Extracting Work from a Single Thermal Bath via Quantum Negentropy

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Classical heat engines produce work by operating between a high temperature energy source and a low temperature entropy sink. The present quantum heat engine has no cooler reservoir acting as a sink of entropy but has instead an internal reservoir of negentropy which allows extraction of work from one thermal bath. The process is attended by constantly increasing entropy and does not violate the second law of thermodynamics.

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The principle of detailed balance, so admirably applied by Einstein to the discovery of stimulated emission and derivation of the Planck distribution, is a mainstay of statistical physics. However, recent studies have shown how to break emission-absorption symmetry, yielding lasers operating without inversion [1] and pointing the way to coherent control of thermodynamic processes [2–4]. In fact, the original state selective masers (SSM) [5] operated by sorting hot (spin up) from cold (spin down) atoms in order to get maser action from a thermal spin distribution.

The present work is an outgrowth of Ref. [4] where it is shown that the internal "spin" states of an atom can be cooled to absolute zero via a SSM scheme, as in Fig. 1(b); see also [6]. We here show how this can be extended to design a quantum heat engine (QHE) based on cycling a single atom through a micromaser [7] cavity many times.

In classical heat engines useful work is produced by drawing energy from a high temperature source and depositing entropy in a low temperature entropy sink. Specifically a working fluid, such as steam, draws energy from a boiler, does work on a piston, and deposits entropy in the cooling water. In the present QHE the atomic spin states play the role of the working fluid, a blackbody holhraum is the energy source, and the atomic spins drive a maser field producing useful work. However, there is no lower temperature entropy sink in the present QHE. Instead the atomic center of mass (c.m.) degrees of freedom are used to provide a source of negentropy [8] which allows the QHE to operate for a finite number of cycles. Engine operation is attended by constantly increasing entropy and does not violate the second law.

To set the stage for the present discussion, consider the operation of the maser cooler [4] of Fig. 1(b). There we depict atoms emerging from a blackbody holhraum in a thermal mixture of spin states $|a\rangle$ and $|b\rangle$ but having a well defined c.m. wave function given by $|\Psi_o\rangle$. The Stern-Gerlach Apparatus (SGA) deflects $|a\rangle$ and $|b\rangle$ atoms along upper (U) and lower (L) paths. The c.m. wave function is peaked about these paths and given by $|\Psi_U\rangle$ and $|\Psi_L\rangle$, respectively. The excited state $|a\rangle$ atoms then pass through a maser cavity and are Rabi flipped to the $|b\rangle$ state with unit probability [4].

The upper and lower path atoms are directed into a box as in Fig. 1 resulting in a spin temperature of $T = 0$. To that end, consider the density matrix describing the spin and the c.m. degrees of freedom just before entering the SGA. It is given by $\rho_o = Z^{-1}\sum_\alpha \exp(-\beta\epsilon_\alpha)\Lambda_\alpha \otimes \Lambda_o$, where $\alpha = a, b$, ϵ_α is the energy of the $|\alpha\rangle$ state, $\beta = 1/kT$, and $Z = \sum_\alpha \exp(-\beta\epsilon_\alpha)$, the projection operators $\Lambda_\alpha = |\alpha\rangle\langle\alpha|$ and $\Lambda_o = |\Psi_o\rangle\langle\Psi_o|$. Just before entering the maser the density matrix is given by

$$\rho_1 = \frac{\exp(-\beta\epsilon_a)}{Z}\Lambda_a\Lambda_U + \frac{\exp(-\beta\epsilon_b)}{Z}\Lambda_b\Lambda_L\,,$$

where $\Lambda_s = |\Psi_s\rangle\langle\Psi_s|$, $S = U, L$. After passing through the maser and flipping the $|a\rangle$ spins to $|b\rangle$, the density matrix is $\rho_2 = Z^{-1}\sum_s \exp(-\beta\epsilon_s)\Lambda_s \otimes \Lambda_b$, where $\epsilon_U = \epsilon_a$ and $\epsilon_L = \epsilon_b$.

The spins are now all in the spin down state and the Boltzmann weighting factors go from being associated with the spin to the c.m. degrees of freedom. Hence, the c.m. motion serves as a kind of "reservoir" into which the spin thermal noise is transferred. In the following we apply this notion to extracting work from a single thermal bath. We first present the one-atom QHE concept and analyze its operation. In the conclusion, we make contact with pertinate previous work and highlight key points and open questions.

The one-atom QHE depicted in Fig. 1, is based on the fact that by using a carefully prepared two-level atom we can cyclically extract maser energy from a single thermal reservoir. In Fig. 1 we see a pulsed atom beam passing through a heat bath consisting of a hot microwave cavity in thermal equilibrium, i.e., a blackbody hohlraum at temperature T. The c.m. motion is essentially unaffected by the radiation heat bath [9], but the $|b\rangle$ spin state is changed to a thermal mixture.

It is important to emphasize that the atom undergoes state change only when in the various cavities, since the atom-field coupling can be made much stronger inside the cavity than outside. This is the central theme of cavity QED [10], wherein we routinely assume the atomic states are very long-lived outside the cavity. As discussed in the figure caption, the atom passes from the hohlraum into an

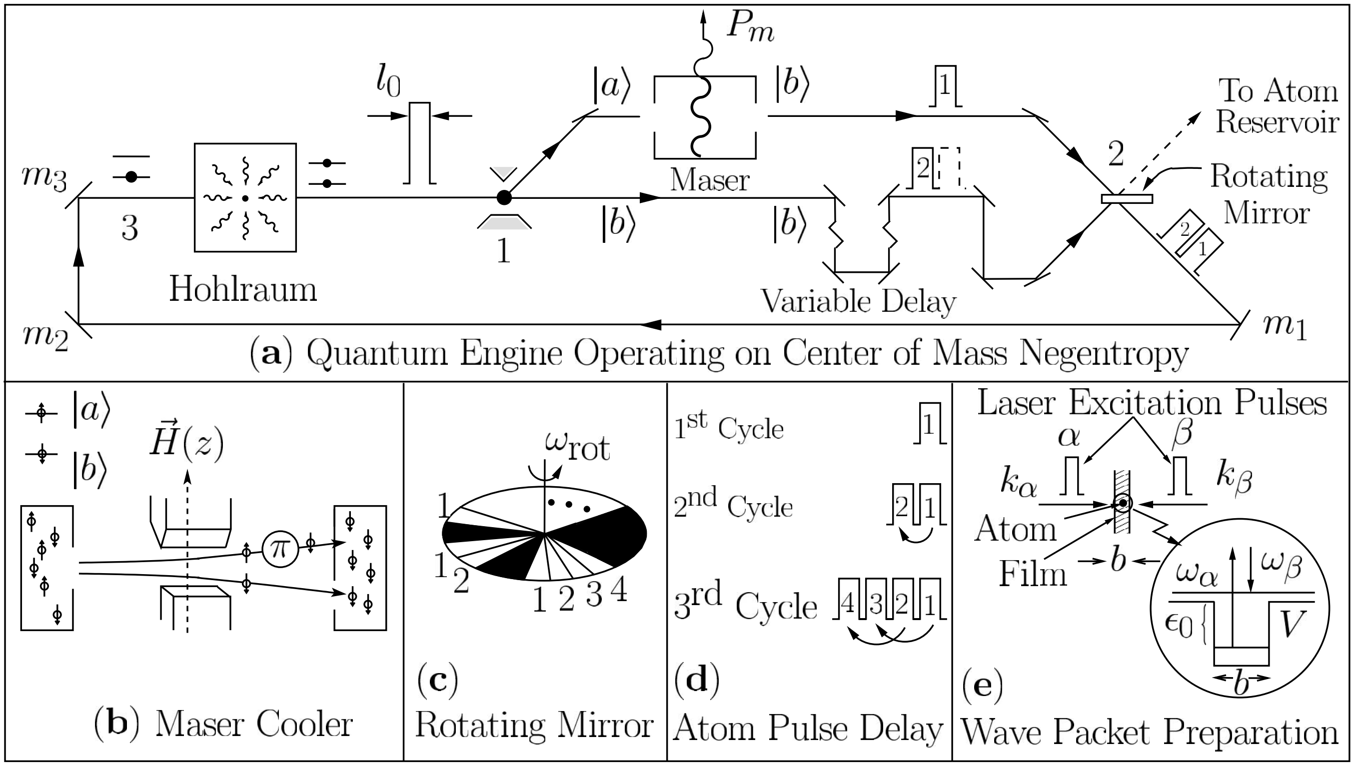

FIG. 1. (a) Two-level atom with c.m. wave packet of width l_0 undergoes state selection by SGA at (1). Excited atom $|a\rangle$ travels on the upper trajectory (UT) ($m_z = 1$) through the maser cavity. Ground state atom ($m_z = 0$) travels on the lower trajectory (LT) and undergoes delay. Hence upper and lower path packets are separated so that UT packet 1 can be transmitted and LT packet 2 can be reflected by the rotating mirror, allowing recombination of the beams by the rotating mirror. Atoms are then directed back to the hohlraum and recycled. This time the packet has width $2l_0$, and upon cycling through the engine it emerges with width $4l_0$, etc. This yields maser power P_m. (b) Beam of spin 1/2 atoms is separated by SGA and spin up atoms are flipped so that all the atoms in the right-hand-side box have spin temperature zero. (c) Rotating mirror arrangement engineered such that a transparent sector of the mirror is timed to coincide with arrival of pulses traveling on the UT. However, when the time delayed LT atoms arrive the mirror has rotated to a reflecting sector and they are reflected. The same thing happens on the second cycle, but now the atomic pulse is twice as long and so the transmitting and reflecting sectors must now subtend twice the angle as on the first cycle, etc. Alternatively a moving film strip having alternate transparent and reflecting sections could be used. Such details are not essential for the present conceptual discussion. (d) The atom pulse delay experienced in traversing the variable delay element in (a). On the second cycle, LT atoms are displaced from 1 to 2 and on the third the delay moves LT atoms from position 1 to 3 and 2 to 4 for a recombined pulse length of width $4\ell_0$. (e) Atoms are directed from point 2 to the reservoir where they are deposited at appropriate sites on a thin film by a sufficiently strong binding potential. Two oppositely directed Raman laser pulses are chosen so that they both contribute a positive momentum kick removing the atom from the film and injecting it into the QHE.

SGA where it is deflected into the two paths determined by the magnetic quantum numbers $m_z = +1, 0$. Please note that the SGA involves a conservative potential and does no work; i.e., the atom leaves the SGA with the same c.m. energy it had on entering [9]. As depicted in Fig. 1(a) an atom in the $|a\rangle$ state passes into the high-Q maser cavity and its spin energy is transferred to the field by stimulated emission.

After many passes, the maser density matrix proceeds from a thermal state, $r_{n,n} = \bar{n}^n/(\bar{n}+1)^{n+1}$, where $\bar{n} = 1/[\exp(\hbar\omega/kT) - 1]$, $\hbar\omega$ is the energy per quantum, and T is the cavity temperature, to a sharply peaked coherent distribution given by the quantum theory of the laser [11].

We recombine the beams following point 2 in Fig. 1(a), by generating a difference in path lengths [see Figs. 1(a) and 1(d)] between the lower and upper trajectories, so that the "L" atoms can be spatially separated from the "U" atoms. It is then possible to recombine the two beams by using the time dependent mirror as in Figs. 1(a) and 1(c).

The atom is now recycled through the hohlraum. But now the c.m. wave packet has twice the width. Hence, for an atom passing on the L path we must increase the path length by $2l_0$ so that the U and L atomic pulses are again totally separated; see Fig. 1(d). We continue in this way, doubling the packet width on each cycle, until the atomic wave function fills the apparatus. Then it is no longer possible to recombine the beams by the time dependent mirror, and the atom will be "lost," as indicated by the dashed trajectory at point 2.

We next turn to an entropy analysis of our QHE. We begin by defining the projection operators for the laser photon states $\Lambda_n = |n\rangle\langle n|$ and the atomic trajectory projection operator $\Lambda_i = |\Psi_i\rangle\langle\Psi_i|$, where Ψ_i is the c.m. atomic state for the atomic pulse in the $1, 2, \ldots, i$th position as depicted in Figs. 1(a) and 1(d).

Consider the situation in which we begin at time t_1 with the atom at point 3 of Fig. 1(a) in the $|b\rangle$ state and the laser in the $|n\rangle$ state [11] so that $\rho_3(t_1) = \Lambda_b \Lambda_n \Lambda_1$. The subscript on ρ indicates the atom location; thus $\rho_3(t_1)$ is the density matrix at point 3 at time t_1. Upon passing through the hohlraum/heat exchanger the atom undergoes irreversible heating and emerges from the heat bath at $t = t_1 + \epsilon \equiv t_1^+$ in a thermal mixture so that Λ_b is now replaced by $\rho_T = \sum_\alpha p_\alpha \Lambda_\alpha$. The density matrix for this configuration is given by $\rho_1(t_1^+) = \rho_T \Lambda_n \Lambda_1$. We emphasize that this does not take place instantaneously. The atom evolves from the pure state $|b\rangle$ to a thermal mixture during its passage through the holhraum.

The working atom now evolves unitarily as it interacts with the laser and undergoes time delay, etc. It is then transferred back to the input of the hohlraum (point 3) at time t_2, where it is irreversibly heated. The time evolution of the density matrix, for the first two cycles, is sketched in Fig. 2.

We calculate the entropy corresponding to the $1, 2, \ldots, N$th cycle by using the von Neumann entropy definition, $S = -k \,\mathrm{Tr}\rho \ln\rho$, and taking $\rho(t)$ from Fig. 2. Thus, we find after N cycles $S^{(N)} = -Nk[\beta \frac{\partial}{\partial\beta} \ln Z - \ln Z]$. Hence entropy is constantly increasing on each cycle as required by the second law. That is, the wave packet describing, quantum mechanically, the c.m. position is becoming broader on each cycle. Thus we are trading in quantum order, i.e., quantum negentropy, on each cycle in order to drive the maser engine.

After N cycles the c.m. wave packet has a length $L = 2^N \ell_0$, where ℓ_0 is the initial width as in Fig. 1. The maximum L is determined by requiring the error in the c.m. position, due to error δv, to be of order ℓ_0 after a time L/v_0. This implies the velocity error limit $\delta v(L/v_0) \leq \ell_0$. The velocity error δv is determined by the quantum uncertainty relation $\delta p \delta x \sim \hbar$. Taking $\delta x = \ell_0$ implies $\delta v \sim \hbar/m\ell_o$. Equating this to the previous error limit $\delta v \sim v(\ell_0/L)$, we find $L \sim (mv\ell_0/\hbar)\ell_0$. And since $L = 2^N \ell_0$, we have $N \sim \ln(mv\ell_0/\hbar)/\ln 2$. Taking reasonable numbers such as $m \sim 200 m_{\mathrm{proton}}$, $v \sim 10^6$ m/sec, $\ell_0 \sim 10^{-6}$ m, we find $N \sim 20$. For a more detailed analysis, see [12].

Thus after ~20 cycles, we must reprepare the c.m. wave packet to its original width ℓ_0, and this will involve an energy W_{prep}. One estimate of this state preparation energy can be gleaned by considering the work to isothermally compress the wave packet from its length after N cycles $L = 2^N \ell_o$ to its original length ℓ_0. This is given by $W_{\mathrm{prep}} = \int P\, dv = kT \ln(L/\ell_o) = NkT \ln 2$. Thus, for this example, $W_{\mathrm{useful}} \sim \frac{N}{2}\epsilon < W_{\mathrm{waste}} \sim N\epsilon \ln 2$. However, other state preparation strategies, e.g., that of Fig. 1(e) can differ significantly. We emphasize that we are here interested in the conceptual design of a new and different kind of heat engine operating off of an entropy cell rather than a battery or energy cell. Detailed analysis of W_{useful} and W_{prep} will be given elsewhere [12], as will the analysis of related QHE's [13].

It is important to note that the spin + c.m. atomic system does not operate in a closed cycle. However, the spin degrees of freedom drive the maser engine and therefore constitute the working fluid. The spin subsystem is returned to the same state, $|b\rangle$, after each passage. And as

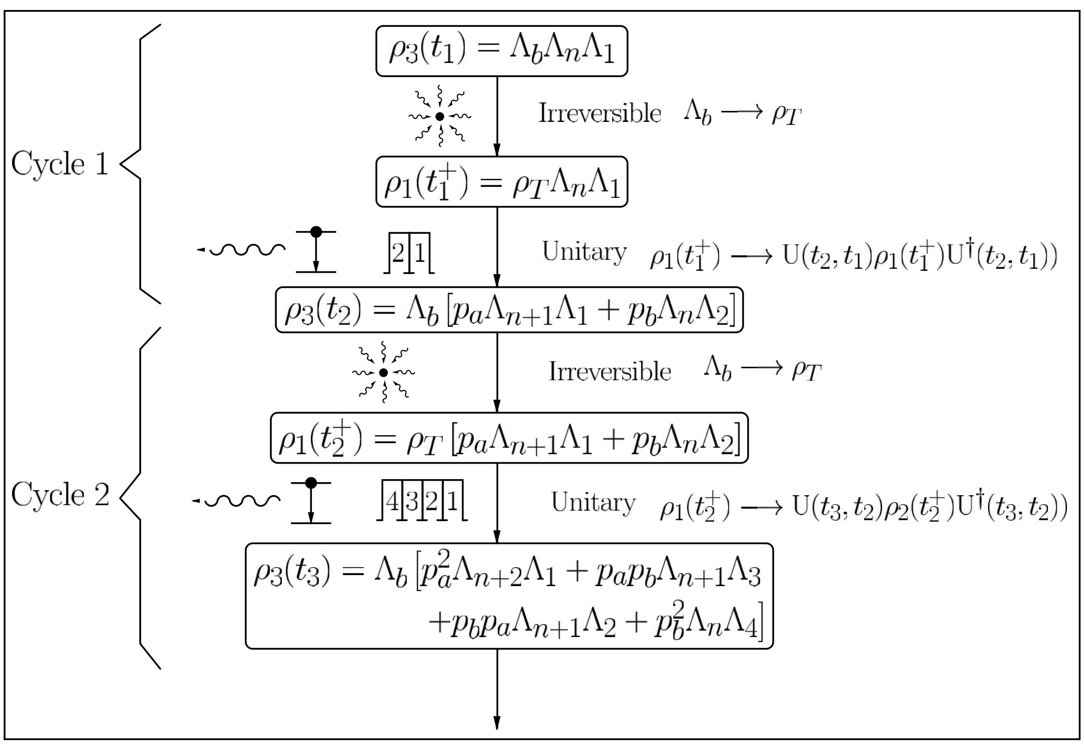

FIG. 2. The first two cycles of the time evolution of the density matrix are depicted.

stated in a recent textbook, "Thermodynamic cycles occur when the working fluid has the same initial and final states" [14].

The similarities and differences with our toy QHE and the Maxwell demon problem are interesting. In particular, our QHE has much in common with the Szilard single atom engine [15]. For example, it is interesting that the estimate $W_{\text{prep}} = NkT \ln 2$ given above is in agreement with the Szilard-Bennett result obtained on the basis of the theory of computing. However, no measurement is made in the operation of our QHE. The present analysis does, however, focus on the quantum information, i.e., quantum negentropy, associated with the atomic c.m. position.

The interplay between the second law of thermodynamics and quantum mechanics has a long history. The pioneering work by Ramsey [16] proved that the Kelvin-Planck statement of the second law had to be revised when (quantum) negative temperatures were introduced. The fact that the laser, driven by three-level atoms, could be viewed as a kind of quantum heat engine was pointed out some time ago [17]. However, the present two-level-state selection engine has more in common with the 1929 classical one-atom engine of Szilard [15].

Finally we point to a recent article [18] stating: 'Our main results are rather dramatic, apparently contradicting the second law: we show that... it is even possible to extract heat from [a single heat] bath by cyclic variations of a parameter ("perpetual mobile").' However, others [19] have expressed doubt. Terhal [19] calls for an analysis of an actual physical system and ventures that, "If they did a more careful analysis based on the physics they would see nothing going on." The present paper is a step in this direction as is the analysis of [12,13].

The purpose of this Letter is to sharpen our understanding of quantum thermodynamics. If it provokes discussion and debate, it will have served its purpose.

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