Spin-Orbit Coupling Effects on Quantum Transport in Lateral Semiconductor Dots

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The effects of interplay between spin-orbit coupling and Zeeman splitting on weak localization and universal conductance fluctuations in lateral semiconductor quantum dots are analyzed: All possible symmetry classes of corresponding random matrix theories are listed and crossovers between them achievable by sweeping magnetic field and changing the dot parameters are described. We also suggest experiments to measure the spin-orbit coupling constants.

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The effects of spin-orbit (SO) coupling on transport phenomena in chaotic quantum dots [1,2] recently attracted attention. Motivated by a puzzling modification of the variance of the mesoscopic conductance fluctuations with applied in-plane magnetic field, Halperin *et al.* [2] suggested that the specific form of the spin-orbit interaction in a 2D electron gas based on semiconductor heterostructures may be responsible for a series of crossovers not considered in the existing literature [3]. It has also been noticed [2,4] that spin relaxation in a quantum dot may be facilitated by the Zeeman field.

The goal of this Letter is threefold: (i) we identify all possible symmetry classes which arise from the interplay between SO coupling and Zeeman splitting in a disordered or chaotic semiconductor quantum dot and describe all the physically achievable parametric dependencies treated as crossovers between distinct symmetry classes; (ii) we provide a complete quantitative theory for the transport characteristics of such a dot based upon both diagrammatic perturbation theory analysis and the use of random matrix theory approach, and (iii) we show that the SO coupling effects depend on the magnetic field orientation in anisotropic dots and discuss possible experiments enabling one to measure directly the ratio between two independent SO constants.

The single-particle Hamiltonian of the system, $H = H_0 + u(\vec{r})$, is the sum of the free-electron dispersion term and a potential, $u(\vec{r})$, consisting of a confining potential and a random potential of impurities. The free-electron term includes spin-orbit coupling, as a combination of a Rashba term and a crystalline anisotropy term [specified for the (001) plane of GaAs], and Zeeman splitting energy due to the in-plane magnetic field [5], $\vec{B} = \vec{l}B$,

$$H_0 = \frac{p^2}{2m} + \frac{\alpha}{m}[\vec{p} \times \vec{n}_z]\frac{\vec{\sigma}}{2} + \frac{\varrho}{m}\left(p_x\frac{\sigma_x}{2} - p_y\frac{\sigma_y}{2}\right) + \vec{l}\,\frac{\vec{\sigma}}{2}\epsilon_Z ,$$

where $\vec{p} = \vec{P} - \vec{A}$ is the kinetic momentum, with $\vec{P}$ being the canonical momentum and $\vec{A} = eB_z[\vec{r} \times \vec{n}_z]/2c$ being the vector potential describing the orbital effect of the magnetic field. Since the (001) plane of GaAs has the symmetry of a square without inversion center, C_{2v}, we choose the coordinate system (x_1, x_2) with axes along crystallographic directions $\vec{e}_1 = [110]$ and $\vec{e}_2 = [1\bar{1}0]$ and rewrite H_0 as

$$H_0 = \frac{1}{2m}\left[\left(p_1 - \frac{\hbar\sigma_2}{2\lambda_1}\right)^2 + \left(p_2 + \frac{\hbar\sigma_1}{2\lambda_2}\right)^2\right] + \vec{l}\,\frac{\vec{\sigma}}{2}\epsilon_Z , \tag{1}$$

where $\hbar\lambda_{1,2}^{-1} = \alpha \pm \varrho$ characterize the length scale associated with the strength of the spin-orbit coupling for electrons moving along principal crystallographic directions [$\sigma_{1,2,3}$ are Pauli matrices, $\sigma_2 = -\sigma_2^T$, $\sigma_{1,3} = \sigma_{1,3}^T$].

The Hamiltonian $H = H_0 + u(\vec{r})$ describes the electron motion in a lateral semiconductor dot coupled to metallic leads via two contacts, l and r, each with $N_{l,r} \gtrsim 1$ open orbital channels. In the present Letter, we focus on chaotic systems in the regime of a hard chaos (i.e., far from integrability and neglecting effects of weakly unstable orbits) and the regime of disordered dots. That is why below we use the universal 0D description (known to be equivalent to the random matrix theory approach [6]) applicable if [7,8]

$$\gamma, \epsilon_Z \ll E_T; \qquad L_{1,2} \ll \lambda_{1,2} . \tag{2}$$

Here, $\gamma/\hbar = (N_l + N_r)\Delta/2\pi\hbar$ stands for the escape rate into the leads via ballistic adiabatic contacts with $N_{r,l}$ reflectionless channels [1,3], $\Delta = 2\pi\hbar^2/m\mathcal{A}$ is the mean level spacing in a dot with area $\mathcal{A} \sim L_1L_2$, and E_T is the conventional Thouless energy. The last inequality in Eq. (2) allows us to treat the SO coupling as weak.

Our purpose now is to identify Hamiltonian (1) with an appropriate random matrix ensemble. Doing it directly, however, is not convenient. The reason for this is that on shell matrix elements of the velocity vanish due to the gauge invariance (the importance of this fact for SO interaction in quantum dots was first noticed in Ref. [2]). It means that if the spin remained fixed during the motion of the electron, the effect of SO coupling would be just a homogeneous shift of the momentum space which could not change observables. To get rid of such terms fixed by gauge invariance, we perform the unitary transformation of the Hamiltonian as $H \to \tilde{H} = U^{\dagger} H U$ with

$$U = \exp\left(\frac{ix_1\sigma_2}{2\lambda_1} - \frac{ix_2\sigma_1}{2\lambda_2}\right). \tag{3}$$

Using the condition $L_{1,2}/\lambda_{1,2} \ll 1$, we expand $\tilde{H}$ up to the second order in the coordinates and obtain

$$\tilde{H} = \frac{1}{2m}(\vec{P} - \vec{A} - \vec{a}_{\perp}\frac{\sigma_z}{2} - \vec{a}_{\|})^2 + h^{(0)} + h^{(1)} + u(\vec{r}),$$
$$\vec{A} = eB_z[\vec{r} \times \vec{n}_z]/2c; \qquad \vec{a}_{\perp} = \hbar[\vec{r} \times \vec{n}_z]/2\lambda_1\lambda_2; \tag{4}$$

$$\vec{a}_{\|} = \frac{\hbar}{6}\frac{[\vec{r} \times \vec{n}_z]}{\lambda_1\lambda_2}\left(\frac{x_1\sigma_1}{\lambda_1} + \frac{x_2\sigma_2}{\lambda_2}\right), \tag{5}$$

$$h^{(0)} = \epsilon_Z \vec{l}\,\frac{\vec{\sigma}}{2}; \qquad h^{(1)} = -\sigma_z\frac{\epsilon_Z}{2}\left(\frac{l_1x_1}{2\lambda_1} + \frac{l_2x_2}{2\lambda_2}\right). \tag{6}$$

Equation (4) indicates that the effects of SO coupling in the leading order at $\epsilon_Z = 0$, and of the orbital magnetic field are somewhat similar. This similarity is not a coincidence—in the leading order, the direction of the spin follows the motion of the electron: for an electron moving along a closed path, its spin spans the closed path, too. Because of the motion in spin space, an electron accumulates extra Berry phase equal to the solid angle spanned by the spin. At weak SO coupling, this area is proportional to the geometrical area encircled by the electron path in the coordinate space resulting in an effect similar to that of Aharonov-Bohm flux. This analogy may be put on a quantitative level by noticing that the two energy scales characterizing both effects,

$$\tau_B^{-1} = \frac{4\pi B_z^2}{\Delta}\langle|M_{\alpha\beta}|^2\rangle = \kappa E_T\left(\frac{2eB_z\mathcal{A}}{c\hbar}\right)^2; \tag{7}$$

$$\epsilon_{\perp}^{so} = \kappa E_T(\mathcal{A}/\lambda_1\lambda_2)^2, \tag{8}$$

have the same dependence on the shape and the disorder in the sample. Here, κ is the coefficient dependent on the geometry and $\mathcal{A}$ is the area of the dot. Random quantities $M_{\alpha\beta}$ are the nondiagonal matrix elements of the magnetic moment of the electron in the dot.

Term (5) is higher order in the SO coupling constant. However, it has a different symmetry from $\vec{a}_{\perp}$; therefore, its retention is legitimate. Its physical significance is to provide the spin flips and, thus, the complete spin relaxation, in contrast to $\vec{a}_{\perp}$ which preserves correlations between spin up and spin down states. Quantitatively, the effect of $\vec{a}_{\|}$ is characterized by the scale

$$\epsilon_{\|}^{so} \sim [(L_1/\lambda_1)^2 + (L_2/\lambda_2)^2]\epsilon_{\perp}^{so} \ll \epsilon_{\perp}^{so}. \tag{9}$$

The effect of the in-plane magnetic field is described by Eq. (6). It includes the homogeneous Zeeman splitting $h^{(0)}$ and the combined effect of the SO interaction and Zeeman splitting described by $h^{(1)}$. The latter can be envisaged as a deflection of the effective magnetic field from the direction given by external $\vec{B}$, and it results in the spin relaxation associated with the energy scale

$$\epsilon_{\perp}^{Z} = \frac{\epsilon_Z^2}{2\Delta}\sum_{i,j=1,2}\frac{l_i}{\lambda_i}\frac{l_j}{\lambda_j}\Xi_{ij}, \qquad \Xi_{ij} = \pi\langle x_i^{\alpha\beta}x_j^{\beta\alpha}\rangle, \tag{10}$$

where $x_{1,2}^{\alpha\beta}$ are the nondiagonal matrix elements of the dipole moment of the electron in the dot. Quantity Ξ_{ij} depends on the geometry and on the disorder in the dot and may be estimated as $\Xi \simeq \Delta L^2/E_T$, so that $\epsilon_{\perp}^{Z} \ll \epsilon_Z$. A similar energy scale has appeared in recent publications [2,4]; however, the symmetry of the corresponding term $h^{(1)}$ was not identified.

Having derived a Hamiltonian free of the gauge invariance constraints on the values of its matrix elements, we identify the symmetries of all relevant limits. The results are summarized in Tables I and II depending on the orbital effect of the magnetic field. In these tables, the conventional parameter β describes time-reversal symmetry of the orbital motion, s is the Kramers degeneracy parameter, and Σ is an additional parameter characterizing the mixing of states with different spins for strong Zeeman splitting. Parameters β, Σ, and s completely characterize the statistical properties of the transport through the system as well as spectral correlations of the isolated dot. The straightforward generalization of the known results, [3], gives the following description of the two-terminal conductance (measured in units of $\frac{e^2}{2\pi\hbar}$) of the dot connected to the leads by ballistic adiabatic contacts with N_l and N_r reflectionless orbital channels [1,3]:

$$\langle g\rangle = \frac{2\Sigma N_l N_r}{(N_l + N_r)\Sigma + (\frac{2}{\beta} - 1)}, \tag{11}$$

$$\langle(\delta g)^2\rangle = \left(\frac{s}{\beta\Sigma}\right)\frac{\Sigma^2 N_l N_r[\Sigma N_l + (\frac{2}{\beta} - 1)][\Sigma N_r + (\frac{2}{\beta} - 1)]}{[(N_l + N_r)\Sigma + (\frac{2}{\beta} - 1)]^2[(N_l + N_r)\Sigma + (\frac{4}{\beta} - 1)][(N_l + N_r)\Sigma + (\frac{2}{\beta} - 2)]}. \tag{12}$$

The parametric dependences of the transport coefficients in Eqs. (11) and (12) can be envisaged as a sequence of crossovers shown below as a function of Zeeman splitting energy and escape rate (increasing along the horizontal and vertical axes, respectively):

TABLE I. Symmetries of the system in the absence of orbital magnetic field effect, $\tau_B \gg \tau_{esc}$.

	Zeeman	Spin orbit	Additional symmetry of $\tilde{H} = \tilde{H}^\dagger$	Symmetry group	β	Σ	s	Applicability
1	$h^{(0,1)} = 0$	$\vec{a}_{\perp,\parallel} = 0$	$\tilde{H}^T = \tilde{H},\ [\tilde{H}, \sigma_{1,2,3}] = 0$	$\frac{\mathrm{O}(N)\otimes\mathrm{O}(N)}{\mathrm{O}(N)}$	1	1	2	$\epsilon_Z,\ \epsilon_\perp^{so} \ll \gamma$
2	$h^{(0,1)} = 0$	$\begin{cases} \vec{a}_\perp \neq 0 \\ \vec{a}_\parallel = 0 \end{cases}$	$\sigma_2 \tilde{H}^T \sigma_2 = \tilde{H},\ [\tilde{H}, \sigma_3] = 0$	$\frac{\mathrm{U}(N)\otimes\mathrm{U}(N)}{\mathrm{U}(N)}$	2	1	2	$\frac{\epsilon_Z^2}{\epsilon_\perp^{so}},\ \epsilon_\parallel^{so} \ll \gamma \ll \epsilon_\perp^{so}$
3	$h^{(0,1)} = 0$	$\vec{a}_{\perp,\parallel} \neq 0$	$\sigma_2 \tilde{H}^T \sigma_2 = \tilde{H}$	$\mathrm{Sp}(2N)$	4	1	2	$\frac{\epsilon_Z^2}{\epsilon_\perp^{so}} \ll \gamma \ll \epsilon_\parallel^{so}$
4	$\begin{cases} h^{(0)} \neq 0 \\ h^{(1)} = 0 \end{cases}$	$\vec{a}_{\perp,\parallel} = 0$	$\tilde{H}^T = \tilde{H},\ [\tilde{H}, \vec{B}\vec{\sigma}] = 0$	$\mathrm{O}(N) \otimes \mathrm{O}(N)$	1	1	1	$\epsilon_\perp^Z,\ \epsilon_\perp^{so} \ll \gamma \ll \epsilon_Z$
5	$\begin{cases} h^{(0)} \neq 0 \\ h^{(1)} = 0 \end{cases}$	$\vec{a}_\perp \neq 0$, $\vec{a}_\parallel = 0$	$\sigma_1 \tilde{H}^T \sigma_1 = \tilde{H}$	$\mathrm{O}(2N)$	1	2	1	$\epsilon_\perp^Z,\ \epsilon_\parallel^{so} \ll \gamma \ll \epsilon_\perp^{so},\ \frac{\epsilon_Z^2}{\epsilon_\perp^{so}}$
6	$\begin{cases} h^{(0)} \neq 0 \\ h^{(1)} = 0 \end{cases}$	$\vec{a}_\perp \neq 0$, $\vec{a}_\parallel \neq 0$	None	$\mathrm{U}(2N)$	2	2	1	$\epsilon_\perp^Z \ll \gamma \ll \epsilon_\parallel^{so},\ \frac{\epsilon_Z^2}{\epsilon_\perp^{so}}$
7	$h^{(0,1)} \neq 0$	$\vec{a}_{\perp,\parallel} = 0$	$\sigma_\perp \sigma_2 \tilde{H}^T \sigma_2 \sigma_\perp = \tilde{H};\ \sigma_\perp = \vec{\sigma}(\vec{l}_z \times \vec{l})$	$\mathrm{O}(2N)$	1	2	1	$\epsilon_\perp^{so} \ll \gamma \ll \epsilon_\perp^Z$
8	$h^{(0,1)} \neq 0$	$\vec{a}_\perp \neq 0$	None	$\mathrm{U}(2N)$	2	2	1	$\gamma \ll \epsilon_\perp^{so},\ \epsilon_\perp^Z$

$$
\begin{array}{ccccc}
1[1u] & \longrightarrow & 4[4u] & \longrightarrow & 7[6u] \\
\uparrow & & \uparrow & & \uparrow \quad \gamma \\
2[2u] & \longrightarrow & 5[5u] & \longrightarrow & 8[6u] \quad \Uparrow \\
\uparrow & & \uparrow & \nearrow & \qquad \Longrightarrow \epsilon_Z \\
3[3u] & \longrightarrow & 6[6u]. & &
\end{array}
\tag{13}
$$

For a large number of channels, $1 \ll N_l + N_r \ll E_T/\Delta$, the result of Eqs. (11) and (12) can be simplified:

$$
g_{wl} = -\frac{1 - \frac{1}{2}\beta}{\beta\Sigma}\,\xi; \qquad \langle \delta g^2 \rangle = \frac{s\xi^2}{16\beta\Sigma},
$$
$$
\xi = \frac{4N_l N_r}{(N_l + N_r)^2} \tag{14}
$$

and the crossover in weak localization (WL) behavior between various symmetry classes in Tables I and II sketched in (13) can be described quantitatively using a diagrammatic perturbation theory [9], as we outline below for the WL correction.

For a spin-$\frac{1}{2}$ particle in a quantum dot with ballistic adiabatic contacts, the WL correction to the conductance can be related to the lowest-lying modes of Cooperons in a singlet $(L = 0)$, $P_C^{00} = \frac{1}{2}\,\mathrm{tr}\langle \sigma_2 \hat{G}_R^T(\varepsilon)\sigma_2 \hat{G}_A(\varepsilon - \omega)\rangle$, and three triplet $(L = 1,2,3)$ channels, $P_C^{LM} = \frac{1}{2}\,\mathrm{tr}\langle \sigma_L \sigma_2 \hat{G}_R^T(\varepsilon)\sigma_2 \sigma_M \hat{G}_A(\varepsilon - \omega)\rangle$, as $g_{wl} \propto P_C^{00} - \sum_{M=1,2,3} P_C^{MM}$ (Ref. [9]). In the absence of SO coupling and Zeeman splitting, Cooperons P_C^{LM} split into completely independent channels: one singlet and three triplet, $\hat{P} = \hat{\delta} P$, where $\hat{\delta} \equiv \delta^{LM}$ and P obeys the diffusion equation. The SO coupling and Zeeman splitting mix up various components [11] and split their spectra, which modifies the diffusion equation into the matrix equations $\hat{\Pi}\hat{P}(\vec{X}, \vec{X}') = \hat{\delta} \cdot \delta(\vec{X}, \vec{X}')$,

$$
\hat{\Pi} = \gamma\hat{\delta} - D\hbar^2\Big(\hat{\delta}\,\partial_{X_1} + i\,\frac{2}{\hbar}A_1\hat{\delta} - i\hat{S}_2\lambda_1^{-1}\Big)^2 - D\hbar^2\Big(\hat{\delta}\,\partial_{X_2} + i\,\frac{2}{\hbar}A_2\hat{\delta} + i\hat{S}_1\lambda_2^{-1}\Big)^2 + i\epsilon_Z\hat{\eta}, \tag{15}
$$

where $\hat{S}_K^{LM} = -i\varepsilon^{KLM}$ are spin-1 operators $(K = 1,2,3)$ and ε^{KLM} is the antisymmetric tensor $(K, L, M = 1,2,3)$. As a 4×4 matrix, $\hat{S}$ also has zero elements when $L = 0$ or $M = 0$. The other relevant matrix is $\hat{\eta}$, defined as $\eta^{LM} = l_L \delta_{0M} + \delta_{0L} l_M$, indicating that coherence between oppositely polarized electrons is lost on the time scale of ϵ_Z^{-1}. D is the classical diffusion coefficient. Equation (15) is supplemented with the boundary condition at the edge of the dot characterized by the normal direction $\vec{n}_\parallel = (n_1, n_2)$,

$$
\Big[\vec{n}_\parallel \cdot \hat{\delta}\Big(\nabla + i\,\frac{2}{\hbar}\vec{A}\Big) - in_1\hat{S}_2\lambda_1^{-1} + in_2\hat{S}_1\lambda_2^{-1}\Big]\hat{P} = 0.
$$

The correspondence between random matrix theory description of a disordered system and diagrams is usually

TABLE II. Symmetries of the system in the presence of orbital magnetic field effect, $\tau_B \ll \tau_{esc}$.

	Zeeman	Spin orbit	Additional symmetry of $\tilde{H} = \tilde{H}^\dagger$	Symmetry group	β	Σ	s	Applicability intervals
1u	$h^{(0,1)} = 0$	$\vec{a}_{\perp,\parallel} = 0$	$[\tilde{H}, \sigma_{1,2,3}] = 0$	$\frac{\mathrm{U}(N)\otimes\mathrm{U}(N)}{\mathrm{U}(N)}$	2	1	2	$\epsilon_Z,\ \epsilon_\perp^{so} \ll \gamma$
2u	$h^{(0,1)} = 0$	$\begin{cases} \vec{a}_\perp \neq 0 \\ \vec{a}_\parallel = 0 \end{cases}$	$[\tilde{H}, \sigma_3] = 0$	$\mathrm{U}(N) \otimes \mathrm{U}(N)$	2	1	1	$\frac{\epsilon_Z^2}{\epsilon_\perp^{so}},\ \epsilon_\parallel^{so} \ll \gamma \ll \epsilon_\perp^{so}$
3u	$h^{(0,1)} = 0$	$\vec{a}_{\perp,\parallel} \neq 0$	None	$\mathrm{U}(2N)$	2	2	1	$\frac{\epsilon_Z^2}{\epsilon_\perp^{so}} \ll \gamma \ll \epsilon_\parallel^{so}$
4u	$\begin{cases} h^{(0)} \neq 0 \\ h^{(1)} = 0 \end{cases}$	$\vec{a}_{\perp,\parallel} = 0$	$[\tilde{H}, \vec{B} \cdot \vec{\sigma}] = 0$	$\mathrm{U}(N) \otimes \mathrm{U}(N)$	2	1	1	$\epsilon_\perp^Z,\ \epsilon_\perp^{so} \ll \gamma \ll \epsilon_Z$
5u	$h^{(0)} \neq 0$	$\vec{a}_\perp \neq 0$	None	$\mathrm{U}(2N)$	2	2	1	$\gamma \ll \epsilon_\perp^{so},\ \frac{\epsilon_Z^2}{\epsilon_\perp^{so}}$
6u	$h^{(0,1)} \neq 0$	$\vec{a}_{\perp,\parallel} = 0$	None	$\mathrm{U}(2N)$	2	2	1	$\epsilon_\perp^{so} \ll \gamma \ll \epsilon_\perp^Z$

transparent in the zero-dimensional (0D) approximation in the diffusion problem, $E_T \to \infty$, when the lowest modes are taken in the coordinate-independent form and coupling to higher modes is treated as a perturbation. Here, the boundary condition requires the use of rotation to a local spin-coordinate system, $\hat{P} = \hat{O}\hat{\tilde{P}}\hat{O}^{-1}$, prior to making the 0D approximation, with $\hat{O} = \exp\{i[\hat{S}_1 X_2 \lambda_2^{-1} - \hat{S}_2 X_1 \lambda_1^{-1}]\}\exp\{-i\varphi_s(\vec{X})\hat{S}_3\}\exp\{-i\varphi_A(\vec{X})\}$ where harmonic functions φ transform the symmetric gauge in Eq. (4) to such a gauge, where vector potentials on the boundary are tangential to it. This eliminates the lowest orders SO coupling terms from the boundary condition, and, in a small dot [8] $L_{1,2} \ll \lambda_{1,2}$, can be followed by a perturbative analysis of extra terms generated by rotation $\hat{O}$ in Eq. (15). This step results in the 0D matrix equation for the Cooperon,

$$\hat{P} = [\gamma\hat{\delta} + i\epsilon_Z\hat{\eta} + (\hat{\delta}\sqrt{\tau_B^{-1}} - \sqrt{\epsilon_\perp^{so}}\hat{S}_3)^2 + \epsilon_\perp^Z(\hat{\delta} - \hat{S}_3^2) + \epsilon_\parallel^{so}(\widehat{\vec{S}^2} - \hat{S}_3^2)]^{-1}. \quad (16)$$

The form of Eq. (16) is applicable beyond the diffusive approximation as it follows from purely the symmetry considerations. The difference in the third term in brackets reflects the addition or subtraction of the Berry and Aharonov-Bohm phases, as was pointed out in Ref. [12]. The expression for the weak localization correction can be found from Eq. (16) as $g_{wl} \propto \mathrm{tr}\{\hat{P}[\hat{\delta} - \widehat{\vec{S}^2}]\}$. In a dot with $\tau_B^{-1} = 0$ and γ, ϵ_Z, $\hbar^2 D/\lambda_{1,2}^2 \ll E_T$ [8], this yields

$$\frac{4g_{wl}}{\xi} \approx -\frac{\gamma}{\gamma + \epsilon_\perp^{so}} - \frac{\gamma}{\gamma + \epsilon_\perp^Z + 2\epsilon_\parallel^{so}} + \frac{\epsilon_\perp^{so}}{\gamma + \epsilon_\perp^{so} + \frac{\epsilon_Z^2}{\gamma}}, \quad (17)$$

where we used the fact that $\epsilon_\perp^Z \ll \epsilon_Z$. It is interesting to notice that the application of the Zeeman field alone (in plane magnetic field [1]) does not suppress the weak localization completely at $\epsilon_Z \ll E_T$. In the opposite case of $\epsilon_Z \gtrsim E_T$, it does (though it has to be studied beyond the universal limit). However, the effect of such a strong in-plane field on orbital motion becomes already sufficient to suppress the weak localization [13].

The form of Eq. (17) in the experimentally easier achievable "high"-energy crossover $1 \to 4 \to 7$ is limited by only the two first terms and suggests a possible procedure for measuring the ratio λ_1/λ_2. By fitting experimental magnetoresistance data to $g_{wl}(\epsilon_Z)$ in Eq. (17), one would determine the characteristic in-plane field $\mathcal{B}$. For a dot with a strongly anisotropic shape, such a parameter would depend on the orientation of an in-plane magnetic field. In particular, $\mathcal{B}$ can be measured for two orientations of $\vec{B} = B\vec{l}$: namely, $\mathcal{B}_{[110]}$ for $\vec{l} = [110]$ and $\mathcal{B}_{[1\bar{1}0]}$ for $\vec{l} = [1\bar{1}0]$. One should also make a simultaneous measurement of two characteristic fields $\mathcal{B}'_{[110]}$ and $\mathcal{B}'_{[1\bar{1}0]}$ in a dot produced on the same chip by rotating the same lithographic mask by 90°. The anisotropy of the SO coupling can then be obtained directly from the ratio

$$(\mathcal{B}_{[110]}\mathcal{B}'_{[110]}/\mathcal{B}_{[1\bar{1}0]}\mathcal{B}'_{[1\bar{1}0]}) = (\lambda_1/\lambda_2)^4,$$

independently of the details of sample geometry.

The other interesting feature may be observed in the weak localization g_{wl} in the regime of a crossover $2 \to 2u$ driven by a weak perpendicular magnetic field for $\epsilon_Z = 0$. Since the SO coupling effect in Eqs. (4) and (16) acts as a homogeneous magnetic field distinguishing between up- and down-spin electrons, the external field can be used to compensate the effect of the SO coupling for one spin component [12], which would produce a dip in the weak localization correction. Indeed, at $\gamma \ll \epsilon_\perp^{so}$, the function

$$\frac{2g_{wl}}{\xi} \approx \frac{\gamma\epsilon_\parallel^{so}}{(\gamma + \tau_B^{-1})(\gamma + \tau_B^{-1} + 2\epsilon_\parallel^{so})} - \frac{\gamma(\gamma + \tau_B^{-1} + \epsilon_\perp^{so} + \epsilon_\parallel^{so})}{(\gamma + \tau_B^{-1} + \epsilon_\perp^{so} + \epsilon_\parallel^{so})^2 - 4\tau_B^{-1}\epsilon_\perp^{so}} \quad (18)$$

has a minimum at the value of the field $2e\mathcal{B}_z = c\hbar/(\lambda_1\lambda_2)$, independently of the sample geometry, which should provide a very accurate measurement of $\lambda_1\lambda_2$.

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[5] Zeeman field is taken parallel to the plane of 2D gas, $\vec{l} = (l_x, l_y)$. The effect of its normal component is negligible when $B_z\mathcal{A} < \Phi_0$, since $\epsilon_Z(\Phi_0/\mathcal{A}) \sim (g^* m/m_e)\Delta \ll \Delta$.
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[8] If $\lambda_{1,2} \ll L_{1,2}$, the use of a gauge transformation is not helpful, and the spin relaxation problem has to be treated as a 2D one. This reproduces the Dykonov-Perel relaxation [10] with the rates $\epsilon_{DP}^{so} \sim \hbar^2 D/\lambda^2 \gg E_T$, associated with the weak localization scenario, where the symplectic ensemble at $\epsilon_Z = 0$ transforms into the unitary ensemble at $\epsilon_Z^2/\epsilon_{DP}^{so} > \gamma$.
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