have been broadened (by the same amount) to simulate the finite resolution of an ARPES experiment. Right: the corresponding spectral function density plots for isotropic (top) and anisotropic $v_F/v_\Delta = 17$ (bottom) cases (to be compared with Figs. 1 and 2 of Ref. [18]).

Inspection of Eq. (8) reveals that Σ_α has an imaginary part only inside the cone defined by $\omega^2 > v_F^2 k_x^2 + v_\Delta^2 k_y^2$; outside this cone Σ''_α vanishes. This implies that TF spectral function plotted as a function of momentum at fixed ω [momentum distribution curve (MDC)] will be very sharp close to the Fermi surface, while the corresponding energy distribution curve (EDC) will be broad. This is illustrated in Fig. 2, where we plot the spectral function $A(\mathbf{k}, \omega) = \pi^{-1} \operatorname{Im}[G_\alpha(\mathbf{k}, \omega)]_{11}$ deduced from Eq. (8). We note that precisely such striking asymmetry between the EDC and MDC cuts is observed in the ARPES data [18].

These qualitative features of the spectral function survive at finite temperature and away from the isotropic limit. Unfortunately, away from this simple limit the precise form of the TF propagator is not known: as soon as the "relativistic" invariance of the $T = 0$ problem (7) is lost, analytic calculations become intractable. We find that, for $T \ll \omega, E_\mathbf{k}$, the self-energy retains its $T = 0$ form [Eq. (8)] with a small temperature correction. On the other hand, when $T \gg \omega, E_\mathbf{k}$, we find $\Sigma''_\alpha \sim T$, qualitatively consistent with the original MFL conjecture $\Sigma''_\alpha \sim \max(\omega, T)$. We note that such a T-linear scattering rate has been deduced from ARPES experiments [18].

In ARPES, one measures the spectral function of *real* electrons, not of TF's. While the inversion of the transformation (3) after the phases have been coarse grained and replaced by the gauge fields is a daunting task, our theory ensures the gauge invariance with respect to a_μ of the true electron propagator. The simplest such gauge invariant propagator is $G_{11}^{\mathrm{elec}}(x, x') \approx \langle \exp(i \int_x^{x'} ds_\mu a_\mu)[\tilde{\Psi}(x)\tilde{\Psi}^\dagger(x')]_{11}\rangle$, where $x = (\mathbf{r}, \tau)$. By employing a gauge in which the line integral of a_μ vanishes [19], we have computed the asymptotic behavior of $G^{\mathrm{elec}}(x, x')$. We find [14] that it exhibits a power law singularity with the exponent $\eta' = 2\eta = -16/3\pi^2 N$. This strongly suggests that the true electron propagator, whose precise form within QED_3 is unknown at present, will exhibit a power law with a small positive exponent.

In conclusion, we argue that the pseudogap regime in cuprates can be modeled as a phase disordered d-wave superconductor. Such an assumption naturally leads to a QED_3 theory for the massless Dirac "topological" fermions interacting with a massless gauge field of vortex berryons. Coupling to the massles gauge field destroys the Fermi liquid pole in the fermion propagator and generates algebraic Fermi liquid. Lacking any energy or length scale this theory can be thought of as being *critical,* independently of the actual doping level x. Below T^* the low energy spectral properties of the fermions are therefore regulated by a *quantum critical line.* In this regime the low energy fermiology, including thermodynamics, transport, and density and current responses are all controlled by the universal properties of topological fermions and vortex berryons encoded in the anisotropic QED_3 Lagrangian (7). Eventually, this peculiar quantum critical behavior gives way to the actual superconducting phase at $T_{\mathrm{SC}}(x)$, and the Fermi liquid character of the nodal quasiparticles is restored as vortices bind into finite loops. At very low doping, hole Wigner crystal, spin-density wave, and other low-T phases become possible, reflecting the strong Mott-Hubbard correlations.

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