Measurement of Lagrangian Velocity in Fully Developed Turbulence

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We have developed a new experimental technique to measure the Lagrangian velocity of tracer particles in a turbulent flow, based on ultrasonic Doppler tracking. This method yields a direct access to the velocity of a single particle at a turbulent Reynolds number $R_\lambda = 740$, with two decades of time resolution, below the Lagrangian correlation time. We observe that the Lagrangian velocity spectrum has a Lorentzian form $E^L(\omega) = u_{\rm rms}^2 T_L/[1 + (T_L\omega)^2]$, in agreement with a Kolmogorov-like scaling in the inertial range. The probability density functions of the velocity time increments display an intermittency which is more pronounced than that of the corresponding Eulerian spatial increments.

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Lagrangian characteristics of fluid motion are of fundamental importance in the understanding of transport and mixing. It is a natural approach for reacting flows or pollutant contamination problems to analyze the motion of individual fluid particles [1]. Another characteristic of mixing flows is their high degree of turbulence. For practical reasons, most of the experimental work concerning high Reynolds number flows has been obtained in the Eulerian framework. Lagrangian measurements are challenging because they involve the tracking of particle trajectories: enough time resolution, both at small and large scales, is required to describe the turbulent fluctuations.

Early Lagrangian information has been extracted from the dispersion of particles, following Taylor's approach. Recently numerical and experimental studies have focused on resolving the motion of individual fluid or tracer particles. The emerging picture is as follows. The one-component velocity autocorrelation function is quasiexponential with a characteristic time of the order of the energy injection scale [2–4]. The velocity power spectrum is supposed to have a scaling $E^L(\omega) \propto \omega^{-2}$, as recently reported [5,6] and expected from Kolmogorov similarity arguments. In the same spirit, the second order structure function should scale as $D_2^L(\tau) = C_0\epsilon\tau$, where ϵ is the power dissipation and C_0 is a universal constant. Measurements of atmospheric balloons [7] have given $C_0 = 4 \pm 2$, and a limit $C_0 \to 7$ has been suggested in stochastic models [8]. Recent experiments [9,10] using high speed optical techniques have shown that the statistics of the Lagrangian acceleration are strongly non-Gaussian.

We have developed a new experimental method, based on sonar techniques [11], in order to study in a laboratory experiment the Lagrangian velocity across the inertial range of time scales. We obtain the first measurement of single particle velocity for times up to the flow large scale turnover time, at high Reynolds number. In this Letter, we report the results of these measurements and compare with previous observations and numerical predictions.

Our technique is based on the principle of a continuous Doppler sonar. A small (2 mm × 2 mm) emitter continuously insonifies the flow with a pure sine wave, at frequency $f_0 = 2.5$ MHz (in water). The moving particle backscatters the ultrasound towards an array of receiving transducers, with a Doppler frequency shift related to the velocity of the particle: $2\pi\Delta f = \mathbf{q}\cdot\mathbf{v}$. The scattering wave vector $\mathbf{q}$ is equal to the difference between the incident and scattered directions. A numerical demodulation of the time evolution of the Doppler shift gives the component of the particle velocity along the scattering wave vector $\mathbf{q}$. It is performed using a high resolution parametric method which relies on an approximated maximum likelihood scheme coupled with a generalized Kalman filter [11]. The study reported here is made with a single array of transducers so that only one Lagrangian velocity component is measured.

The turbulent flow is produced in the gap between two counterrotating disks [12]. This setup has the advantage of generating a strong turbulence in a compact region of space, with no mean advection. In this way, particles can be tracked during times comparable to the large eddy turnover time. Disks of radius $R = 9.5$ cm are used to set water into motion inside a cylindrical vessel of height $H = 18$ cm. To ensure inertial entrainment, the disks are fitted with eight blades with height $h_b = 5$ mm. In the measurement reported here, the power input is $\epsilon = 25$ W/kg. It is measured on the experimental cooling system, from the injection-dissipation balance. The integral Reynolds number is $\mathrm{Re} = R^2\Omega/\nu = 6.5 \times 10^4$, where Ω is the rotation frequency of the disks (7.2 Hz), and $\nu = 10^{-6}$ m^2/s is the kinematic viscosity of water. A conventional turbulent Reynolds number can be computed from the measured rms amplitude of velocity fluctuations ($u_{\rm rms} = 0.98$ m/s) and an estimate of the Taylor microscale ($\lambda = \sqrt{15\nu u_{\rm rms}^2/\epsilon} = 0.88$ mm); we obtain $R_\lambda = 740$. This value is consistent with earlier studies in the same geometry; it corresponds to the range of turbulent Reynolds numbers where measurements of particle acceleration have been reported [10].

The flow is seeded with a small number of neutrally buoyant (density 1.06) polystyrene spheres with diameter

$d = 250\ \mu$m. It is expected that the particles follow the fluid motion up to characteristic times of the order of the turbulence eddy turnover time, at a scale corresponding to their diameter, i.e., $\tau_{\min} \sim d/u_d \sim \epsilon^{-1/3} d^{2/3}$, using standard Kolmogorov phenomenology. For beads of diameter 250 μm, one estimates $\tau_{\min} \sim 1.3$ ms. This value is within the resolution of the demodulation algorithm whose cutoff frequency is at 3 kHz. Note that the Kolmogorov dissipative time ($\tau_\eta = \sqrt{\nu/\epsilon} = 0.2$ ms) is smaller, so that we do not expect to resolve the dissipative region. The statistical quantities are calculated from 3×10^6 velocity data points, taken at a sampling frequency equal to 6500 Hz. The acoustic measurement zone is in central region of the flow, 10 cm thick in the axial direction and almost spanning the cylinder cross section. In this region the flow is a good approximation to isotropic and homogeneous conditions: at all points, the mean velocity is nonzero, but equal to about one-tenth of its rms value.

We first consider the Lagrangian velocity autocorrelation function:

$$R^L(\tau) = \frac{\langle v(t)v(t+\tau)\rangle_t}{\langle v^2\rangle}. \tag{1}$$

We observe (see Fig. 1a) that it has a slow decrease which can be modeled by an exponential function $\rho_v(\tau) \sim e^{-\tau/T_L}$. This expression defines an integral Lagrangian time scale $T_L = 22$ ms. For comparison, the period of rotation of the disks is 140 ms and the sweeping period of the blades is 17 ms. The measured Lagrangian time scale thus appears as a time characteristic of the energy injection. The exponential reproduces extremely well the variation of the autocorrelation function, from about $5\tau_\eta$ at small scales to $4T_L$. These limits coincide with the upper and lower resolution of the technique, so that we observe an exponential decay over the entire range of our measurement. However, as the variance of the acceleration must be finite [10] there has to be some lower cutoff to this behavior, at times of order τ_η. These observations extend and confirm previous numerical and experimental studies at moderate Reynolds numbers [2,3,6]. Note that the exponential decay of the Lagrangian velocity autocorrelation is a key feature of stochastic models of dispersion since it appears as a linear drift term in a Langevin model of particle dynamics [1,8].

We show in Fig. 1b the velocity power spectrum, computed both from the data and as the Fourier transform of the exponential decay of the autocorrelation function:

$$E^L_{\rm fit}(\omega) = \frac{u^2_{\rm rms} T_L}{1 + (T_L\omega)^2}. \tag{2}$$

We observe a clear range of power law scaling $E^L(\omega) \propto \omega^{-2}$. This is in agreement with a Kolmogorov K41 picture in which the spectral density at a frequency ω is a dimensional function of ω and ϵ: $E^L(\omega) \propto \epsilon\omega^{-2}$. To our knowledge, this is the first time that it is directly observed at high Reynolds number and in a laboratory experiment,

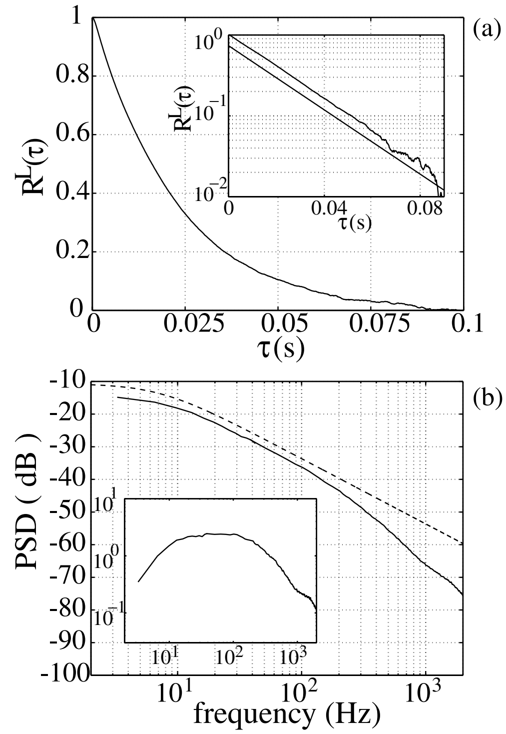

FIG. 1. (a) Velocity autocorrelation function. A best exponential fit is $\rho_v^L(\tau) = 1.03e^{-45.7\tau}$. It is shown, slightly shifted for clarity, as the linear curve in the inset. (b) Corresponding power spectrum; the upper curve is the Lorentzian function calculated from the exponential fit of the autocorrelation function (shifted for clarity). The inset shows the compensated spectrum $\omega^2 E^L\omega$.

although it has been reported in oceanic studies [5] and in lower Reynolds number direct numerical simulations [6]. Departure from the Kolmogorov behavior is observed at low frequencies in agreement with the exponential decay of the autocorrelation. At high frequencies, the spectrum deviates from the Lorentzian form due to the particle response. Note in Fig. 1b that the measurement is made over a dynamical range of about 60 dB.

We now consider the second order structure function of the velocity increment,

$$D^L_2(\tau) = \langle[v(t+\tau) - v(t)]^2\rangle_t = \langle(\Delta_\tau v)^2\rangle. \tag{3}$$

We emphasize that these are time increments, and not space increments as in the Eulerian studies. The profile $D^L_2(\tau)$ is shown in the inset of Fig. 2. It is linked to the autocorrelation by $D^L_2(\tau) = 2u^2_{\rm rms}[1 - R^L(\tau)]$: at small times one observes the trivial scaling $D^L_2(\tau) \propto \tau^2$ and at large times $D^L_2(\tau)$ saturates at $2u^2_{\rm rms}$ [as $v(t)$ and $v(t+\tau)$ become uncorrelated].

In between these two limits, one expects an inertial range of scales with a Kolmogorov-like scaling

$$D^L_2(\tau) = C_0\epsilon\tau, \tag{4}$$

where C_0 is a "universal" constant. Such a behavior is consistent with dimensional analysis and with an ω^{-2} scaling range in the velocity power spectrum. Figure 2 shows

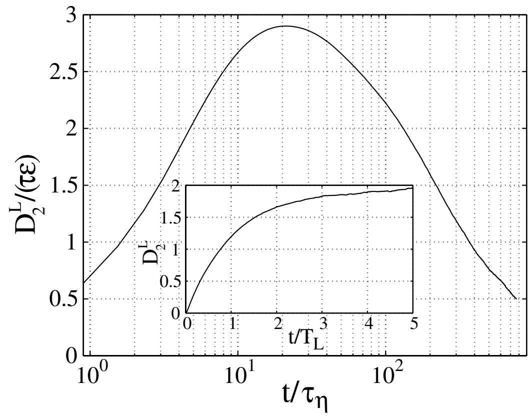

FIG. 2. Second order structure function. Inset: profile $D_2^L(\tau)$ as a function of time, nondimensionalized T_L. In the main figure the second order structure function is nondimensionalized by the Kolmogorov scaling $\epsilon\tau$.

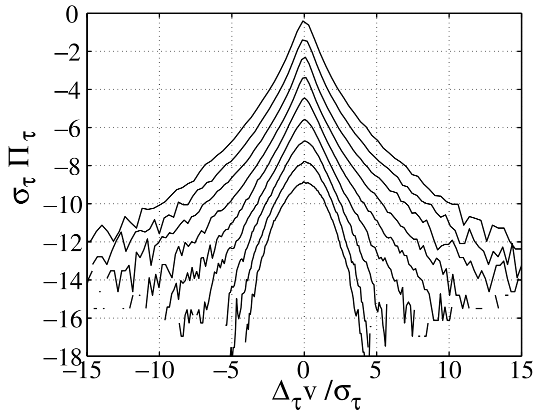

FIG. 3. PDF $\sigma_\tau \Pi_\tau$ of the normalized increment $\Delta v_\tau/\sigma_\tau$. The curves are shifted for clarity. From top to bottom: $\tau = 0.15$, 0.3, 0.6, 1.2, 2.5, 5, 10, 20, and 40 ms.

$D_2^L(\tau)/\epsilon\tau$; a plateau with a constant C_0 is not observed. Note that this is also the case in Eulerian measurements when the third order structure function is represented in linear coordinates [13]. The function reaches a maximum at $20\tau_\eta$, for which $C_0 \sim 2.9$. This value is in agreement with the estimation $C_0 = 4 \pm 2$ in [7] and in the range of values (between 3 and 7) used in stochastic models for particle dispersion [14]. In our case there may also be a bias at small times due to particle effects. However, if we assume the exponential fit for the velocity autocorrelation function to be valid down to the smallest scales, we obtain a value $C_0 = 3.5$ as an upper bound for the maximum of $D_2^L(\tau)/\epsilon\tau$. In our set of measurements between $R_\lambda = 100$ and $R_\lambda = 1100$, we have observed an increase of C_0 (defined in the same way) from 0.5 to 4. We point out that in the absence of an equivalent of the Kármán-Howarth relationship for the Lagrangian time increments, a limit value of C_0 is not *a priori* fixed. Dimensional analysis yields $D_2^L(\tau) = C_0(\mathrm{Re})\epsilon\tau$ and similarity arguments give $C_0(\mathrm{Re}) \to$ const or $C_0(\mathrm{Re}) \to \mathrm{Re}^\alpha$ in the limit of infinite Reynolds numbers.

To further describe the statistics of the Lagrangian velocity fluctuations, we have analyzed the statistics of the velocity increments $\Delta_\tau v$. Their probability density function (PDF) Π_τ for τ covering the accessible range of time scales is shown in Fig. 3. To emphasize the functional form, the velocity increments have been normalized by their standard deviation so that all PDFs have unit variance. A first observation is that the PDFs are symmetric, in agreement with the local symmetries this flow. Another is that the PDFs are almost Gaussian at integral time scales and progressively develop stretched exponential tails for small time increments. At the smallest increment, the stretched exponential shape is in agreement with measurements of the PDF of Lagrangian acceleration at identical Reynolds numbers [10]. In our case, the limit form of the velocity increments PDF is not as wide as that of the acceleration because the Kolmogorov scale is not resolved. Note that in regards of the evolution of the PDF, the intermittency is at least as developed in the Lagrangian frame as it is in the Eulerian one [15].

The continuous evolution with scale can be quantified using the flatness factor. We show in Fig. 4 the variation of the excess kurtosis $K(\tau) = \langle(\Delta_\tau v)^4\rangle/\langle(\Delta_\tau v)^2\rangle^2 - 3$. It is null at integral scale as expected from the Gaussian shape of the PDF and increases steeply at small scales. Below about $5\tau_\eta$, the increase is limited by the cutoff of the particle; an extrapolation of the trend to τ_η yields $K(\tau_\eta) \sim 40$ in agreement with acceleration measurements in [10].

More generally, one can choose to describe the evolution of the PDFs by the behavior of their moments (or "structure functions") $D_q^L(\tau) = \langle|\delta_\tau v|^q\rangle$. Indeed, a consequence of the change of shape of the PDFs with scale is that their moments, as the flatness factor above, vary with scale. Classically in the Eulerian picture, one expects scaling in the inertial range, $D_q^E(r) \propto r^{\zeta_q}$, at least in the limit of very large Reynolds numbers. At the finite Reynolds

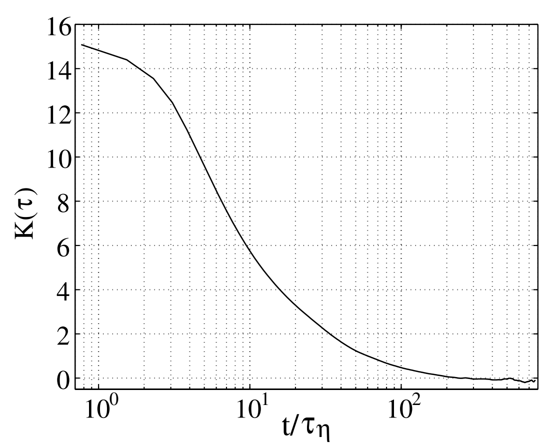

FIG. 4. Evolution of the excess kurtosis factor $K(\tau) = \langle(\Delta_\tau v)^4\rangle/\langle(\Delta_\tau v)^2\rangle^2 - 3$ for the PDFs of the time velocity increments.

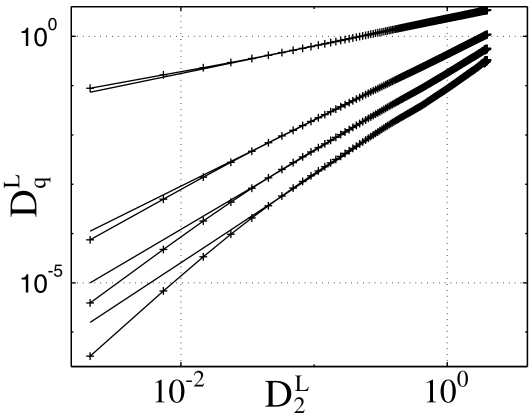

FIG. 5. ESS plots of the structure function variation (in double log coordinates). The solid curves are best linear fits with slopes equal to $\xi_q^L = 0.56 \pm 0.01$, 1.34 ± 0.02, 1.56 ± 0.06, and 1.8 ± 0.2 for $p = 1$, 3, 4, and 5 from top to bottom. Coordinates in arbitrary units.

number where most experiments are made, the lack of a true inertial range is usually compensated by studying the relative scaling of the structure functions—the extended self-similarity, or ESS, ansatz [16]. We use the second order structure function as a reference. Indeed, the dimensional estimation of D_2^L (as that of D_3^E) depends linearly on the increment and on the dissipation. Figure 5 shows that, as in the Eulerian frame, a relative scaling is observed for the Lagrangian structure functions of orders 1 to 5, $D_q^L(\tau) \propto D_2^L(\tau)^{\xi_q}$. We observe that the relative exponents follow a sequence close to, but more intermittent than the corresponding Eulerian quantity. Indeed, we obtain $\xi_1^L/\xi_3^L = 0.42$, $\xi_2^L/\xi_3^L = 0.75$, $\xi_4^L/\xi_3^L = 1.17$, $\xi_5^L/\xi_3^L = 1.28$ to be compared to the commonly accepted Eulerian values [17] $\xi_1^E/\xi_3^E = 0.36$, $\xi_2^E/\xi_3^E = 0.70$, $\xi_4^E/\xi_3^E = 1.28$, $\xi_5^E/\xi_3^E = 1.53$.

In conclusion, using a new experimental technique, we have obtained a Lagrangian velocity measurement that covers the inertial range of scales. Our results are consistent with Kolmogorov-like dimensional predictions for second order statistical quantities. At higher orders, the observed intermittency is very strong. How the Lagrangian intermittency is related to the statistical properties of the energy transfers is an open question. From a dynamical point of view, the Navier-Stokes equation in Lagrangian coordinates could be modeled using stochastic equations. Work is currently underway to compare the dynamics of the Lagrangian velocity to predictions of Langevin-like models.

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