# New Measurement of $K_{e4}^+$ Decay and the $s$-Wave $\pi\pi$-Scattering Length $a_0^0$

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A sample of $4 \times 10^5$ events from the decay $K^+ \rightarrow \pi^+\pi^-e^+\nu_e$ ($K_{e4}$) has been collected in experiment E865 at the Brookhaven Alternating Gradient Synchrotron. The analysis of these data yields new measurements of the $K_{e4}$ branching ratio $[(4.11 \pm 0.01 \pm 0.11) \times 10^{-5}]$, the $s$-wave $\pi\pi$ scattering length $[a_0^0 = 0.216 \pm 0.013(\text{stat}) \pm 0.004(\text{syst}) \pm 0.005(\text{theor})]$, and the form factors $F$, $G$, and $H$ of the hadronic current and their dependence on the invariant $\pi\pi$ mass.

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More than 30 years ago it was recognized that measurements of the properties of $K_{e4}$ decay $[K^\pm \rightarrow \pi^+\pi^-e^\pm\nu_e(\bar{\nu}_e)]$ would provide important information about both the weak and strong interactions. This four-body semileptonic decay is particularly interesting because the two pions are the only hadrons in the final state. It allows studies over a broad kinematic range of several form factors describing both the vector and axial vector hadronic currents, and uniquely of the low energy $\pi\pi$ interaction in an environment without the presence of other hadrons.

While experimental studies of $K_{e4}$ held promise of significant physics insight, the small branching ratio of about 0.004% has made precise measurements of the decay parameters difficult [1,2]. For instance, while the possibility of extracting the isospin zero, angular momentum zero scattering length $a_0^0$ has long been recognized [3], it was not until 1977, when the Geneva-Saclay experiment [2] gathered about 30 000 events, that a measurement was made of this quantity to 20% accuracy.

On the theoretical side, chiral QCD perturbation theory (ChPT) [4] makes firm predictions for the scattering length. The tree level calculation in ChPT $a_0^0 = 0.156$ (in units of $m_\pi$) [5]. The one-loop ($a_0^0 = 0.201 \pm 0.01$ [6]) and two-loop calculations ($a_0^0 = 0.217$ [7]) show a satisfactory convergence. The most recent calculation [8] matches the known chiral perturbation theory representation of the $\pi\pi$ scattering amplitude to two loops [7] with a phenomenological description that relies on the Roy equations [9,10], resulting in the prediction $a_0^0 = 0.220 \pm 0.005$.

The analysis of the Geneva-Saclay experiment [2] combined with the Roy equations and the inclusion of peripheral $\pi N \rightarrow \pi\pi N$ data led to the presently accepted value of $a_0^0 = 0.26 \pm 0.05$ [11]. It has been argued, that, if the central experimental value $a_0^0 = 0.26$ would be confirmed with a smaller error, such a large value can be explained only by a significant reduction of the quark condensate $\langle 0|\bar{u}u|0\rangle$, as is possible in generalized chiral perturbation theory [12]. On the other hand, a higher precision measurement of $a_0^0$ would allow one to reduce the bounds on this parameter [13].

The analysis outlined here is based on data recorded at the Brookhaven Alternating Gradient Synchrotron (AGS), employing the E865 detector. The apparatus, described in detail in [14], is shown in Fig. 1. The detector resided in a 6 GeV/$c$ unseparated $K^+$ beam directly downstream of

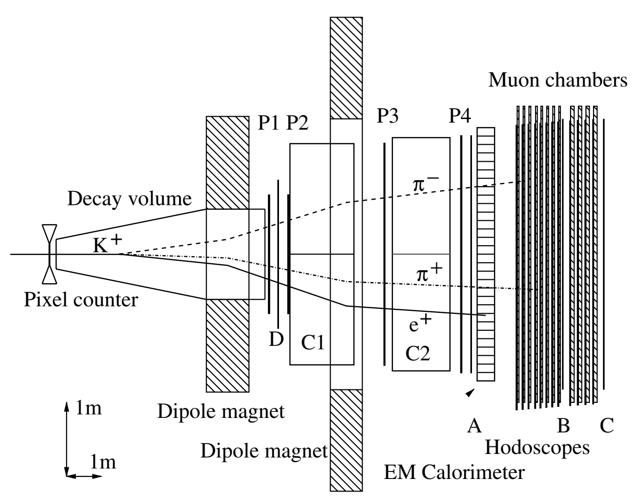


FIG. 1. Plan view of the E865 detector. A $K_{e4}$ event is superimposed.

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a 5 m long evacuated decay volume. A first dipole magnet separated the $K^+$ decay products by charge. A second dipole magnet sandwiched between four proportional wire chambers (P1–P4) served as spectrometer. Two gas Čerenkov counters C1 and C2, filled with $CH_4$ at atmospheric pressure, and an electromagnetic calorimeter distinguished $\pi^\pm$ and $\mu^\pm$ from $e^\pm$. $\pi^\pm$ are separated from $\mu^\pm$ by a set of 12 muon chambers. Four hodoscopes were added to the detector for trigger purposes. In our analysis we determined the $K^+$ momentum using the beam line as a spectrometer, the position of the decay vertex, and the information from the pixel counter installed just upstream of the decay volume.

The first level trigger selected three charged particle tracks based on coincidences between the A and D hodoscopes and the calorimeter. The second level trigger indicated the presence of an $e^+$ not accompanied by an $e^-$. It required signals in both right side counters and only minimal light in both left side Čerenkov counters. In this we discriminated against two of the most common background channels: (i) $K^+ \rightarrow \pi^+\pi^+\pi^-(K_\tau)$ and (ii) $K^+ \rightarrow \pi^+\pi^0$ followed by $\pi^0 \rightarrow e^+e^-\gamma(K_{dal})$.

The off-line analysis selected events containing three charged tracks with a vertex within the decay volume of acceptable quality, a summed momentum of less than 5.87 GeV/$c$, and a timing spread between the tracks consistent with the resolution of 0.5 ns. Even after particle identification criteria were applied, the remaining sample still contained background events mainly from $K_\tau$ decay with a misidentification of a $\pi^+$ as an $e^+$ and accidentals. Requiring that the $K^+$ reconstructed from the three charged daughter particles does not track back to the target reduced the background from $K_\tau$ to the level of 1.3±0.3%, since for $K_{e4}$ the undetected neutrino made the reconstruction incomplete. The dominating accidental background was a combination of a $\pi^+\pi^-$ pair from a $K_\tau$ decay with an $e^+$ from either the beam or a coincident decay with an $e^+$ in its final state. A likelihood method was employed to reduce this background to a level of 2.4 ± 1.2%. Because of the excellent particle identification capabilities of our detector all other backgrounds were negligible.

After the event selection 406 103 events remained, of which we estimate 38 8270 ± 5025 to be $K_{e4}$ events.

To determine the branching ratio, the form factors, and other related quantities a Monte Carlo simulation is needed. Our code, based on GEANT, takes into account the detector geometry as well as the independently measured efficiencies of all detector elements. $K_{e4}$ decays are modeled by ChPT on the one-loop level [15,16]. Radiative corrections are included following Diamant-Berger [17]. With this apparatus, we generated $81.6 \times 10^6$ $K_{e4}$ events, resulting in $2.9 \times 10^6$ accepted events. The agreement between data and Monte Carlo in all control variable distributions is very good, as, e.g., evidenced by the plots shown in Fig. 3.

The $K_{e4}$ branching ratio is measured with respect to $K_\tau$ decay. $K_\tau$ events were collected in a minimum bias prescaled trigger together with $K_{e4}$ events. With $B(\tau) = (5.59 \pm 0.05)\%$ [18], the $K_{e4}$ branching ratio is calculated to be

$$B(K_{e4}) = [4109 \pm 8(\text{stat}) \pm 110(\text{syst})] \times 10^{-8}.$$

This result agrees well with the average of previous experiments [18]: $(3.91 \pm 0.17) \times 10^{-5}$. The systematic uncertainties are dominated by the uncertainties in the Čerenkov counter efficiencies and background contributions.

The kinematics of $K_{e4}$ decay can be fully described by five variables [19]: (i) $s_\pi = M^2_{\pi\pi}$ and (ii) $s_e = M^2_{e\nu}$, the invariant mass squared of the dipion and the dilepton, respectively; (iii) $\theta_\pi$ and (iv) $\theta_e$, the polar angles of $\pi^+$ and $e^+$ in the dipion and dilepton rest frames measured with respect to the flight direction of dipion and dilepton in the $K^+$ rest frame, respectively; (v) $\phi$, the azimuthal angle between the dipion and dilepton planes. The FWHM resolution of the apparatus for these five variables is estimated to be 0.001 33 GeV$^2$ ($s_\pi$), 0.003 61 GeV$^2$ ($s_e$), 147 mrad ($\theta_\pi$), 111 mrad ($\theta_e$), and 404 mrad ($\phi$).

The matrix element in terms of the hadronic vector and axial vector current contributions $V^\mu$ and $A^\mu$ is given by

$$M = \frac{G_F}{\sqrt{2}} V^*_{us}\overline{u}(p_\nu)\gamma_\mu(1 - \gamma_5)v(p_e)(V^\mu - A^\mu), \quad (1)$$

$$A^\mu = FP^\mu + GQ^\mu + RL^\mu, \quad V^\mu = H\epsilon^{\mu\nu\rho\sigma}L_\nu P_\rho Q_\sigma, \quad (2)$$

where $P = p_1 + p_2$, $Q = p_1 - p_2$, and $L = p_e + p_\nu$, and $p_1$, $p_2$, $p_e$, and $p_\nu$ are the four-momenta of the $\pi^+$, $\pi^-$, $e^+$, and $\nu_e$ in units of $M_K$, respectively.

TABLE I. Form factors and phase shifts $\delta \equiv \delta^0_0 - \delta^1_1$ (in units of $10^{-3}$) for the six bins in $M_{\pi\pi}$. The number of degrees of freedom for each fit is 4796. The first uncertainty is statistical, the second systematical with dominant contributions from background and Čerenkov efficiency.

| $M_{\pi\pi}$ ($\overline{M}_{\pi\pi}$) (MeV) | $F$ | $G$ | $H$ | $\delta$ | $\chi^2$/NdF |
|---|---|---|---|---|---|
| 280–294 (285.2) | 5832 ± 13 ± 80 | 4703 ± 89 ± 69 | −3740 ± 800 ± 180 | −16 ± 40 ± 2 | 1.07 |
| 294–305 (299.5) | 5875 ± 14 ± 83 | 4694 ± 62 ± 67 | −3500 ± 520 ± 190 | 68 ± 25 ± 1 | 1.08 |
| 305–317 (311.2) | 5963 ± 14 ± 90 | 4772 ± 54 ± 70 | −3550 ± 440 ± 200 | 134 ± 19 ± 2 | 1.07 |
| 317–331 (324.0) | 6022 ± 16 ± 94 | 5000 ± 51 ± 82 | −3630 ± 410 ± 230 | 160 ± 17 ± 2 | 1.10 |
| 331–350 (340.4) | 6145 ± 17 ± 96 | 5003 ± 49 ± 83 | −1700 ± 410 ± 240 | 212 ± 15 ± 3 | 1.09 |
| >350 (381.4) | 6196 ± 20 ± 83 | 5105 ± 50 ± 74 | −2230 ± 480 ± 330 | 284 ± 14 ± 3 | 1.03 |

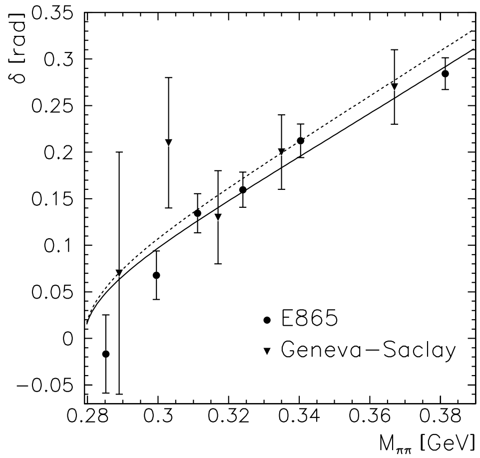


FIG. 2. Phase shift difference $\delta$ as a function of dipion mass. The dashed line represents the fit to Eq. (4) for the Geneva-Saclay data [2] and the solid line for our data with the scattering length $a_0^0$ as free parameter.

The form factors $F$, $G$, $R$, and $H$ are dimensionless complex functions of $s_\pi$, $s_e$, and $\theta_\pi$. The expressions for the decay rate derived from this matrix element have been given in Ref. [20].

Amorós and Bijnens recently developed a parametrization of these form factors, based on a partial wave expansion in the variable $\theta_\pi$ [21]:

$$F = (f_s + f_s' q^2 + f_s'' q^4 + f_e s_e) e^{i\delta_0^0} + \tilde{f}_p (Q^2/s_\pi)^{1/2} (P \cdot L) \cos\theta_\pi e^{i\delta_1^1},$$
$$G = (g_p + g_p' q^2 + g_e s_e) e^{i\delta_1^1}, \qquad H = (h_p + h_p' q^2) e^{i\delta_1^1}, \tag{3}$$

where $q = [s_\pi/(4M_\pi^2) - 1]^{1/2}$ is the pion momentum in $\pi\pi$ rest frame. The form factor $R$ enters the decay distribution multiplied by $m_e^2$ and can therefore be neglected. This parametrization yields ten new form factors $f_s$, $f_s'$, $f_s''$, $f_e$, $\tilde{f}_p$, $g_p$, $g_p'$, $g_e$, $h_p$, and $h_p'$, which do not depend on any kinematic variables, plus the phases $\delta_0^0$ and $\delta_1^1$, which are functions of $s_\pi$.

The phase shifts can be related to the scattering lengths. A recent analysis [10] used the parametrization proposed by Schenk [22]:

$$\tan\delta_\ell^I = \sqrt{1 - \frac{4M_\pi^2}{s}} \sum_{k=0}^{3} A_{\ell k}^I q^{2(\ell+k)} \left( \frac{4M_\pi^2 - s_\ell^I}{s - s_\ell^I} \right). \tag{4}$$

The Roy equations [9] are then solved numerically, expressing the parameters $A_{\ell k}^I$ and $s_\ell^I$ as functions of the scattering lengths $a_0^0$ and $a_0^2$. The possible values of the scattering lengths are restricted to a band in the $a_0^0$ versus $a_0^2$ plane. The centroid of this band, the *universal curve* [23] relates $a_0^0$ and $a_0^2$:

$$a_0^2 = -0.0849 + 0.232\, a_0^0 - 0.0865\, (a_0^0)^2 [\pm 0.0088], \tag{5}$$

where the error given in the bracket indicates the width of the band permitted by analyticity [10]. This width reduces considerably, if chiral symmetry constraints are imposed. One then obtains [13]

$$a_0^2 = -0.0444 + 0.236(a_0^0 - 0.22) - 0.61(a_0^0 - 0.22)^2 - 9.9(a_0^0 - 0.22)^3 [\pm 0.0008]. \tag{6}$$

For the fits we divided our data into six bins in $s_\pi$, five in $s_e$, ten in $\cos\theta_\pi$, six in $\cos\theta_e$, and 16 in $\phi$. In the $\chi^2$ minimization procedure, the number of measured events in each bin $j$ is compared to the number of expected events given by

$$r_j = B(K_{e4}) \frac{N^K}{N^{\rm MC}} \sum \frac{J_5(F,G,H)^{\rm new}}{J_5(F,G,H)^{\rm MC}}, \tag{7}$$

where the sum runs over all Monte Carlo events in bin $j$. $N^K$ is the number of $K^+$ decays derived from the number of $K_\tau$ events. $N^{\rm MC}$ is the number of generated events. $J_5(F,G,H)^{\rm MC}$ ($\equiv I$ [20]) is the five-dimensional phase space density generated at the momentum $q = q^{\rm MC}$ with the form factors $F$, $G$, and $H$ used to simulate the event. $J_5(F,G,H)^{\rm new}$ is calculated at $q^{\rm MC}$ with $F$, $G$, $H$ evaluated from the parameters of the fit. Thus, we apply the parameters on an event by event basis, and, at the same time, we divide out a possible bias caused by the matrix element, making the fit independent of the ChPT ansatz used to generate the MC.

For the fit, we have assumed that $F$, $G$, and $H$ do not depend on $s_e$ and that $F$ contributes to $s$ waves only, i.e., $f_e = g_e = \tilde{f}_p = 0$. Our first set of fits is done independently for each bin in $s_\pi$. The above assumptions then leave one parameter each to describe $F$, $G$, and $H$ aside from the phase difference $\delta \equiv \delta_0^0 - \delta_1^1$. The results are listed in Table I. The centroids of the bin ($\overline{M}_{\pi\pi}$) are determined following Lafferty and Wyatt [24]. If the six phase shifts in Table I are fit using Eqs. (4) and (5), one obtains $a_0^0 = 0.229 \pm 0.015$ ($\chi^2/\text{NdF} = 4.8/5$). The resulting curve is shown in Fig. 2.

We have also made a single fit to the entire data sample. In this second fit we substituted $\delta$ in Eq. (3) by the expression of Eq. (4). With the relation between $a_0^0$ and $a_0^2$ given by Eq. (5) or Eq. (6) only $f_s$, $f_s'$, $f_s''$, $g_p$, $g_p'$, $h_p$, and $a_0^0$ then remain as free parameters. The results, listed

TABLE II. Form factors (in units of $10^{-2}$) and scattering length $a_0^0$ in the parametrization of Eq. (3) using either Eq. (5) or Eq. (6). The sequence of errors given is statistical, systematic, and theoretical. ($\chi^2/\text{NdF} = 30\,963/28\,793$.)

| | | | | | |
|---|---|---|---|---|---|
| $f_s$: $575 \pm 2 \pm 8$ | $f_s'$: $106 \pm 10 \pm 40$ | $f_s''$: $-59 \pm 12 \pm 40$ | $g_p$: $466 \pm 5 \pm 7$ | $g_p'$: $67 \pm 10 \pm 4$ | $h_p$: $-295 \pm 19 \pm 20$ |
| $a_0^0$: $0.228 \pm 0.012 \pm 0.004^{+0.006}_{-0.012}$ [Eq. (5)] | | | $a_0^0$: $0.216 \pm 0.013 \pm 0.004 \pm 0.005$ [Eq. (6)] | | |

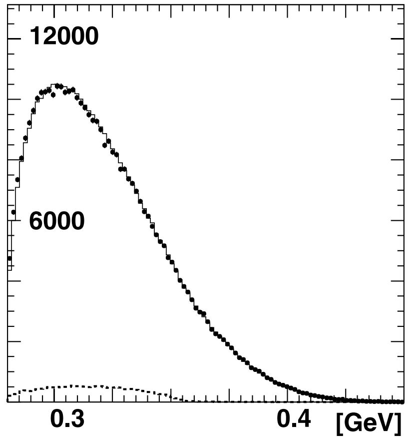


FIG. 3. Invariant mass distribution describing the $K_{e4}$ decay. The histogram is the Monte Carlo distribution while the markers with the error bars represent the data. The dashed histogram indicates the non-$K_{e4}$ background.

in Table II are in an excellent agreement with the ones derived in the previous paragraph.

To check the assumption $f_e = g_e = \tilde{f}_p = 0$ we also allowed these form factors to vary, one at a time, in our second fit. The results ($\tilde{f}_p = -4.3 \pm 1.3 \pm 3.4$, $f_e = -4.1 \pm 1.3 \pm 3.1$, $g_e = 0.5 \pm 4.4 \pm 11.3$) show that within the experimental uncertainties all three form factors are consistent with zero.

The quality of the fits is demonstrated in Fig. 3, where the invariant mass ($s_\pi$) distribution from data is compared to the reweighted Monte Carlo distributions [Eq. (7)].

To summarize, experiment E865 has collected a $K_{e4}$ event sample more than 10 times larger than all previous experiments combined. From the model independent analysis of this data the momentum dependence of the form factors of the hadronic currents as well as $\pi\pi$ scattering phase shifts have been extracted. The form factors and phase shifts serve as an important input in the program to determine the couplings of the effective Hamiltonian of chiral QCD perturbation theory at low energies [25]. From a preliminary communication of these results already tight bounds on the value of the quark condensate have been extracted [13]. Using the relations between $a_0^0$ and $a_0^2$ given by the Roy equations [10] and chiral symmetry constraints [13], we have extracted the most precise value of the $\pi\pi$ scattering length $a_0^0$, namely, [$a_0^0 = 0.216 \pm 0.013(\text{stat}) \pm 0.004(\text{syst}) \pm 0.005(\text{theor})$]. This value agrees well with predictions obtained in the framework of ChPT [8].

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