Coarsening and Slow Dynamics in Granular Compaction

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We address the problem of the microscopic reorganization of a granular medium under a compaction process in the framework of Tetris-like models. We point out the existence of regions of spatial organization which we call *domains,* and study their time evolution. It turns out that after an initial transient, most of the activity of the system is concentrated on the boundaries between domains. One can then describe the compaction phenomenon as a coarsening process for the domains, and a progressive reduction of domain boundaries. We discuss the link between the coarsening process and the slow dynamics in the framework of a model of active walkers on active substrates.

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The phenomenon of granular compaction involves the increase of the density of a granular medium [1] subject to shaking or tapping. Triggered by experimental results of the Chicago group [2], that suggested that compaction follows an inverse logarithmic law with the tapping number, several models have been proposed to explain the slow relaxation features of granular media [3–10]. Although in all these different cases a very slow relaxation (eventually logarithmic) is reproduced, an explicit connection between the above models and a *real* granular medium is however still rather tenuous.

The aim of this paper is to elucidate the origin of the very slow relaxation studying explicitly the microscopic response of a granular medium subject to shaking. We address this problem within the framework of the recently introduced class of Tetris-like models [6] which are known to reproduce several features observed experimentally in granular materials such as slow dynamics, segregation, aging, and hysteresis.

We find, quite surprisingly, that the system reorganizes under the shaking dynamics into several ordered regions (see [11,12] for other examples). We call these *domains,* and study their time evolution. After a short transient, most of the activity of the system is concentrated along the boundaries between domains (we note that this concerns only a small part of the entire system). Under shaking, the domain boundaries move throughout the system and free the vacancies they encounter leading to a progressive densification. Moreover, when two domain boundaries meet, they annihilate. One can thus describe the compaction phenomenon in this system as a coarsening [13] (i.e., a domain growth) process. As the system compactifies, the domains coarsen, and the boundary regions are reduced; thus the process becomes slower. We give a quantitative description of this phenomenon studying the behavior of the space-time correlation function that is expressed by $C(r,t) \simeq f(r/\xi(t))$ where $\xi(t)$ is the correlation length, say the typical size of a domain. This coarsening of domains is related to the slow compaction process by measuring the *persistence* [14] exponent of the phenomenon, as well as by measuring the activity and the motion of the domain boundaries.

Let us briefly recall the definition of the Tetris model. Although this class of models allows for an infinity of particles types, shapes, and sizes, here we use, without loss of generality for the main features, a system of elongated particles. These occupy the sites of a square lattice tilted by 45°, with periodic boundary conditions in the horizontal direction (cylindrical geometry) and a rigid plane at its bottom. Particles cannot overlap and this condition produces strong constraints (frustration) on their relative positions. The system is initialized by inserting grains at the top of the system, one at a time, letting them fall down, performing under the effect of gravity, an oriented random walk on the lattice until they reach a stable position, i.e., a position from which they cannot fall further. The effect of vibrations is implemented by means of a two-step Monte Carlo algorithm mimicking a tapping procedure. The role of the tapping amplitude is played by a parameter $0 < x < 1$ that describes the strength of the bias in the particle movement, induced by the gravity (we refer to [6] for the details).

In the simplest version, the Tetris model consists of a single rodlike type of particle (rectangles of uniform size $a \times b$ with $a = 0.75$ mesh units and $b = 0$) with two possible orientations (along the principal axis of the lattice) chosen to be equally probable. A generic configuration can be described by assigning to each site of the lattice (x,y) ($0 < x < L_x$ is the horizontal coordinate and $0 < y < L_y$ is the vertical one), a variable $\sigma(x,y,t)$, whose value is 0 if the site is empty and ± 1 if the site is occupied by a particle with one of the two possible orientations. At every density the system can be resolved into domains, i.e., regions in which the staggered magnetization keeps a definite sign and each domain presents an antiferromagnetic order with vacancies [$+1$ particles on odd (even) rows and -1 particles on even (odd) rows]. One can then observe the evolution of the compaction dynamics in terms of the evolution of the domains.

At the beginning, after pouring the grains into the container, i.e., in the so-called loose density state, the system presents a disordered structure with an alternation of the two types of domains, even though the domain boundaries do not as yet span the system from top to bottom. At this stage the number of domain boundaries depends on the aspect ratio (height by width) of the container: the smaller the aspect ratio (wider is the system) the larger the number of domains. The domain size is of the order of the height of the system, and is almost independent of the horizontal size of the system L_x, as long as $L_x > L_y$.

The compaction can now be seen as a slow elimination of the voids frozen in the different domains. Since the system changes only at the domain boundaries, a void in the bulk of the domain can be freed only when it comes in contact with a domain boundary. The domain boundaries are then the only regions where the activity of the system is concentrated. Figure 1 shows an example of the time evolution of the Tetris model resolved in antiferromagnetic domains [15]. It is important to note how narrow systems ($L_x \ll L_y$) may display a pathological behavior (blocking) if the system has an almost single domainlike packing.

Let us now describe the coarsening dynamics in a more quantitative way. We have monitored the evolution of the (longitudinal) correlation function defined as

$$C(r,t) = \frac{1}{N_p} \sum_{y=0}^{L_y/2} \sum_{x=0}^{L_x} \sigma(x,y,t)\sigma(x+r,y,t), \quad (1)$$

where N_p is the number of particles in the bottom half of the system. A pair of particles inside the same domain gives a positive contribution to $C(r,t)$ while a pair in different domains gives a negative contribution. With this definition, the correlation function is not sensitive to density changes and reflects only the evolution of the domain sizes.

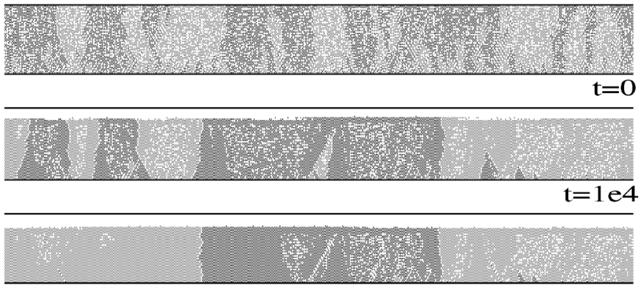

FIG. 1. Time evolution of the Tetris model with $x = 0.5$ resolved in antiferromagnetic domains at times (number of taps) $t = 0$, 10^4, and 10^7 from top to bottom. White squares are voids, while light gray and dark gray squares represent particles belonging to the two possible domain types.

We perform extensive simulations of wide systems (to avoid blocking), and we try a scaling collapse of several $C(r,t)$ curves at different times: i.e., we look for a characteristic length $\xi(t)$ such that $C(r,t) = f(r/\xi(t))$, the length $\xi(t)$ representing the average (horizontal) domain size. As in standard coarsening dynamics, $\xi(t)$ grows in time and when $\xi(t) \approx L_x$ the growth stops (blocking for single domain systems).

However, the height, L_y, of the system is another characteristic scale, and depending on whether ξ is smaller or larger than L_y, two different regimes can be observed. A quantitative analysis of ξ versus time reveals that for $\xi(t) < L_y$, $\xi(t) \propto t^{0.25}$, whereas for $\xi(t) > L_y$, we observe a faster growth: $\xi(t) \propto t^{0.5}$ (see Fig. 2).

These results can be interpreted according to the following scenario. One can imagine in general that the size $S(t)$ (area) of the domains grows as a power of time. The correlation length $\xi(t)$ is a measure of the lateral size of the domains. Now, as long as the domains grow in an isotropic way, we can expect the area to scale as $S(t) \sim \xi(t)^2$. In this regime the domain walls, though biased by gravity, do not yet span the system in the y direction. Later on, when the domain walls span the system from top to bottom, the coarsening dynamics is dominated by the diffusion of these almost vertical walls, which eventually collide and annihilate each other. At this stage we expect the area scales as $S(t) \sim L_y \xi(t)$. The crossover we observe is then compatible with a growth $S(t) \sim t^{1/2}$ which gives $\xi(t) \sim t^{1/4}$

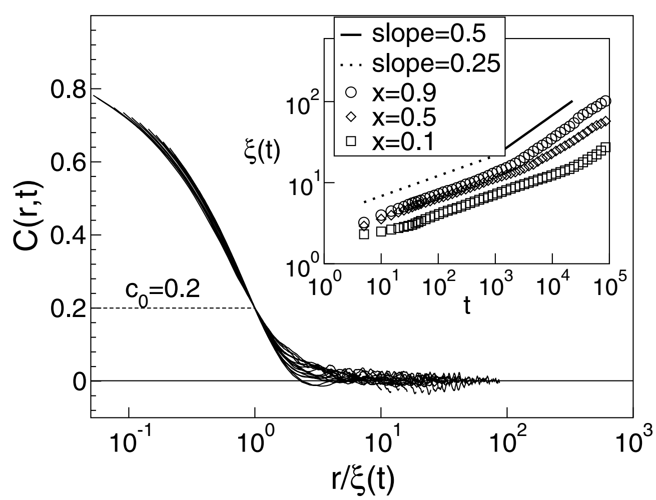

FIG. 2. Scaling collapse of (1) for a system with $L_x = 800$ and $L_y = 50$, simulated up to a time $t = 10^5$ with $x = 0.1$, and averaged over 200 different dynamics. $\xi(t)$ is chosen such that $C(\xi(t),t) = C_0$ for all curves obtained at different times for $C_0 = 0.2$ (one gets the same result for a large range of values of C_0). In the inset is shown $\xi(t)$ vs t for different values of $x = 0.1$, 0.5, and 0.9. The crossover occurs at an x-dependent t^* such that $\xi(t^*) \simeq L_y$. In the second regime one observes an exponent equal to 0.5 only for $x = 0.9$ while slightly smaller exponents are observed for smaller x. We believe these deviations from an exponent 0.5 are transients evolving towards the asymptotic diffusive value.

in the early regime followed by the asymptotic behavior $\xi(t) \sim t^{1/2}$. The crossover between the first and the second regime is evident in the first two pictures of Fig. 1.

How do we relate this coarsening behavior to the density relaxation? The motion of the domain walls occurs in a background of vacancies. We observe that regions swept through by the domain walls compactify by triggering particle rearrangements, while regions not yet swept through remain disordered. Thus within each domain there is a compactified region through which the domain wall has made several forays, and a disordered loose region through which the domain wall has not yet swept through. The compaction process is related to the growth of the dense ordered regions rather than to the characteristic length of the domains. The fast dynamics of the domain walls is thus not in contrast with the slow dynamics of the bulk density.

In order to support this picture we have measured the fraction of persistent sites (or persistence probability) $R(t)$ [14], i.e., the fraction of sites that never changed their status up to time t. If the triggering process mentioned above was perfectly efficient, i.e., every time a domain wall passes through a vacancy one triggers a process increasing the density locally, one would expect that $1 - \rho(t)$ is described by $R(t)$. Otherwise the behavior of $1 - \rho(t)$ is slower than $R(t)$. In analogy with recent studies [14] in standard coarsening models, a very slow algebraic decrease of such quantities is observed as a function of time; we obtain, in particular, that $R(t)$ (see Fig. 3) scales as $t^{-\theta}$ with $\theta \simeq 0.15$. The behavior of $[1 - \rho(t)]$ vs t is shown in the inset of Fig. 4. We observe that it is consistent with an algebraic decay $1 - \rho(t) \simeq t^{-0.10}$ with an exponent smaller than the persistence exponent θ which is compatible with our discussion above.

Additional information on the system can be obtained by monitoring the following quantities. (1) $A(t)$: activity in the system measured as the cumulative number of successful moves; (2) $M(t)$ is intended to measure the mobility of the domain walls. It is obtained by the cumulative sum of the absolute value of the derivative of the staggered magnetization in thin vertical stripes, averaged over all the stripes. From Fig. 4, which reports the results for both quantities, one deduces two main things: the activity of the system is concentrated on the domain walls [since $A(t)$ and $M(t)$ have the same functional form up to a constant] and both $A(t)$ and $M(t)$ scale as $[1 - \rho(t)]^{-\beta}$.

In order to get better insight into the above mentioned phenomenology, it would be very interesting to have a quantitative understanding of the link between the coarsening process and the very slow global density relaxation. Under the hypothesis that the system is translationally invariant in the direction of gravity, the medium is described as a one dimensional (y-averaged) density profile, $\rho(x, t)$, in which particles (the domain wall) move. To comply with our previous description, we assume that the density ρ is only susceptible to increase at the positions occupied by the walkers, and remains quenched elsewhere. Moreover, in order to follow more closely what happens in the Tetris model we consider a situation where the motion of the walkers is coupled to the environment, i.e., the local density via a potential field depending only on the density. The problem can be cast in terms of two coupled equations: one describing the overdamped motion of the walkers and another describing the evolution of the local density as

$$\frac{dX}{dt} = -\frac{\partial V[\rho(x,t)]}{\partial \rho(x,t)}\bigg|_{x=X(t)} \frac{\partial \rho(x,t)}{\partial x}\bigg|_{x=X(t)} + \Gamma(t), \qquad \frac{\partial \rho(x,t)}{\partial t} = f(\rho(x,t))\delta(x - X(t)), \tag{2}$$

where $\Gamma(t)$ is an uncorrelated Langevin force with $\langle \Gamma(t) \rangle = 0$ and $\langle \Gamma(t)\Gamma(t') \rangle = q\delta(t - t')$. The potential V attracts the walker to regions where activity has been intense, and repels it from unvisited regions.

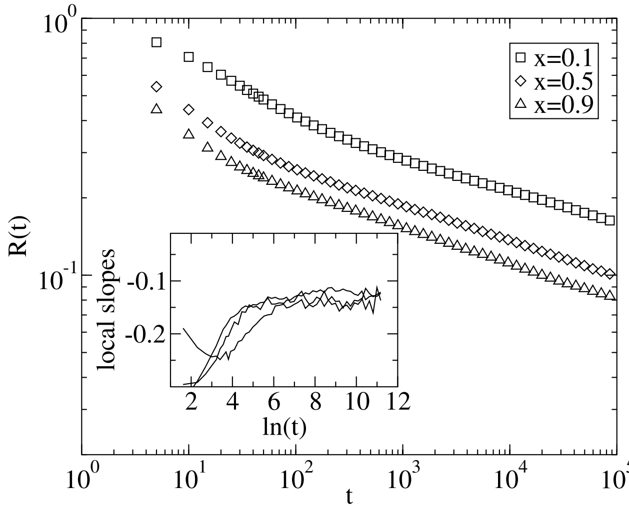

FIG. 3. $R(t)$ (see text for the definition) vs time for different values of x. The inset shows the local logarithmic slopes.

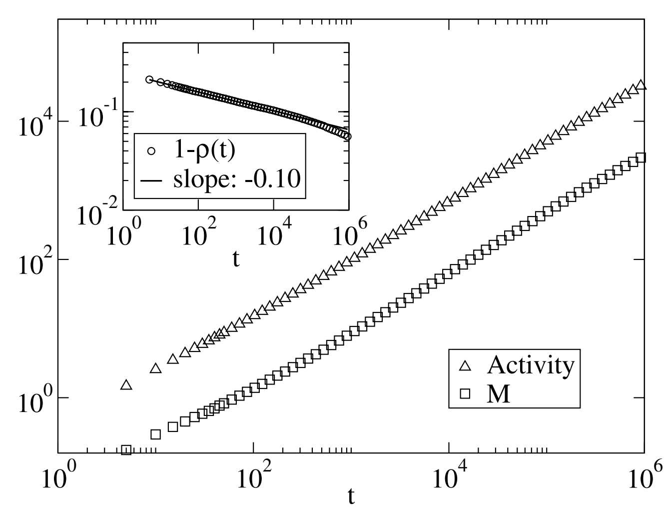

FIG. 4. $A(t)$ and $M(t)$ (see text for the definition) vs t for $x = 0.5$, $L_x = 800$, and $L_y = 50$. Similar results have been obtained with $x = 0.1$ and $x = 0.9$. The inset reports $[1 - \rho(t)]$ vs t [16].

A detailed treatment of the above defined equations is presented elsewhere [17]. Here we summarize the main features. The function f should be such that ρ approaches unity for long times, and hence $f(\rho) \to 0$ as $\rho \to 1$. An example of such an f is $f(\rho) = (1 - \rho)^a$, where a is a positive exponent. The potential V should provide a drift toward high density regions. A simple suitable form is $V(\rho) = -\rho^\gamma$. In fact, it is possible to show that the latter functional form is inessential, provided $V(\rho)$ behaves linearly in ρ as $\rho \to 1$. In this way $\frac{\partial V[\rho(x,t)]}{\partial \rho(x,t)}$ becomes constant, i.e., unimportant, as ρ tends to 1. With these definitions Eqs. (2) can be recast in the form

$$\frac{dX}{dt} = F^{-1-b}\nabla F + \Gamma(t), \qquad \frac{dF}{dt} = \delta(x - X(t)), \tag{3}$$

where $b = 1/(a - 1)$ and we have introduced the function $F = [1/(a - 1)](1 - \rho)^{1-a}$ which, according to Eq. (3), represents the cumulative activity on the site x [in general $dF/d\rho = f(\rho)$]. In this way the choice of the function f is consistent with the results obtained for $A(t)$ and $M(t)$ in the Tetris model. The main idea behind this kind of modeling is that the high density regions (i.e., the potential wells) tend to trap the walkers that, in their turn, are able to change the environment, i.e., the local density, though their efficiency decreases with the increase of the density. From the combination of these two effects a drastic slowing down is expected. The way the walkers escape from the potential wells is to progressively carve their way out by pushing the potential barrier and so enlarging the compactified region.

Different aspects come into play in the compaction process. One of them is related to the fact that initially a large number of walkers is to be introduced. Since our modeling concerns only the regime where a one dimensional description is adapted, the typical distance between walkers is proportional to L_y. However, when two walkers meet, they annihilate, and thus the subsequent increase of the density becomes less and less effective. The quantification of this effect has been done in a previous section, through the pair correlation function of the domains. By itself, this single aspect is not sufficient to account for the slow densification observed numerically. The second aspect concerns the densification due to a single walker. Starting from a low density for the medium, we observe that the density does not remain uniform. Starting from any site, at low temperature, the domain walls first drill a potential well where they lie. However, as the density approaches unity, the densification becomes less and less efficient. The only option for the walkers is to expand in lateral size through a progressive translation of the well boundaries. Specific solutions of this regime can be obtained as solitary waves [17]. In this second regime, the densification rate is controlled by the velocity of the latters. Finally, the wells tend to coalesce and the mean density decays as $1 - \rho \sim t^{-1/(a-1)}$. Though the variety of the different phenomena involved in the density evolution (number of walkers, width and depth of potential wells, late stages crossover phenomena) renders difficult the identification of the F function in Eq. (3), we retain that the main features observed in the Tetris model are captured by this walkers modeling.

It is also important to stress that the equivalence observed between the activity and the motion of the domain boundaries implies that the same treatment could also be carried out for particles with random shapes [6]. Here the existence of domains is no longer evident but the activity still remains confined and it is not spread out uniformly over the system. Finally it is worth remarking that our analysis could be easily exported in an experimental setup such as the one proposed in [18].

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[1] *Physics of Dry Granular Media,* edited by H. J. Herrmann *et al.* (Kluwer Academic Publishers, Dordrecht, 1998).

[2] J. B. Knight, C. G. Fandrich, C. N. Lau, H. M. Jaeger, and S. R. Nagel, Phys. Rev. E **51**, 3957 (1995).

[3] G. C. Barker and A. Mehta, Phys. Rev. E **47**, 184 (1993).

[4] E. Ben-Naim, J. B. Knight, E. R. Nowak, H. M. Jaeger, and S. R. Nagel, Physica (Amsterdam) **123D**, 380 (1998); P. L. Krapivsky and E. Ben-Naim, J. Chem. Phys. **100**, 6778 (1994).

[5] T. Boutreux and P. G. de Gennes, Physica (Amsterdam) **244A**, 59 (1997).

[6] E. Caglioti, V. Loreto, H. J. Herrmann, and M. Nicodemi, Phys. Rev. Lett. **79**, 1575 (1997). For random particle shapes, see A. Barrat and V. Loreto, J. Phys. A **33**, 4401 (2000).

[7] M. Nicodemi, A. Coniglio, and H. J. Herrmann, Phys. Rev. E **55**, 3962 (1997).

[8] J. J. Brey, A. Prados, and B. Sanchez-Rey, Phys. Rev. E **60**, 5685 (1999).

[9] D. A. Head, Phys. Rev. E **62**, 2439 (2000).

[10] S. N. Majumdar, D. S. Dean, and P. Grassberger, Phys. Rev. Lett. **86**, 2301 (2001).

[11] E. Caglioti, A. Coniglio, H. J. Herrmann, V. Loreto, and M. Nicodemi, Europhys. Lett. **43**, 591 (1998).

[12] S. Krishnamurthy, V. Loreto, H. J. Herrmann, S. S. Manna, and S. Roux, Phys. Rev. Lett. **83**, 304 (1999); S. Krishnamurthy, V. Loreto, and S. Roux, Phys. Rev. Lett. **84**, 1039 (2000).

[13] A. Bray, Adv. Phys. **43**, 357 (1994).

[14] See S. N. Majumdar, Curr. Sci. **77**, 370 (1999) for a brief review.

[15] See cond-mat/0110026 for an animation.

[16] However, we have indication of a second crossover in the temporal behavior of $R(t)$ and $\rho(t)$ at very large times. In particular, $[1 - \rho(t)]$ seems to undergo an acceleration.

[17] A. Baldassarri, S. Krishnamurthy, V. Loreto, and S. Roux (to be published).

[18] C. J. Olson, C. Reichhardt, M. McCloskey, and R. J. Zieve, cond-mat/0011508.