

Branching Fraction and Photon Energy Spectrum for $b \rightarrow s\gamma$

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We have measured the branching fraction and photon energy spectrum for the radiative penguin process $b \rightarrow s\gamma$. We find $\mathcal{B}(b \rightarrow s\gamma) = (3.21 \pm 0.43 \pm 0.27^{+0.18}_{-0.10}) \times 10^{-4}$, where the errors are statistical, systematic, and from theory corrections. We obtain first and second moments of the photon energy spectrum above 2.0 GeV, $\langle E_\gamma \rangle = 2.346 \pm 0.032 \pm 0.011$ GeV, and $\langle E_\gamma^2 \rangle - \langle E_\gamma \rangle^2 = 0.0226 \pm 0.0066 \pm 0.0020$ GeV², where the errors are statistical and systematic. From the first moment, we

obtain (in the modified minimal subtraction renormalization scheme, to order $1/M_B^3$ and $\beta_0\alpha_s^2$) the heavy quark effective theory parameter $\bar{\Lambda} = 0.35 \pm 0.08 \pm 0.10$ GeV.

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The standard model (SM) expression for the $b \rightarrow s\gamma$ branching fraction has been determined to next-to-leading order [1–3]. The numerical value originally given [2] was $(3.28 \pm 0.33) \times 10^{-4}$. Recently, Gambino and Misiak [4] argue for the use of a different choice for the charm quark mass, and obtain $(3.73 \pm 0.30) \times 10^{-4}$. The theoretical literature on the connections between $b \rightarrow s\gamma$ and beyond-SM physics is extensive [5]. Thus a measurement different from the SM value would indicate beyond-SM physics, and a measurement close to the SM value would constrain the parameters of SUSY, Technicolor, and other beyond-SM physics options.

In contrast, the photon energy spectrum is insensitive to beyond-SM physics. The photon mean energy $\langle E_\gamma \rangle$ is, to a good approximation, equal to half the b quark mass m_b , while the mean square width of the energy distribution, $\langle E_\gamma^2 \rangle - \langle E_\gamma \rangle^2$, depends on the mean square momentum of the b quark within the B meson. A knowledge of the b quark mass and momentum allows a determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element V_{cb} from the B semileptonic width, as shown in the following Letter [6]. Further, the spectrum provides information allowing a better determination of the CKM matrix element V_{ub} from the yield of leptons near the endpoint of $b \rightarrow u\ell\nu$ [3,7,8].

CLEO's previously published measurement [9] of the $b \rightarrow s\gamma$ branching fraction, $(2.32 \pm 0.57 \pm 0.35) \times 10^{-4}$, was based on 3.0 fb^{-1} of luminosity, on resonance plus off resonance. The photon energy spectrum then obtained was not sufficiently precise to be of interest. Here we present a new study of $b \rightarrow s\gamma$, based on 9.1 fb^{-1} on the $Y(4S)$ resonance and 4.4 fb^{-1} 60 MeV below the resonance (below $B\bar{B}$ production threshold). We present the branching fraction and the first and second moments of the photon energy spectrum. From the first moment, we obtain [10,11] the heavy quark effective theory (HQET) parameter $\bar{\Lambda}$ (physically, the energy of the light quark and gluon degrees of freedom).

The characteristic feature of the $b \rightarrow s\gamma$ decay is the high energy photon, roughly monoenergetic, with $E_\gamma \approx m_b/2 \approx 2.3$ GeV. In our previous measurement we considered the spectrum above 2.2 GeV, incurring a model-dependent correction for the fraction of the spectrum below 2.2 GeV. Here we use the spectrum down to 2.0 GeV, which includes $\sim 90\%$ of the $b \rightarrow s\gamma$ yield.

The data used for this analysis were taken with the CLEO detector [12] at the Cornell Electron Storage Ring (CESR), a symmetric e^+e^- collider. We select hadronic events containing a high energy photon in the central region of the calorimeter ($|\cos\theta_\gamma| < 0.7$). The photon must not form a π^0 or η with any other photon in the event.

There is a large background of high energy photons from continuum processes: initial state radiation, photons from decays of π^0 , η , and other hadrons. We use several techniques to suppress the continuum background, measure what survives with the below-resonance data sample, and subtract it from the on-resonance sample. There is also a significant background of photons from B decay processes other than $b \rightarrow s\gamma$, particularly in the lower portion of our window, 2.0–2.2 GeV. We determine the dominant background sources, photons from unvetted $\pi^0 \rightarrow \gamma\gamma$ and $\eta \rightarrow \gamma\gamma$, by directly measuring the π^0 and η spectra, estimate the K_L^0 and \bar{n} backgrounds using the shower shape, and estimate the remaining background sources via Monte Carlo techniques [13].

Part of our continuum suppression comes from eight event shape variables: normalized Fox-Wolfram [14] second moment R_2 , S_\perp (a measurement of the momentum transverse to the photon direction [15]), R_2' (the value of R_2 in the primed frame, the rest frame of e^+e^- following an assumed initial state radiation of the high energy photon, with R_2 evaluated excluding the photon), $\cos\theta'$ (θ' the angle, in the primed frame, between the photon and the thrust axis of the rest of the event), and the energies in 20° and 30° cones, parallel and antiparallel to the high energy photon direction. We combine the eight variables into a single variable r which tends towards +1 for $b \rightarrow s\gamma$ events and tends towards -1 for continuum background events. A neural network is used for this task [15].

We obtain further continuum suppression from “pseudo-reconstruction,” and from the presence of leptons. In pseudoreconstruction, we search events for combinations of particles that reconstruct to a $B \rightarrow X_s\gamma$ decay. For X_s we use a $K_S^0 \rightarrow \pi^+\pi^-$ or a charged track consistent with a K^\pm , and 1–4 pions, of which at most one may be a π^0 . We calculate the candidate B momentum P , energy E , and beam-constrained mass $M \equiv \sqrt{E_{\text{beam}}^2 - P^2}$. A reconstruction is deemed acceptable if it has $\chi_B^2 < 20$, where

$$\chi_B^2 \equiv \left(\frac{E - E_{\text{beam}}}{\sigma_E} \right)^2 + \left(\frac{M - M_B}{\sigma_M} \right)^2,$$

where, $\sigma_E = 40$ MeV, $\sigma_M = 4.0$ MeV. If an event contains more than one acceptable reconstruction, the one with the lowest χ_B^2 is chosen. (It is not important that the reconstruction be correct in detail, as we are only using it for continuum suppression. It is correct 40% of the time.)

For events with an acceptable reconstruction, we add χ_B^2 , and $|\cos\theta_{\ell\ell}|$, where $\theta_{\ell\ell}$ is the angle between the thrust axis of the candidate B and the thrust axis of the rest of the event, to the list of variables for continuum suppression. If the event contains a lepton (muon or electron), then we add the momentum of the lepton, P_ℓ , and the angle between

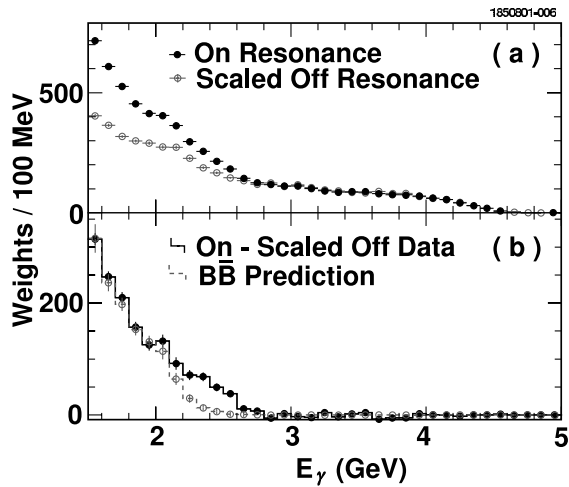


FIG. 1. Photon energy spectra (weights per 100 MeV). The upper plot (a) shows the on $Y(4S)$ and scaled off-resonance spectra. The lower plot (b) shows their difference and the spectrum estimated for B decay processes other than $b \rightarrow s\gamma$ and $b \rightarrow d\gamma$.

lepton and high energy photon, $\theta_{\ell\gamma}$, to the list. We thus have events with both a pseudoreconstruction and a lepton, for which we use r , χ_B^2 , $|\cos\theta_{ll}|$, P_ℓ , and $\theta_{\ell\gamma}$ for continuum suppression; events with only a pseudoreconstruction, for which we use r , χ_B^2 , and $|\cos\theta_{ll}|$; events with only a lepton, for which we use r , P_ℓ , and $\theta_{\ell\gamma}$; and events with neither, for which we use r . In each of the four cases, the available variables are combined using a neural net. All nets are trained on signal and continuum Monte Carlo. We convert the output of each of the four nets, r_j , into weights, $w(r_j) = s(r_j)/[s(r_j) + (1+a)b(r_j)]$, where $s(r_j)$ is the expected signal yield and $b(r_j)$ is the expected continuum background yield for that value of the net output r_j , and a is the luminosity scale factor between on-resonance and off-resonance data samples (≈ 2.0). Weights so defined minimize the statistical error on the $b \rightarrow s\gamma$ yields after off-resonance subtraction.

Were we to use the weights as just defined, the efficiency would be higher for low-mass X_s states than for high-mass X_s states. We thus modify the weights for those cases where there is a reconstruction, and thus a measurement of M_{X_s} , to render the efficiency independent of the X_s mass. With these weights, the statistical error is slightly

worse, but a systematic error has been reduced. The dependence of efficiency on M_s for events for which there is no pseudoreconstruction remains.

We sum weights on and off resonance, and subtract the off-resonance sum, scaled by \mathcal{L}/E_{cm}^2 and with momenta scaled by the on-off energy ratio, from the on-resonance sum (the on-off subtraction). Biases in this subtraction procedure were estimated with Monte Carlo to be $(0.5 \pm 0.5)\%$ of the subtraction. The on-resonance yields and scaled off-resonance yields are shown in Fig. 1a, and tabulated in Table I. The on-off subtracted yields are shown in Fig. 1b; they consist of a component from $b \rightarrow s\gamma$, and a component from other B decay processes.

We investigated the component from other B decay processes with a $B\bar{B}$ Monte Carlo sample that included contributions from $b \rightarrow u$ and $b \rightarrow sg$ processes, as well as the dominant $b \rightarrow c$ decay. We found that the overwhelming source of background ($\sim 90\%$) is photons from π^0 or η decay. Consequently we carefully tuned the Monte Carlo to match the data in π^0 and η yields, and also η' and ω . We included radiative ψ decays, $\rho \rightarrow \pi\gamma$, $a_1 \rightarrow \pi\gamma$, and final state radiation. We determined the background from neutral hadrons, K_L^0 and \bar{n} , using information from the measured lateral distribution of the showers. The yield from B decay processes is shown in Fig. 1b and tabulated in Table I. The error on this yield is dominated by the error on measured on-off-subtracted π^0 and η yields, and by uncertainty in K_L^0 and \bar{n} backgrounds from fits to shower shape.

The fully subtracted spectrum, on-off-other B decay processes, is shown in Fig. 2. There is a clear peak near 2.3 GeV, indicative of $b \rightarrow s\gamma$. The region of interest for $b \rightarrow s\gamma$ is 2.0–2.8 GeV. The region above 3.0 GeV thus serves as a control, indicating how well we have subtracted the continuum background, while the region 1.5–2.0 GeV serves as another control, indicating how well we have accounted for the other B decay processes. The yields in both regions are consistent with zero. Subtracting 5% more or less $B\bar{B}$ background causes the B background control region to have a deficit or excess, at the 1σ level. We take $\pm 5\%$ of the $B\bar{B}$ subtraction as the systematic error on the subtraction.

We determine the efficiency, weights per $b \rightarrow s\gamma$ event generated with photon energy above 2.0 GeV, via Monte

TABLE I. Yields (summed weights) with statistical errors for three photon energy intervals. Given are the yields on $Y(4S)$ resonance, scaled off-resonance yields, on minus scaled off, estimated backgrounds from B decay processes other than $b \rightarrow s\gamma$ and $b \rightarrow d\gamma$, and on minus scaled off minus B backgrounds, the putative $b \rightarrow s\gamma$ plus $b \rightarrow d\gamma$ signal.

Energy Range	1.5–2.0 GeV	2.0–2.7 GeV	3.0–5.0 GeV
On	2718.8 ± 17.2	1861.7 ± 16.5	1083.2 ± 8.0
$a \times$ off	1665.6 ± 14.6	1399.4 ± 15.31	1101.4 ± 11.6
On $- a \times$ off	1053.2 ± 22.6	462.3 ± 22.5	-18.2 ± 14.1
B backgrounds	1033.0 ± 35.4	228.6 ± 21.6	1.5 ± 1.1
On $- a \times$ off $- B$	20.2 ± 42.0	233.6 ± 31.2	-19.8 ± 14.2

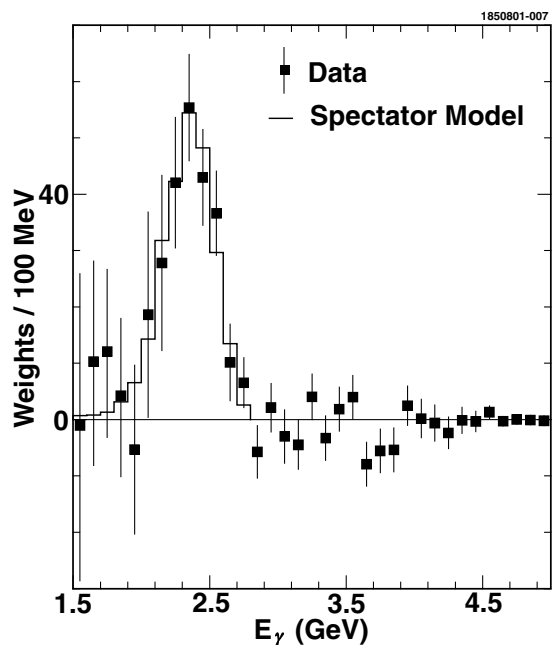


FIG. 2. Observed laboratory frame photon energy spectrum (weights per 100 MeV) for on minus scaled off minus B backgrounds, the putative $b \rightarrow s\gamma$ plus $b \rightarrow d\gamma$ signal. No corrections have been applied for resolution or efficiency. Also shown is the spectrum from Monte Carlo simulation of the Ali-Greub spectator model with parameters $\langle m_b \rangle = 4.690$ GeV, $P_F = 410$ MeV/ c , a good fit to the data.

Carlo simulation. To model the decay $b \rightarrow s\gamma$, i.e., $B \rightarrow X_s\gamma$, we use the spectator model of Ali and Greub [16], which includes gluon bremsstrahlung and higher-order radiative effects. Rather than considering the Fermi momentum parameter P_F and the b quark average mass $\langle m_b \rangle$ as quantities given from first principles, we treat them as free parameters which allow us to vary the mean and width of the photon energy spectrum. We fit our measured spectrum over the range 2.0–2.8 GeV, and use the values of the parameters so obtained, and their errors, to define our model of $B \rightarrow X_s\gamma$. For hadronization of $s\bar{q}$ into X_s , we have compared two approaches. In the first, we have taken several K^* resonances and combined them in proportions to approximate the X_s mass distribution given by the Ali-Greub model. In the second, we let JETSET hadronize the $s\bar{q}$, again giving the Ali-Greub X_s mass distribution. We include in the systematic errors on the efficiency an uncertainty in the modeling of the $B \rightarrow X_s\gamma$ decay (P_F , $\langle m_b \rangle$, hadronization), an uncertainty in the simulation of the detector performance (track-finding, photon-finding, resolutions), and an uncertainty in the modeling of the other B .

As an alternative to the Ali-Greub model for $B \rightarrow X_s\gamma$ decay, we have used the description of Kagan and Neubert [3], with their simple two-parameter shape function, with the two parameters m_b and a adjusted to fit our measured photon spectrum. We obtain results very similar to those obtained using Ali-Greub, both for efficiency and for the

moments. (Because we determine our $B \rightarrow X_s\gamma$ model from a fit to data, our results are independent of the nominal description, as long as that description allows us two adjustable parameters to match data on mean and width.)

To obtain the $b \rightarrow s\gamma$ branching fraction, we take the yield between 2.0 and 2.7 GeV, $233.6 \pm 2 \pm 13.4$ weights, where the first error is statistical and the second is systematic, from $B\bar{B}$ ($\pm 5\%$) and continuum ($\pm 0.5\%$) subtraction. The efficiency is $(3.93 \pm 0.15 \pm 0.17) \times 10^{-2}$ weights per event, where the first error is from the model dependence of the $B \rightarrow X_s\gamma$ decay and the second error is from detector simulation and model dependence of the decay of the other B . Our sample contains 9.70 million $B\bar{B}$ pairs ($\pm 2\%$). We obtain an uncorrected branching fraction of $(3.06 \pm 0.41 \pm 0.26) \times 10^{-4}$.

This uncorrected branching fraction, which we directly measure, is the branching fraction for $b \rightarrow s\gamma$ plus $b \rightarrow d\gamma$, for B -rest-frame photon energies above 2.0 GeV. We apply two “theory” corrections to obtain a branching fraction of more direct interest. Using a model for $b \rightarrow d\gamma$ similar to that used for $b \rightarrow s\gamma$, we find the efficiency for $b \rightarrow d\gamma$ to be the same as that for $b \rightarrow s\gamma$. (Pseudo-reconstruction favors $b \rightarrow s\gamma$ over $b \rightarrow d\gamma$ but shape variables and photon energy spectrum favor $b \rightarrow d\gamma$ over $b \rightarrow s\gamma$.) The SM expectation is that the $b \rightarrow d\gamma$ and $b \rightarrow s\gamma$ branching fractions are in the ratio $|V_{td}/V_{ts}|^2$. Using $|V_{td}/V_{ts}| = 0.20 \pm 0.04$ [17] (taking $V_{ts} = -V_{cb}$ from unitarity), we correct the branching fraction down by $(4.0 \pm 1.6)\%$ of itself, to remove the $b \rightarrow d\gamma$ contribution. The fraction of $b \rightarrow s\gamma$ decays with photon energies above 2.0 GeV is sensitive to the b mass and Fermi momentum. We use $0.915^{+0.027}_{-0.055}$, as given by Kagan and Neubert [3,18], to extrapolate our branching fraction to the full energy range (actually, to energies above 0.25 GeV). With these two corrections, we have

$$\mathcal{B}(b \rightarrow s\gamma) = (3.21 \pm 0.43 \pm 0.27^{+0.18}_{-0.10}) \times 10^{-4},$$

for the branching fraction for $b \rightarrow s\gamma$ alone, over all energies. The first error is statistical, the second is systematic, and the third is from the theory corrections. This result is in good agreement with the standard model prediction. It is also in agreement with our previously measured result [9], which it supersedes.

We calculate the first and second moments of the photon energy spectrum, in two ways. In the first, we correct the raw spectrum for the energy dependence of the efficiency, calculate moments from that spectrum, and apply corrections for experimental resolution and for the transformation from lab frame to B rest frame. We then apply a final, empirical correction, obtained by carrying out the above procedure on Monte Carlo. In the second way, we take our best fit Monte Carlo model, with P_F and $\langle m_b \rangle$ determined from a fit to the spectrum, and take the moments given by that model. The two ways of obtaining moments agree, and we take their average. We thus obtain moments in

the B rest frame, for $E_\gamma(\text{rest frame}) > 2.0$ GeV:

$$\langle E_\gamma \rangle = 2.346 \pm 0.032 \pm 0.011 \text{ GeV}.$$

$$\langle E_\gamma^2 \rangle - \langle E_\gamma \rangle^2 = 0.0226 \pm 0.0066 \pm 0.0020 \text{ GeV}^2.$$

Heavy quark effective theory and the operator product expansion, with the assumption of parton-hadron duality,

allow inclusive observables to be written as double expansions in powers of α_s and $1/M_B$. The expressions [10,11] for the moments of the photon energy spectrum in $B \rightarrow X_s \gamma$, for $E_\gamma > 2.0$ GeV, in the modified minimal subtraction renormalization scheme, to order $\beta_0 \alpha_s^2$ and $1/M_B^3$, are given in Eqs. (1) and (2):

$$\langle E_\gamma \rangle = \frac{M_B}{2} \left\{ 1 - 0.385 \frac{\alpha_s}{\pi} - 0.620 \beta_0 \left(\frac{\alpha_s}{\pi} \right)^2 - \frac{\bar{\Lambda}}{M_B} \left[1 - 0.954 \frac{\alpha_s}{\pi} - 1.175 \beta_0 \left(\frac{\alpha_s}{\pi} \right)^2 \right] - \frac{13\rho_1 - 33\rho_2}{12M_B^3} - \frac{\mathcal{T}_1 + 3\mathcal{T}_2 + \mathcal{T}_3 + 3\mathcal{T}_4}{4M_B^3} - \frac{\rho_2 C_2}{9M_B M_D^2 C_7} + \mathcal{O}(1/M_B^4) \right\}, \quad (1)$$

$$\langle (E_\gamma - \langle E_\gamma \rangle)^2 \rangle = M_B^2 \left\{ \frac{-\lambda_1}{12M_B^2} + \left[0.00815 \frac{\alpha_s}{\pi} + 0.01024 \beta_0 \left(\frac{\alpha_s}{\pi} \right)^2 \right] - \frac{\bar{\Lambda}}{M_B} \left[0.05083 \frac{\alpha_s}{\pi} + 0.05412 \beta_0 \left(\frac{\alpha_s}{\pi} \right)^2 \right] - \frac{2\rho_1 - 3\rho_2}{12M_B^3} - \frac{\mathcal{T}_1 + 3\mathcal{T}_2}{12M_B^3} + \mathcal{O}(1/M_B^4) \right\}, \quad (2)$$

where C_2 and C_7 are Wilson coefficients and β_0 is the one-loop QCD β function. The $1/M_B^3$ parameters ρ_i , \mathcal{T}_i are estimated, from dimensional considerations, to be $\sim (0.5 \text{ GeV})^3$. Using Eq. (1), we obtain

$$\bar{\Lambda} = 0.35 \pm 0.08 \pm 0.10 \text{ GeV},$$

where the first error is from the experimental error in the determination of the first moment, and the second error is from the theoretical expression, in particular from the neglected $1/M_B^3$ terms, and the uncertainty of the scale to use for α_s , which we take to be from $m_b/2$ to $2m_b$.

The expression for the second moment converges slowly in $1/M_B$, and so we make no attempt to extract expansion parameters from it.

In summary, we measured the branching fraction for $b \rightarrow s \gamma$ to be $(3.21 \pm 0.53) \times 10^{-4}$, in good agreement with the standard model expectation. We measured the first and second moments of the B -rest-frame photon energy spectrum above 2.0 GeV to be $\langle E_\gamma \rangle = 2.346 \pm 0.034$ GeV and $\langle E_\gamma^2 \rangle - \langle E_\gamma \rangle^2 = 0.0226 \pm 0.0069$ GeV². From the first moment, we obtained a value for the HQET parameter $\bar{\Lambda} = 0.35 \pm 0.08 \pm 0.10$ GeV.

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