Test of Low-Energy Theorems for ^{1}H($\vec{\gamma}$, π^0)^{1}H in the Threshold Region

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The photon asymmetry in the reaction ^{1}H($\vec{\gamma}$, π^0)^{1}H close to threshold has been measured for the first time with the photon spectrometer TAPS using linearly polarized photons from the tagged-photon facility at the Mainz Microtron MAMI. The total and differential cross sections were also measured simultaneously with the photon asymmetry. This allowed determination of the S-wave and all three P-wave amplitudes. The values obtained at threshold are $E_{0+} = [-1.33 \pm 0.08(\text{stat}) \pm 0.03(\text{syst})] \times 10^{-3}/m_{\pi^+}$, $P_1 = [9.47 \pm 0.08(\text{stat}) \pm 0.29(\text{syst})] \times 10^{-3} q/m_{\pi^+}^2$, $P_2 = [-9.46 \pm 0.1(\text{stat}) \pm 0.29(\text{syst})] \times 10^{-3} q/m_{\pi^+}^2$, and $P_3 = [11.48 \pm 0.06(\text{stat}) \pm 0.35(\text{syst})] \times 10^{-3} q/m_{\pi^+}^2$.

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In the early 1970s, low-energy theorems (LETs) were derived for the amplitudes of pion photoproduction from the nucleon at threshold [1,2]. Based on fundamental principles, like gauge invariance and the partially conserved axial current, the LETs predict the value of the S-wave threshold amplitude E_{0+} in a power series in $\mu = m_\pi/m_N$, the ratio of the masses of the pion and nucleon. The LETs represent tests of effective degrees of freedom in the nonperturbative domain of QCD and, therefore, their investigation is of considerable interest for an understanding of QCD at low momentum transfers. Only the development of high duty factor accelerators enabled the first precise measurements of the photoproduction of neutral pions from the proton at Saclay [3] and Mainz [4]. The experimental values for E_{0+} at threshold were in conflict with the LET prediction. Most calculations also failed to predict the strong dependence of E_{0+} on the photon energy between the π^0 threshold (144.7 MeV) and 160 MeV, where a unitarity cusp due to the two-step process $\gamma p \rightarrow \pi^+ n \rightarrow \pi^0 p$ [5] was seen in the Mainz measurement [4]. These disagreements motivated several theoretical and experimental investigations. New experiments were performed at Mainz [6] and Saskatoon [7], measuring the total and differential cross sections close to threshold. The extracted values of E_{0+} confirmed the strong energy dependence and were again nearly a factor of 2 smaller than the LET prediction at threshold. This discrepancy was explained by Bernard, Kaiser, Gasser, and Meißner [8], who investigated threshold pion photoproduction in the framework of heavy-baryon chiral perturbation theory (ChPT), which showed that additional contributions due to pion loops in μ^2 have to be added to the old LET.

In the following years, refined calculations within heavy-baryon ChPT [9] led to descriptions of the four relevant amplitudes at threshold by well-defined expansions up to order p^4 in the S-wave amplitude E_{0+} and p^3 in the P-wave combinations P_1, P_2, and P_3, where p denotes any small momentum or pion mass, the expansion parameters in heavy-baryon ChPT. To that order, three low-energy constants (LEC) due to the renormalization counter terms appear, two in the expansion of E_{0+} and an additional LEC b_P for P_3, which have to be fitted to the data or estimated by resonance saturation. However, two combinations of the P-wave amplitudes, P_1 and P_2, are free of low-energy constants. Their expansions in μ converge rather well leading to new LETs for these combinations. Therefore, the P-wave LETs offer a significant test of heavy-baryon ChPT. However, for this test the S-wave amplitude E_{0+} and the three P-wave combinations P_1, P_2, and P_3 have to be separated. This separation can be achieved by measuring the photon asymmetry using linearly polarized photons, in addition to the measurement of the total and differential cross sections.

The ^{1}H($\vec{\gamma}$, π^0)^{1}H experiment [10], reported in this Letter, was performed at the Mainz Microtron MAMI [11] using the Glasgow/Mainz tagged photon facility [12,13] and the photon spectrometer TAPS [14]. The MAMI accelerator delivered a continuous wave beam of 405-MeV electrons. Linearly polarized photons were produced via coherent bremsstrahlung in a 100-μm-thick diamond radiator [15,16] with degrees of polarization of up to 50%. The diamond radiator was mounted on a goniometer [16], which was adjusted so that the linearly polarized photons lay in the energy region between π^0

threshold and 166 MeV. The energy of the photons was determined by measuring the energy of the electron after the bremsstrahlung process with the tagging spectrometer. The resolution was approximately 1 MeV at intensities of up to 5×10^5 photons $\mathrm{s}^{-1}\,\mathrm{MeV}^{-1}$.

Neutral pions were produced in a liquid hydrogen target of cylindrical shape with a length of 10 cm and a diameter of 4 cm. The neutral pion decay photons were detected in TAPS, consisting of six blocks of hexagonally shaped BaF_2 scintillation crystals each arranged in a matrix of 8×8 detectors. The blocks were mounted in a horizontal plane around the target at polar angles of ±50°, ±100°, and ±150° with respect to the photon beam direction. A forward wall, consisting of 120 phoswich telescopes [17], covered polar angles between 5° and 20°. Further details of the experimental setup are found in Ref. [18].

The identification of neutral pions relies on the coincident detection of the two photons from π^0 decay in the TAPS detector (the $\pi^0 \to \gamma\gamma$ branching ratio is $\approx 99.8\%$). The photons were identified with the help of charged-particle veto detectors, a pulse shape, and a time-of-flight analysis. An invariant mass analysis was performed to identify neutral pions and a resolution of $\simeq 19$ MeV (FWHM) was achieved. Accidental coincidences between TAPS and the tagging spectrometer were subtracted using scaled distributions of background events outside the prompt coincidence time window. For each event a missing energy analysis was performed for an unambiguous identification of neutral pions in the threshold region. The missing energy resolution for π^0 mesons close to threshold was approximately 5 MeV (FWHM). The acceptance of TAPS for neutral pions and the analyzing efficiency were determined by a Monte Carlo simulation using the GEANT3 code [19] in which all relevant properties of the setup and the TAPS detectors were taken into account.

The differential cross sections can be expressed in terms of the S- and P-wave multipoles, assuming that close to threshold neutral pions are produced only with angular momenta l_π of zero and one. Because of parity and angular momentum conservation only the S-wave amplitude E_{0+} ($l_\pi = 0$) and the P-wave amplitudes M_{1+}, M_{1-}, and E_{1+} ($l_\pi = 1$) can contribute, and it is convenient to write the differential cross section and the photon asymmetry in terms of the three P-wave combinations $P_1 = 3E_{1+} + M_{1+} - M_{1-}$, $P_2 = 3E_{1+} - M_{1+} + M_{1-}$, and $P_3 = 2M_{1+} + M_{1-}$. The c.m. differential cross section is

$$\frac{d\sigma(\theta)}{d\Omega} = \frac{q}{k}[A + B\cos(\theta) + C\cos^2(\theta)], \qquad (1)$$

where θ is the c.m. polar angle of the pion with respect to the beam direction and q and k denote the c.m. momenta of pion and photon, respectively. The coefficients $A = |E_{0+}|^2 + |P_{23}|^2$, $B = 2\,\mathrm{Re}(E_{0+}P_1^*)$, and $C = |P_1|^2 - |P_{23}|^2$ are functions of the multipole amplitudes with $P_{23}^2 = \frac{1}{2}(P_2^2 + P_3^2)$. Earlier measurements of the total and differential cross sections already allowed determination of E_{0+}, P_1, and the combination P_{23}. In order to obtain E_{0+} and all three P waves separately, it is necessary to measure, in addition to the cross sections, the photon asymmetry Σ,

$$\Sigma = \frac{d\sigma_\perp - d\sigma_\parallel}{d\sigma_\perp + d\sigma_\parallel}, \qquad (2)$$

where $d\sigma_\perp$ and $d\sigma_\parallel$ are the differential cross sections for photon polarizations perpendicular and parallel to the reaction plane defined by the pion and proton. The asymmetry is proportional to the difference of the squares of P_3 and P_2:

$$\Sigma(\theta) = \frac{q}{2k}(P_3^2 - P_2^2)\sin^2(\theta)/[d\sigma(\theta)/d\Omega]. \qquad (3)$$

Thus, the measurement of the total and differential cross sections together with Σ allows a separate determination of P_2 and P_3 and hence a test of the new LET of ChPT [9].

In the present work the total and differential cross sections were measured over the energy range from π^0 threshold to 168 MeV. Figure 1 shows the results for the total cross section which agrees with the data of Ref. [7]; the results of Ref. [6] are systematically lower, at least in the incident photon energy range of 153–162 MeV. This discrepancy may be due to a better elimination of pions produced in the target cell windows, performed in the analysis of the present data, combined with the improved detector acceptance for forward and backward angles. The different slope in the total cross section of [6] compared to the other experiments results in a steeper energy dependence for the real part of E_{0+} and slightly smaller values for P_1 and P_{23} (see Table I). The results for the photon asymmetry are shown in Fig. 2 in comparison to the values of ChPT [9] and to a prediction of a dispersion theoretical calculation (DR) by Hanstein, Drechsel, and Tiator [20]. The photon asymmetry was determined from all the data between threshold and 166 MeV for which the mean energy was 159.5 MeV. The theoretical predictions are shown for

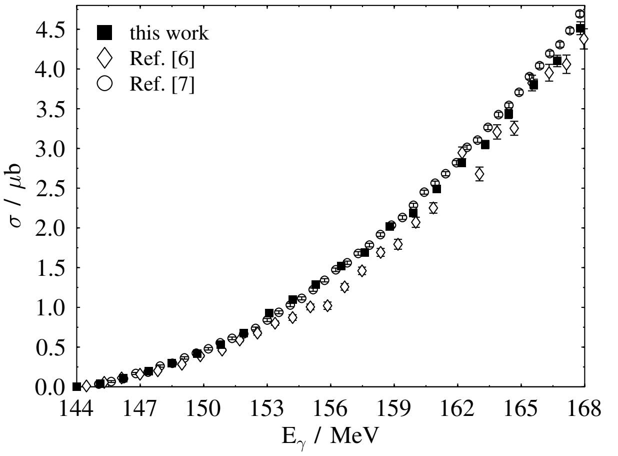

FIG. 1. Total cross sections for π^0 photoproduction close to threshold with statistical errors (without systematic error of 5%) as a function of incident photon energy (solid squares, this work; open circles, Ref. [7]; open diamonds, Ref. [6]).

TABLE I. Results of both fits (qk fit and q fit) for ReE_{0+} at the π^0 and π^+ threshold (unit: $10^{-3}/m_{\pi^+}$), for the parameter β of ImE_{0+} (unit: $10^{-3}/m_{\pi^+}^2$) and for the three combinations of the P-wave amplitudes (unit: $q \times 10^{-3}/m_{\pi^+}^2$) with statistical and systematic errors in comparison to the results of previous experiments ([7] and [6,21], only with statistical errors) and to the predictions of ChPT [9,25] ($\mathcal{O}(p^3)$) and of a dispersion theoretical approach (DR [20]).

	This work		Bergstrom[a]	Fuchs[a]	ChPT	DR[a]
	qk fit[a]	q fit	qk fit	qk fit		
$E_{0+}(E_{\rm thr}^{p\pi^0})$	$-1.23 \pm 0.08 \pm 0.03$	$-1.33 \pm 0.08 \pm 0.03$	-1.32 ± 0.05	-1.31 ± 0.2	-1.16	-1.22
$E_{0+}(E_{\rm thr}^{n\pi^+})$	$-0.45 \pm 0.07 \pm 0.02$	$-0.45 \pm 0.06 \pm 0.02$	-0.52 ± 0.04	-0.34 ± 0.03	-0.43	-0.56
β	$2.43 \pm 0.28 \pm 1.0$	$5.2 \pm 0.2 \pm 1.0$	$3.0-3.8$	2.82 ± 0.32	2.78	3.6
P_1	$9.46 \pm 0.05 \pm 0.28$	$9.47 \pm 0.08 \pm 0.29$	9.3 ± 0.09	9.08 ± 0.14	9.14 ± 0.5	9.55
P_2	$-9.5 \pm 0.09 \pm 0.28$	$-9.46 \pm 0.1 \pm 0.29$			-9.7 ± 0.5	-10.37
P_3	$11.32 \pm 0.11 \pm 0.34$	$11.48 \pm 0.06 \pm 0.35$			10.36	9.27
P_{23}	10.45 ± 0.07	10.52 ± 0.06	10.53 ± 0.07	10.37 ± 0.08	11.07	9.84

[a] Values of the P-wave combinations converted into the unit $q \times 10^{-3}/m_{\pi^+}^2$.

the same energy. The energy dependence of the ChPT prediction for the photon asymmetry was explored in the range threshold to 166 MeV and found to have a very small effect on the average, e.g., <2% at 90°.

The values for the real and imaginary parts of E_{0+} and the three P-wave combinations were extracted via two multipole fits to the cross sections and the photon asymmetry simultaneously using the following minimal model assumptions. The parametrizations of ReE_{0+} and ImE_{0+} take into account the strong energy dependence of E_{0+} below and above π^+ threshold ($E_{\rm thr}^{n\pi^+} = 151.4$ MeV) due to the two-step process $\gamma p \rightarrow \pi^+ n \rightarrow \pi^0 p$ [21]:

$$E_{0+}(E_\gamma) = A^{p\pi^0}(E_\gamma) + i\beta q_{\pi^+}\,, \tag{4}$$

where q_{π^+} is the π^+ c.m. momentum. E_{0+} is a sum of two parts, $A^{p\pi^0}$ due to the direct process and a second part, arising from the two-step process. Below π^+ threshold, one must analytically continue $q_{\pi^+} \rightarrow i|q_{\pi^+}|$. Thus E_{0+} is purely real and has the value $E_{0+} = A^{p\pi^0} - \beta|q_{\pi^+}|$, where β is the product of the S-wave amplitude $E_{0+}^{n\pi^+}$ for π^+ production and the scattering length $a_{n\pi^+\rightarrow p\pi^0}$. Above π^+ threshold, E_{0+} is complex with $E_{0+} = A^{p\pi^0} + i\beta|q_{\pi^+}|$ and Im$E_{0+} = \beta|q_{\pi^+}|$, the cusp function. In the threshold region the imaginary parts of the P waves are negligible because of the small πN-phase shifts. The two multipole fits differ in the energy dependence of the real parts of the P-wave combinations. For the first fit the usual assumption of a behavior proportional to the product of q and k was adopted (qk fit, χ^2/d.o.f. = 1.28). The assumption made for the second fit is an energy dependence of the P-wave amplitudes proportional to q (q fit, χ^2/d.o.f. = 1.29). This is the dependence which ChPT predicts for the P-wave amplitudes in the near-threshold region, but at higher energies the prediction is between the q and qk energy dependence.

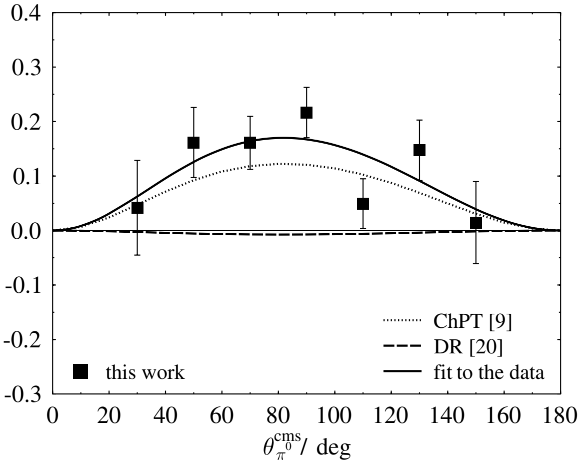

FIG. 2. Photon asymmetry for π^0 photoproduction for a photon energy of 159.5 MeV with statistical errors (without systematic error of 3%) as a function of the polar angle θ (solid line: fit to the data) in comparison to ChPT [9] (dotted line) and DR [20] (dashed line).

The results of both multipole fits for ReE_{0+} as a function of the incident photon energy are shown in Fig. 3 and compared with the predictions of ChPT and of DR. The results for the threshold values of ReE_{0+} (at the π^0 and π^+ thresholds), for the parameter β of ImE_{0+} and for the values of the threshold slopes of the three P-wave combinations of the qk fit and the q fit are summarized in Table I together with the results of [7] and [6,21]. To obtain the threshold slope of the qk fits the values of the P-wave combinations of these fits (unit: $qk \times 10^{-3}/m_{\pi^+}^3$) must be multiplied by the threshold value of the photon momentum k. In addition, the results are compared to the ChPT and DR predictions, where the errors of ChPT refer to a 5% theoretical uncertainty.

The extracted value for β and thus ImE_{0+} of the q fit is larger than the value of β obtained with the qk fit. This result can be explained by the observation that A is the best measured of the three coefficients of the differential cross section, and by noting that this determines the absolute value of E_{0+} in addition to the dominant P-wave contribution. Since ReE_{0+} is determined from the B coefficient, this gives ImE_{0+} after a subtraction of the P-wave contribution to the A coefficient. If one assumes a smaller energy dependence in the P-wave amplitudes (q fit), a stronger energy dependence for ImE_{0+} will result. However, the values of both fits for ReE_{0+} and the values of the

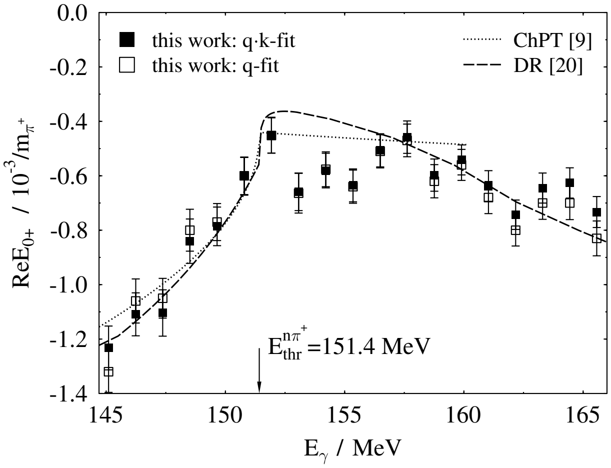

FIG. 3. Results for $\mathrm{Re}E_{0+}$ with statistical errors as a function of incident photon energy E_γ for an assumed energy dependence of the P-wave amplitudes proportional to qk (solid squares) and q (open squares) in comparison to ChPT [9] (dotted line) and DR [20] (dashed line).

three P-wave combinations at threshold are in remarkable agreement. Assuming for $E_{0+}^{n\pi^+}$ the prediction of ChPT, which agrees with the result of [22], taking for $a_{n\pi^+\to p\pi^0}$ the measured value of [23] and thus fixing the parameter β to the expected unitarity value of $3.61 \times 10^{-3}/m_{\pi^+}^2$, the values of the P-wave combinations for both fits change by less than 3%.

The main experimental uncertainty is the value of β. The systematic error of β in Table I includes the experimental uncertainty in the energy dependence of the P-wave amplitudes. The average value for $\beta = (3.8 \pm 1.4) \times 10^{-3}/m_{\pi^+}^2$ of the two fit results, obtained in this experiment, is consistent with the unitarity value. To determine β more accurately will require a direct measurement of $\mathrm{Im}E_{0+}$ in the $^1\overrightarrow{\mathrm{H}}(\gamma, \pi^0)^1\mathrm{H}$ reaction with a polarized target [24].

For both fits the low-energy theorems of ChPT [$\mathcal{O}(p)^3$] for P_1 and P_2 agree with the measured experimental results within their systematic and statistical errors. The experimental value for P_3 is higher than the value of ChPT, which can be explained by the smaller total and differential cross sections of Ref. [6], used by ChPT to determine the dominant low-energy constant $b_P = 13.0\ \mathrm{GeV}^{-3}$ [25] for this multipole.

A new fourth-order calculation in heavy-baryon ChPT by Bernard *et al.,* introduced in [26] and compared to the new MAMI data presented in this Letter, shows that the potentially large Δ-isobar contributions are canceled by the fourth-order loop corrections to the P-wave low-energy theorems. This gives confidence in the third-order LET predictions for P_1 and P_2, which are in agreement with the present MAMI data. With the new value of $b_P = 14.84\ \mathrm{GeV}^{-3}$ [26], fitted to the present MAMI data, the ChPT calculation is in agreement with the measured photon asymmetry.

To summarize, the total and differential cross sections and the photon asymmetry for the reaction $^1\mathrm{H}(\vec{\gamma}, \pi^0)^1\mathrm{H}$ have been measured simultaneously for the first time in the threshold region. Using a multipole fit to the physical observables the threshold values of the S-wave amplitude E_{0+} and all three P-wave amplitudes were extracted. The main conclusion is that the calculations of heavy-baryon ChPT for P_1 and P_2 are in agreement with the experimental results.

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