

Top-Quark Spin Correlations at Hadron Colliders: Predictions at Next-to-Leading Order QCD

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The collider experiments at the Tevatron and the LHC will allow for detailed investigations of the properties of the top quark. This requires precise predictions of the hadronic production of $t\bar{t}$ pairs and of their subsequent decays. In this Letter we present for the reactions $p\bar{p}, pp \rightarrow t\bar{t} + X \rightarrow \ell^+\ell'^- + X$ the first calculation of the dilepton angular distribution at next-to-leading order in the QCD coupling, keeping the full dependence on the spins of the intermediate $t\bar{t}$ state. The angular distribution reflects the degree of correlation of the t and \bar{t} spins which we determine for different choices of t and \bar{t} spin bases. In the case of the Tevatron, the QCD corrections are sizable, and the distribution is quite sensitive to the parton content of the proton.

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The top quark is by far the heaviest fundamental fermion discovered [1] to date. It is an excellent probe of the fundamental interactions in the high energy regime that will be explored by the upgraded Fermilab Tevatron collider and by the CERN large hadron collider LHC. It is expected that very large numbers of top quarks will be produced with these colliders: eventually about 10^4 top quark-antiquark ($t\bar{t}$) pairs per year at the Tevatron and more than about 10^7 $t\bar{t}$ pairs per year at the LHC. This will make feasible precise investigations of the interactions of top quarks.

Because of their extremely short lifetime top quarks find no time to form hadronic bound states: they are highly unstable particles whose interactions are governed by short-distance dynamics [2]. As a consequence, the properties of the top quark and antiquark, in particular phenomena associated with their spins, are reflected directly in the distributions and the corresponding angular correlations of the jets, W bosons, or leptons into which the t and \bar{t} decay. In fact the top quark is the only quark for which this is true. Therefore, this quark provides a unique probe of the spin-related properties and interactions of quarks. The top quark decay distributions are determined by the t and \bar{t} polarizations and spin correlations induced by the production mechanism(s). Furthermore, they depend on the interactions responsible for the top (anti-)quark decay. Hence the analysis of these distributions will be an important tool, once large data samples are available [3], to obtain detailed information about top-quark production and decay.

For hadronic pair production the spin correlations of $t\bar{t}$ pairs were studied to leading order in the coupling α_s of quantum chromodynamics (QCD) in Ref. [4,5]. There exists also an extensive literature, for example [6] and references therein, on how to exploit top-quark spin phenomena at hadron colliders in the search for new interactions. In order to test known or to search for new interactions these spin effects should be known as precisely as possible within the standard model (SM) of particle physics. An investigation at next-to-leading order (NLO) in the QCD

coupling, which has not been performed so far, is therefore mandatory. In this Letter we report on the results of such an analysis. We study the hadronic production of $t\bar{t}$ pairs and their subsequent decays to order α_s^3 , keeping the full information on the spin configuration of the $t\bar{t}$ state. Specifically we consider the channels where both t and \bar{t} decay semileptonically,

$$p\bar{p}, pp \rightarrow t\bar{t} + X \rightarrow \ell^+\ell'^- + X, \quad (1)$$

($\ell = e, \mu, \tau$), and we predict the dileptonic angular distribution that encodes the $t\bar{t}$ spin correlations. In the case of the Tevatron the QCD corrections turn out to be sizable, and we observe that the angular distribution analyzed below is quite sensitive to the parton distribution functions (PDF). For the LHC we find that the QCD corrections to the leading order distribution are small.

For the reactions (1) we analyze the following double leptonic distribution,

$$\frac{1}{\sigma} \frac{d^2\sigma}{d\cos\theta_+ d\cos\theta_-} = \frac{1}{4} (1 + B_1 \cos\theta_+ + B_2 \cos\theta_- - C \cos\theta_+ \cos\theta_-), \quad (2)$$

with σ being the cross section for the channel under consideration. In Eq. (2) $\theta_+(\theta_-)$ denotes the angle between the direction of flight of the lepton $\ell^+(\ell'^-)$ in the $t(\bar{t})$ rest frame [7] and a reference direction $\hat{\mathbf{a}}$ ($\hat{\mathbf{b}}$). The directions $\hat{\mathbf{a}}, \hat{\mathbf{b}}$ can be chosen arbitrarily. Different choices will yield different values for the coefficients $B_{1,2}$ and C . The physical interpretation of these coefficients is well known [4,5]: The coefficient C in Eq. (2) reflects spin correlations of the $t\bar{t}$ intermediate state. A more detailed discussion will be given below Eq. (6). For our choices of the directions $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ [cf. Eq. (8)] QCD interactions yield vanishing coefficients B_1, B_2 [8].

In principle one could measure the angular distribution of every possible decay product of the top (anti-)quark. In the SM, where the main top-quark decay modes are

$t \rightarrow bW \rightarrow bq\bar{q}', b\ell\nu_\ell$, the most powerful analyzers of the polarization of the top quark are the charged leptons, or the jets that originate from quarks of weak isospin $-1/2$ produced by the decay of the W boson. Here we restrict ourselves to the double leptonic distribution.

To predict the “dilepton + jets” distribution (2) at NLO accuracy we have to consider the following parton subprocesses:

$$gg, q\bar{q} \xrightarrow{t\bar{t}} b\bar{b}\ell^+\ell'^-\nu_\ell\bar{\nu}_{\ell'}, \quad (3)$$

$$gg, q\bar{q} \xrightarrow{t\bar{t}} b\bar{b}\ell^+\ell'^-\nu_\ell\bar{\nu}_{\ell'} + g, \quad (4)$$

$$g + q(\bar{q}) \xrightarrow{t\bar{t}} b\bar{b}\ell^+\ell'^-\nu_\ell\bar{\nu}_{\ell'} + q(\bar{q}). \quad (5)$$

At the Tevatron the cross section is dominated by quark-antiquark annihilation while at the LHC gluon-gluon fusion is predicted to be the dominant production process.

In view of the fact that the total width Γ_t of the top quark is much smaller than its mass m_t [$\Gamma_t/m_t = \mathcal{O}(1\%)$], one may analyze the above reactions using the so-called leading pole approximation [9]. This amounts to expanding the amplitudes of Eqs. (3)–(5) around the poles of the unstable t and \bar{t} quarks. Only the leading term of this expansion, i.e., the residue of the double poles is kept here. The radiative corrections to the respective lowest-order amplitudes can be classified into so-called factorizable and nonfactorizable corrections. We take into account the factorizable corrections to the above reactions for which the squared matrix element \mathcal{M} is of the form $|\mathcal{M}|^2 \propto \text{Tr}[\rho R \bar{\rho}]$. Here R denotes the respective spin density matrix for the production of on-shell $t\bar{t}$ pairs, and ρ ($\bar{\rho}$) is the t (\bar{t}) decay density matrix. As far as the nonfactorizable NLO QCD corrections are concerned which were calculated in [10], we expect [11] that their contribution to the coefficient C in Eq. (2) is considerably smaller than those of the factorizable corrections given below.

To obtain a theoretical prediction for the distribution in Eq. (2) at NLO accuracy we use our recent results [12] on the $t\bar{t}$ production spin-density matrices at NLO QCD. These results extend previous calculations [13] of the differential $t\bar{t}$ cross section with spins summed over and allow the calculation of the cross section for a specific spin configuration of the $t\bar{t}$ state. In particular, the quantization axes can be chosen arbitrarily.

The decay density matrix ρ ($\bar{\rho}$) required for computing (2) describes the normalized angular distribution of the decay of a polarized $t(\bar{t})$ quark into $\ell^+(\ell^-) + \text{anything}$ in the rest frame of the $t(\bar{t})$ quark. The matrix ρ has the form $2\rho_{\alpha'\alpha} = (\mathbb{1} + \kappa_+ \sigma \cdot \hat{\mathbf{q}}_+)_{\alpha'\alpha}$ where $\hat{\mathbf{q}}_+$ describes the direction of flight of ℓ^+ in the rest frame of the t quark and σ_i denote the Pauli matrices. The decay matrix $\bar{\rho}$ is obtained from ρ by replacing $\hat{\mathbf{q}}_+$ by $-\hat{\mathbf{q}}_-$ and κ_+ by κ_- . The factor κ_+ (κ_-) signifies the top-spin analyzing power of the charged lepton. It is equal to one to lowest order in the SM, that is, for $V - A$ charged currents. Its value including the order α_s corrections can be extracted from

the results of [14] and turns out to be very close to one: $\kappa_+ = \kappa_- = 1 - 0.015\alpha_s$. Using the general expressions for ρ , $\bar{\rho}$ and the fact that the factorizable contributions are of the form $\text{Tr}[\rho R \bar{\rho}]$ one obtains the following formula for the correlation coefficient C in Eq. (2):

$$C = 4\kappa_+\kappa_-\langle(\hat{\mathbf{a}} \cdot \mathbf{s}_t)(\hat{\mathbf{b}} \cdot \mathbf{s}_{\bar{t}})\rangle, \quad (6)$$

where $\mathbf{s}_t, \mathbf{s}_{\bar{t}}$ denote the t and \bar{t} spin operators. The expectation value in Eq. (6) is defined with respect to the matrix elements for the hadronic production of $t\bar{t}X$. It is related to the more familiar double spin asymmetry

$$4\langle(\hat{\mathbf{a}} \cdot \mathbf{s}_t)(\hat{\mathbf{b}} \cdot \mathbf{s}_{\bar{t}})\rangle = \frac{N(\uparrow\uparrow) + N(\downarrow\downarrow) - N(\uparrow\downarrow) - N(\downarrow\uparrow)}{N(\uparrow\uparrow) + N(\downarrow\downarrow) + N(\uparrow\downarrow) + N(\downarrow\uparrow)}, \quad (7)$$

where $N(\uparrow\uparrow)$, etc., denote the number of $t\bar{t}$ pairs with t and \bar{t} spin parallel—or antiparallel—to $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$, respectively. From Eq. (7) one can see that the axes $\hat{\mathbf{a}}, \hat{\mathbf{b}}$ introduced through the angles θ_\pm in Eq. (2) can be interpreted as quantization axes of the intermediate $t\bar{t}$ state within our approximation. Equation (6) generalizes the lowest-order results of [4,5] and holds for factorizable contributions to all orders in the QCD coupling [10].

For definiteness we consider here the following spin bases:

$$\hat{\mathbf{a}} = \hat{\mathbf{k}}_t, \hat{\mathbf{b}} = \hat{\mathbf{k}}_{\bar{t}} \text{ (helicity basis)}, \quad (8a)$$

$$\hat{\mathbf{a}} = \hat{\mathbf{p}}, \hat{\mathbf{b}} = \hat{\mathbf{p}} \text{ (beam basis)}, \quad (8b)$$

$$\hat{\mathbf{a}} = \hat{\mathbf{d}}_t, \hat{\mathbf{b}} = \hat{\mathbf{d}}_{\bar{t}} \text{ (off-diagonal basis)}. \quad (8c)$$

Here $\hat{\mathbf{k}}_t(\hat{\mathbf{k}}_{\bar{t}})$ denotes the direction of flight of the $t(\bar{t})$ quark in the parton center-of-mass frame, and $\hat{\mathbf{p}}$ is the unit vector along one of the hadronic beams in the laboratory frame. Furthermore $\hat{\mathbf{d}}_t$ is the axis constructed in Ref. [5] with respect to which the spins of t and \bar{t} produced by $q\bar{q}$ annihilation are 100% correlated [15] to leading order in α_s . (For $gg \rightarrow t\bar{t}$ one can show that no spin basis with this property exists.)

Table I contains our results for C at leading and next-to-leading order in α_s using the parton distribution functions CTEQ5L (LO) and CTEQ5M (NLO) of [16], where μ_R and μ_F denotes the renormalization and factorization scale, respectively. (These numbers and the results given below were obtained by integrating over the full phase space. Results with cuts included will be given elsewhere [11].) For $p\bar{p}$ collisions at $\sqrt{s} = 2$ TeV the helicity basis is not the best choice because the t, \bar{t} quarks are only

TABLE I. Coefficient C of Eq. (6) to leading (LO) and next-to-leading order (NLO) in α_s for the spin bases of Eq. (8). The parton distribution functions of [16] were used and we choose $\mu_R = \mu_F = m_t = 175$ GeV [17].

	$p\bar{p}$ at $\sqrt{s} = 2$ TeV		pp at $\sqrt{s} = 14$ TeV	
	LO	NLO	LO	NLO
C_{hel}	-0.456	-0.389	0.305	0.311
C_{beam}	0.910	0.806	-0.005	-0.072
C_{off}	0.918	0.813	-0.027	-0.089

moderately relativistic in this case. Table I shows that the dilepton spin correlations at the Tevatron are large both in the off-diagonal and in the beam basis. In fact they are almost identical. The QCD corrections decrease the LO results for these correlations by about 10%. Since the gg initial state dominates $t\bar{t}$ production with $p\bar{p}$ collisions at $\sqrt{s} = 14$ TeV the beam and off-diagonal bases are no longer useful. Here the helicity basis is a good choice and gives a spin correlation of about 30%. In this case the QCD corrections are small. The large difference between the LO and NLO results for the correlation in the beam basis at the LHC is due to an almost complete cancellation of the contributions from the $q\bar{q}$ and gg initial state at LO.

We now discuss the uncertainties of our predictions. It is well known that the inclusion of the QCD corrections reduces the dependence of the $t\bar{t}$ cross section σ_t on the renormalization and factorization scales significantly. The same is true for the product $\sigma_t C$. In Figs. 1a and 1b we demonstrate this with $\sigma_t C_{\text{beam}}$ and $\sigma_t C_{\text{hel}}$ evaluated at Tevatron and LHC energies, respectively, as functions of μ/m_t , putting $\mu = \mu_R = \mu_F$. The corresponding figure for $\sigma_t C_{\text{off}}$ is almost identical to Fig. 1a.

To leading order in α_s the coefficient C depends only on the factorization scale μ_F , while at NLO it depends on both scales μ_R and μ_F . Table II shows our NLO results for the three choices $\mu_R = \mu_F = m_t/2, m_t, 2m_t$, again using the PDF of [16]. An extension of this work, which is, however, beyond the scope of this Letter, would be the resummation of Sudakov-type logarithms at the next-to-

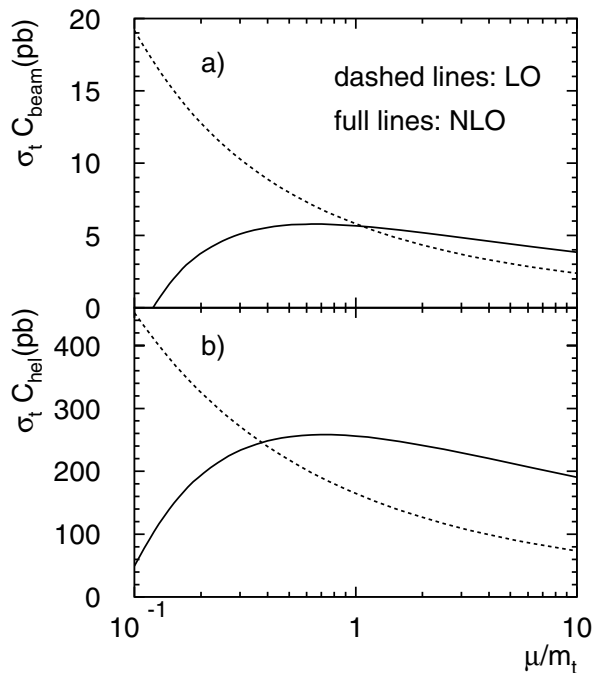


FIG. 1. Dependence of the LO (dashed line) and NLO (solid line) results on $\mu = \mu_R = \mu_F$. (a) $\sigma_t C_{\text{beam}}$ for $p\bar{p}$ collisions at $\sqrt{s} = 2$ TeV and (b) $\sigma_t C_{\text{hel}}$ for pp collisions at $\sqrt{s} = 14$ TeV, with PDF of [16].

TABLE II. Dependence of the correlation coefficients, computed with the PDF of [16], on $\mu = \mu_R = \mu_F$ at NLO.

$\mu_R = \mu_F$	$p\bar{p}$ at $\sqrt{s} = 2$ TeV			pp at $\sqrt{s} = 14$ TeV
	C_{hel}	C_{beam}	C_{off}	C_{hel}
$m_t/2$	-0.364	0.774	0.779	0.278
m_t	-0.389	0.806	0.813	0.311
$2m_t$	-0.407	0.829	0.836	0.331

leading logarithmic level. This was performed in Ref. [18] for the total cross section σ_t and it stabilizes the predictions for σ_t with respect to variations of μ_R and μ_F .

In Table III we compare results for C using different sets of PDF. In the case of $p\bar{p}$ collisions at $\sqrt{s} = 2$ TeV, the spread of the results is larger than the scale uncertainty given in Table II. To a considerable extent this is due to an interesting feature of C, namely the $q\bar{q}$ and gg initial states contribute to C with opposite signs. Therefore the spin correlations are quite sensitive to the relative weights of $q\bar{q}$ and gg initiated $t\bar{t}$ events. These weights depend in particular on the chosen set of PDF. For example, one finds the following individual NLO contributions for the helicity, beam, and off-diagonal correlation at the upgraded Tevatron: for the GRV98 (MRST98) PDF $C_{\text{hel}}^{q\bar{q}} = -0.443$ (-0.486), $C_{\text{hel}}^{gg} = +0.124$ (+0.075), $C_{\text{beam}}^{q\bar{q}} = +0.802$ (+0.879), $C_{\text{beam}}^{gg} = -0.068$ (-0.042), and $C_{\text{off}}^{q\bar{q}} = +0.810$ (+0.889), $C_{\text{off}}^{gg} = -0.073$ (-0.044). This suggests that accurate measurements of the dilepton distribution (2), using different spin bases, at the upgraded Tevatron may provide additional constraints in the continuing effort to improve the knowledge of the PDF.

Finally we have studied the dependence of the C coefficients on the top quark mass. For this we have used again the CTEQ5 PDF and set $\mu = m_t$. In the case of $p\bar{p}$ collisions at $\sqrt{s} = 2$ TeV, a variation of m_t from 170 to 180 GeV changes C_{hel} from -0.378 to -0.397, C_{beam} from 0.790 to 0.817, and C_{off} from 0.797 to 0.822. At LHC energies, C_{hel} changes by less than a percent.

The extension of our results to the “lepton + jets” and “all jets” decay channels [11] is straightforward. The “lepton + jets” channels should be particularly useful for detecting $t\bar{t}$ spin correlations: although one loses top-spin analyzing power one gains in statistics and the experimental reconstruction of the t and \bar{t} rest frames may also be facilitated.

TABLE III. Correlation coefficients C_{hel} , C_{beam} , and C_{off} at NLO for $\mu_R = \mu_F = m_t$ and different sets of parton distribution functions: GRV98 [19], CTEQ5 [16], and MRST98 (c-g) [20].

PDF	$p\bar{p}$ at $\sqrt{s} = 2$ TeV			pp at $\sqrt{s} = 14$ TeV
	C_{hel}	C_{beam}	C_{off}	C_{hel}
GRV98	-0.325	0.734	0.739	0.332
CTEQ5	-0.389	0.806	0.813	0.311
MRST98	-0.417	0.838	0.846	0.315

In conclusion, we have analyzed, at next-to-leading order in α_s , the hadronic production of $t\bar{t}$ quarks in a general spin configuration and have computed the dileptonic angular correlation coefficients C that reflect the degree of correlation between the t and \bar{t} spins. Our results for the Tevatron show that the scale and in particular the PDF uncertainties in the prediction of the dileptonic angular distribution must be reduced before $t\bar{t}$ spin correlations can be used in a meaningful way to search for relatively small effects of new interactions that are, for example, not distinguished by violating parity or CP invariance. Our results may also be useful to learn more about the parton distributions in the proton at high energies. For pp collisions at $\sqrt{s} = 14$ TeV the theoretical uncertainties in the prediction of this distribution are smaller and one may adopt the optimistic view that at the time the LHC will be turned on further theoretical progress will have turned top quark spin correlations into a precision tool for the analysis of $t\bar{t}$ events.

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