Rayleigh-Taylor Instabilities in Thin Films of Tapped Powder

Jacques Duran*
LMDH, UMR 7603 CNRS, Université Pierre et Marie Curie, 4 place Jussieu, 75252 Paris Cedex 05, France
(Received 2 March 2001; revised manuscript received 19 September 2001; published 30 November 2001)

We observe powder "droplets" forming when tapping repeatedly a horizontal flat plate initially covered with a monolayer of fine powder particles. Starting from a simple model involving both the air flow through the porous cake and avalanche properties, we set up an analytical model which satisfactorily fits the experimental results. We observe a close analogy between the governing equations of the phenomenon and the basic physics of wetting liquids, including the equivalent of the Laplace law and the surface tension parameter leading to the well known Rayleigh-Taylor instability.

DOI: 10.1103/PhysRevLett.87.254301 PACS numbers: 45.70.Qj, 45.70.Mg, 47.20.Dr, 67.70.+n

In a recent past, most of the theoretical, experimental, and simulated works dealing with the physics of granular materials have considered collections of large solid particles (i.e., typically larger than 100 μm) or smaller particles under vacuum. This simplification allowed one to neglect the complex interaction of the surrounding gases or fluids with the moving solid particles [1]. On the other hand, the dynamical behavior of fine powders interacting with gases or liquids is recognized as the keystone of a large number of technological processes, e.g., in fine chemicals and pharmaceuticals, ceramics, and food industry. In nature, huge fields of well known patterns such as dunes and ripples result from sand-wind interaction in deserts or sand-water interaction on sea shores.

In this spirit, an increasing number of current works deals with the interaction of granular species with interstitial fluids [2] and with the effects of reduced particle size (e.g., [3–5]). Among others, a recent paper [6] enumerates the complex series of harmonic patterns obtained when vibrating deep beds of relatively small size particles. The authors mentioned that the patterns depend on the particle size (ranging between 60 and 1000 μm).

The situation is different here. It follows a recently published paper [5] reporting experiments and a model of the steady state patterns generated by tapping repeatedly, at a low pace and from below a flat container half-filled with a *deep bed* of fine powder of tiny silica particles in the range of 10 μm. Using fine powder particles ensures a significant air-granulate interaction because the free fall velocity v_f of these small size particles ($v_f = D^2\rho g/18\eta$, where D and ρ are the diameter and density of the particles, η is the air viscosity, and g the gravitational acceleration) is of the same order of the forced velocity of the particles due to the external perturbation. Moreover, using taps at a low pace provides a simplification because it allows the system to relax between successive excitation avoiding intricate coupling to vibrational modes. We reported that, under these circumstances, a quasiperiodic and steady state corrugated pattern spreads out with a characteristic wavelength proportional to the amplitude of the taps. We explained that the instability happens because, contrary to intuition, particles are more easily ejected by the air blow from the tops than from the sides of the heaplets (Fig. 1) inducing a sort of "volcano effect." The resulting pattern was analyzed in terms of a cutoff length which characterizes the competition between the downstream avalanches and the upcoming particles ejected by the trapped air flow.

Keeping along the same line, we consider now a *thin slice* of a fine powder (particle diameter: 10 to 50 μm) spread out over a flat plate. When gently tapping repeatedly and at constant intensity onto the plate, we observe the formation of a collection of separate rounded conical heaps looking like droplets of powder evenly spread over the plate. The resulting pattern strikingly reminds one of the Rayleigh-Taylor instability illustrated by the droplet structure obtained when turning up a glass plate initially covered with a thin film of a wetting liquid. As we will show in the following, this analogy is not fortuitous. It results from an underlying similarity between the equations governing the wetting properties of liquids and the behavior of powder piles interacting with a surrounding fluid.

Several basic characteristic features of the instability of a tapped thin film of powder can be readily observed starting from a simple tabletop experiment: Using a small leucite ruler equipped with thin spacers (thickness slightly larger than the diameter of the particles D), we spread a quasimonolayer slice of powder (dry silica beads SDS, diameter $D = 10$ to 50 μm) over a flat glass plate (size

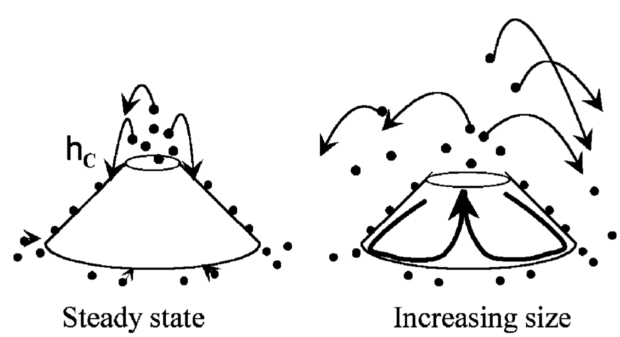

FIG. 1. Sketch of the trajectories of the powder particles participating in the intrinsic convection process when the heaplet is ejected above the plate resulting from either taps or air blowing from below.

60×90 mm^2). This glass plate is kept horizontal and secured on its periphery using a latex band which allows a certain degree of freedom for up and down motions. Using a small metallic or plastic rod, we knock gently and repeatedly at a very low pace (e.g., a tap per second) and at a constant intensity against one corner of the glass plate, applying vertically as brief taps as possible. After a few taps (about 10 to 20), the surface, initially flat, smooth, and horizontal, separates into a collection of tiny rounded conical heaps (Fig. 2). Starting from the same initial conditions but tapping more energetically while keeping the intensity as constant as possible from one tap to the next, induces a pattern with bigger heaplets separated by a larger distance. Note that humidity or excessive grain-grain cohesive forces prevents the observation of these patterns.

Setting a CCD (charge coupled device) camera above the plate in order to record and process the patterns allows one to get reliable data. A magnetically driven tapping device and a commercial Bruer and Kjaer accelerometer stuck on the plate in the vicinity of the sample is used in order to monitor the acceleration of the taps. Typical experimental results are reported in Fig. 3 which exhibits both the results of the mean separation distance measurements between adjacent heaplets as a function of the taps acceleration as well as a best fit with the theoretical model described below [7].

First we look for a relationship between the height of the approximately identical conical piles and the mean distance separating them. Consider the initial situation when a thin slice of powder of thickness e made of small spherical beads (diameter D) is evenly spread over a horizontal flat surface whose area is S. Suppose now that the powder has been gathered in a number of N disjointed identical conical piles having an angle θ to horizontal and culminating at altitude h. These piles are evenly distributed over the area S. Because of volume conservation, the number N of these piles is approximately given by $N = 3Se\tan^2\theta/\pi h^3 \propto h^{-3}$. The mean separation distance Λ of this pattern is the square root of the mean area occupied by each pile

$$\Lambda = \sqrt{\frac{\pi}{3e\tan^2\theta}}\, h^{3/2} \propto h^{3/2}. \tag{1}$$

In connection with Faraday's [8] and more recent authors' [9] experimental observation of the powder heaping and the associated convection, we have shown [5] that when a conical powder pile undergoes a ballistic flight and falls down, we can distinguish between two regions, delimited by a circle at altitude h_C (Fig. 1). The lowest region is stable against the upcoming air flux because it is stabilized by the lateral avalanches ($0 \leq h < h_C$). Around the apex we found an unstable part ($h_C \leq h \leq h_T$) (T for top) where grains are expelled by the upcoming air flux. The dimensionless parameter C measures the proportion of the unstable part of the heap, so that $C = (h_T - h_C)/h_T$. C has been conjectured to be independent of the shock acceleration [5]. In other words, the steady state of a pattern sketched in the left hand side part of Fig. 1 results from the balance between the number of expelled particles near the apices and the number of particles which are reinjected into the bulk of the heaps at every tap.

Considering a particle sitting at altitude h_C on the side of the conical heaplets, we found that the velocity of the upcoming air flux at altitude h_C required to eject this particle is given by

$$v_{h_C} = \frac{K\Delta P}{\eta h_C} = v_f\,\frac{C}{1-C}\,\frac{h_C p \sin\theta}{D}, \tag{2}$$

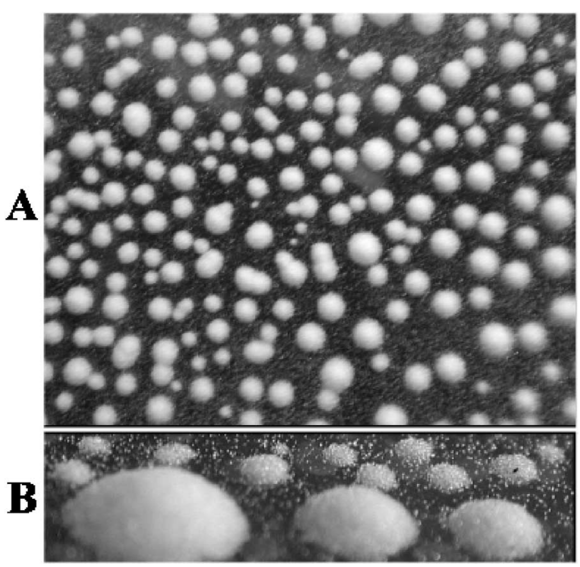

FIG. 2. (A) Bird's eye view of the pattern obtained after 40 taps over a plate initially covered with a nearly uniform film of powder particles (dia. about 30 μm). The mean separation distance between neighboring heaps is about 5 mm. (B) The snapshot shows an enlarged (5×) oblique view of a few small heaps. It exhibits the rounded shape of the apices due to the "volcano effect."

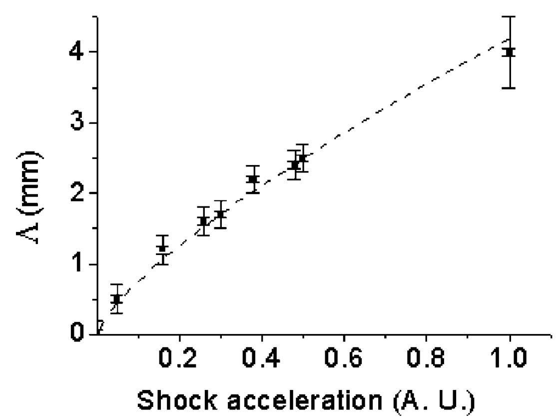

FIG. 3. Experimental results obtained with a monolayer slice of silica powder (particle size about 35 μm). The dashed line is a theoretical best fit to Eq. (6) where ΔP is expected to be proportional to the taps acceleration because the heaplets undergo a ballistic flight. Error bars correspond to five series of experiments using the same material and the same powder thickness.

where K is the permeability of the powder and ΔP the pressure difference acting over the granular porous cake, due to the air compression when the pile falls down. p is the number (typically 5) of sheets possibly involved in the avalanches. Thus we find the basic equation governing the problem:

$$\frac{K\Delta P}{Dh_C} = \rho\left(\frac{1}{18}\,\frac{C}{1 - C}\,p\,\sin\theta\right)gh_C\,. \tag{3}$$

Written in this form, Eq. (3) can be seen as describing the balance between two antagonistic pressures as follows.

(i) A "hydrostatic" pressure $P_g = \rho^* g h_C$ which accounts for the screening effect of the avalanche properties of the powder where $\rho^* = \rho(\frac{1}{18}\frac{C}{1-C}p\sin\theta)$ is the normalized density of the particles participating in the avalanches. Note that ρ^* explicitly depends on the micromechanical characteristics of the particles because of the presence of both C and p in this equation.

(ii) The equivalent of a Laplace-Young pressure, P_l (describing the pressure difference at the interface of two liquids) which can be written

$$P_l = \frac{K\Delta P}{Dh_C} = \gamma^*\left(\frac{2}{h_C}\right), \tag{4}$$

where γ^* plays the role of a surface tension and is defined by $\gamma^* = \frac{K\Delta P}{2D}$.

In brief, Eq. (3) describes the equilibrium of the analog of a wetting liquid droplet [10] on a horizontal plate. Thus, we treat a conical powder heaplet as a half spherical wetting material of height h_C and curvature $2/h_C$. This ersatz displays a surface tension (or capillary forces) γ^*. This analog to a surface tension can be seen as resulting from the convective forces [11] which drag powder particles from the surrounding surface(s) and subsequently inject them into the powder pile. This sort of sucking effect mimics the effect of capillary forces in liquids which tend to gather liquid films into droplets or bubbles. Therefore, the equivalent surface tension of the powder pile has a purely dynamical origin in this situation since it results from the convective forces related to the volcano effect. From Eq. (3), we get h_C from the following relationship:

$$h_C \simeq \left(\frac{K\Delta P}{D}\,\frac{1}{\rho^* g}\right)^{1/2} = \left(\frac{2\gamma^*}{\rho^* g}\right)^{1/2}. \tag{5}$$

Going on with the analogy to wetting liquids [10], we can also define the usual capillary length λ equating the hydrostatic pressure and the Laplace-Young pressure so that $\lambda = (\gamma^*/\rho^* g)^{1/2} = h_C/\sqrt{2}$ and a related Bond number $Bo = (\rho^* g h_C^2/\gamma^*)$.

Now, using Eq. (1), we find

$$\Lambda = \sqrt{\frac{\pi}{3e\tan^2\theta}}\left(\frac{K\Delta P}{D}\,\frac{1}{\rho^* g}\right)^{3/4} = \sqrt{\frac{\pi}{3e\tan^2\theta}}\left(\frac{2\gamma^*}{\rho^* g}\right)^{3/4}. \tag{6}$$

Figure 3 reports a best fit of a series of experimental results obtained with the previously cited powder to this equation.

Here a numerical estimation of the involved parameters is imperative. We calculate an approximate value for the surface tension γ^* starting from Eq. (6) using typical values for $\Lambda = 5$ mm, $e = 20\ \mu$m, and ρ^* obtained for $C = 5\%$. We get $\gamma^* \simeq 2.3 \times 10^{-5}\ \mathrm{N\,m^{-1}}$ which means that this constant is about 3000 times smaller than the surface tension of pure water. As expected, λ and h_C are in the order of 1 mm. Moreover, using Eq. (4) we can get an estimated value for the pressure difference between the altitude h_C and the base. First, we consider that the permeability of the granular material is a fraction of the cross sectional area of a single particle. Thus, we get ΔP in the order of 3 Pa. This quantity should be a fraction of the maximum possible air pressure due to the total weight of the powder pile leaning on the basis surface S. Indeed, this maximum air pressure is found to be about 10 Pa which is a correct order of magnitude.

Table I summarizes the analogy between the basic equations governing the powder heap equilibrium and the equations governing the equilibrium of liquid droplets.

Now, starting from this analogy and using Eq. (3), we can transcribe the classical demonstration of the Rayleigh-Taylor instability for wetting liquids. The standard analysis consists in examining the evolution of an infinitesimal sinusoidal distortion of the initially flat surface. Note that the basic calculation for liquids (found in textbooks) leads to a wavelength (the mean separation distance, here) dependence $\Lambda \propto (\gamma/\rho g)^{1/2}$. Here the distortion is by no means infinitesimal. We rather introduced the volume conservation condition which leads to $\Lambda \propto (\gamma^*/\rho^* g)^{3/4}$. However, except for this difference, the underlying phenomenology of the blown powder mimics the standard Rayleigh-Taylor instability.

Along the same lines, still proceeding with the analogy of the inner pressure within a powder heap given by Eq. (4) which mimics the Laplace-Young law, we predict that if two powder heaps of unequal sizes are sitting next to each other, the smaller one would be sucked into the larger one

TABLE I. Basic equations for a wetting liquid and a blown powder.

Wetting liquid	Equation	Blown powder heap	Equation
Surface tension	$\gamma = \frac{dF}{dl}$	Convective forces	$\gamma^* = \frac{K\Delta P}{2D}$
Droplet radius	R	Heap height	h_C
Laplace law	$\Delta P = \frac{2\gamma}{R}$	Eq. (4)	$\Delta P^* = \frac{2\gamma^*}{h_C}$
Droplet equilibrium	$\frac{2\gamma}{R} = \rho g R$	Blown heap equilibrium	$\frac{2\gamma^*}{h_C} = \rho^* g h_C$

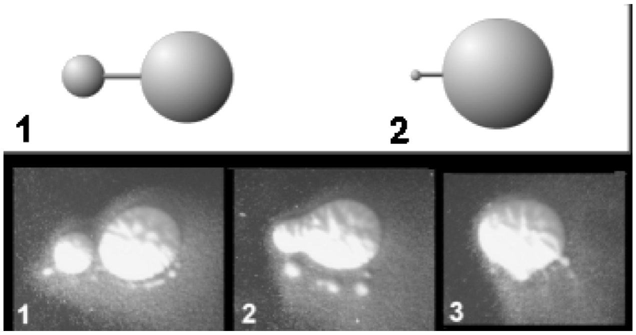

FIG. 4. Above, sketch of a basic experiment showing that the inner gas pressure is larger in a smaller droplet. When two bubbles are connected by a small pipe, the small bubble is sucked into the largest one. Below, the bird's eye view of an experiment showing the fusion mechanism among tapped powder heaps. The smaller heaps are sucked into the largest heap in agreement with Eq. (4).

just as this occurs between two communicating bubbles. An experimental result shows this in Fig. 4.

Even if it has the merit to establish a connection between the (yet unknown) description of blown powder properties and the (already known) wetting liquid behavior, our theoretical explanation certainly lays itself open to several criticisms. In particular, it does not convey any information regarding the development of the surface instability. Such an analysis would involve the introduction of the equivalent of a powder viscosity, which is not considered in the present model dealing with the steady state of the process. A time-resolved scrutiny of the pattern growth would probably convey information about this question. I postpone the description of this study to a forthcoming paper.

I am grateful to R. Jacobs, E. Raphael, I. Aronson, and the granular group in Jussieu for stimulating discussions.

*Email address: jd@ccr.jussieu.fr

[1] H. K. Pak, E. Van Doorn, and R. P. Behringer, Phys. Rev. Lett. **74**, 4643–4646 (1995).
[2] J. Schleier-Smith and H. A. Stone, Phys. Rev. Lett. **86**, 3016–3019 (2001).
[3] A. Castellanos, J. M. Valverde, A. T. Perez, A. Ramos, and P. K. Watson, Phys. Rev. Lett. **82**, 1156 (1999).
[4] E. Falcon, K. Krumar, K. M. S. Bajaj, and J. K. Bhattacharjee, Phys. Rev. E **59**, 5716 (1999).
[5] J. Duran, Phys. Rev. Lett. **84**, 5126 (2000).
[6] T. Shinbrot, L. Lomelo, and F. J. Muzzio, Granular Matter **2**, 65–69 (2000).
[7] A few additional pictures and movies can be seen on the URL: www.espci.fr/DirectionJD/.
[8] M. Faraday, Philos. Trans. R. Soc. London **52**, 299 (1831).
[9] C. Laroche, S. Douady, and S. Fauve, J. Phys. **50**, 699–706 (1989).
[10] P.-G. de Gennes, Rev. Mod. Phys. **57**, 827–861 (1985).
[11] Figure 1 and a cartoon are in www.espci.fr/DirectionJD/