Stochastic Resonances in an Optical Two-Order Parameter Vectorial System

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We show experimentally that optical two-dimensional systems such as vectorial lasers can exhibit novel stochastic resonances. All optical noise and modulation of this system allows the isolation of so-called inhibitional and rotational stochastic resonances. In particular, incoherent rotational tunneling is shown to be sensitive enough to be also induced by Faraday noise and by quantum noise, i.e., external spontaneous emission.

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The stochastic resonances observed in a wide range of physical systems are generally based on one-dimensional two-well potentials. Since the work of Benzi *et al.* [1], who first proposed stochastic resonance (SR) to explain the geophysical dynamics of the earth, there has been increasing interest in this subject mainly due to experimental observations in many different fields including lasers, neurology, chemistry, and magnetism. All these developments are summarized in an exhaustive review by Gammaitoni *et al.* [2]. Physically, the signature of a stochastic resonance is that the addition of random noise can enhance the response of a periodically or aperiodically modulated system, relative to that observed without noise. The behaviors of all the systems considered thus far are well modeled by the nonlinear cooperation between *scalar* physical quantities. The scalar model correctly explains the stochastic resonance observed in some vectorial systems exhibiting two bistable polarization states, such as vertical-cavity surface-emitting lasers submitted to electrical pump noise [3]. However, two orthogonal order parameters, namely, the polarization states for the electromagnetic field, compete with each other in these systems, in analogy with the phases in perovskite-type crystals submitted to a uniaxial stress [4]. Indeed, in the latter case with two competing order parameters, Müller *et al.* have shown the necessity of introducing spatially generalized potentials to describe the structural phase transitions. One may, hence, wonder whether such generalized potentials can give rise to new dynamics in two-dimensional optical systems when modulation and noise are also *vectorial* in nature. It is the aim of this Letter to investigate stochastic resonances in an all-optical two-order parameter vectorial system. We intend to explore experimentally an optical system fulfilling the following conditions: (i) It exhibits two orthogonal polarization states, (ii) the associated order parameters are competing with an adjustable strength, and (iii) both modulation and noise are vectorial in nature.

These conditions are satisfied by the experimental setup depicted in Fig. 1(a). The vectorial bistable system is a longitudinally and transversely monomode laser. An adjustable linear phase anisotropy $\Delta\Phi_{xy}$ is introduced inside the laser cavity and its role is twofold. First, it lifts the cavity isotropy, yielding a two-dimensional system defined by two linearly polarized eigenstates $\vec{E}_x$ and $\vec{E}_y$ aligned along the x and y directions. These oscillators (with eigenfrequencies ν_x and ν_y, respectively) are mutually exclusive stable states, due to the strong nonlinear coupling inside the active medium. Second, depending upon the magnitude of $\Delta\Phi_{xy}$, a first-order phase transition between the two oscillators $\vec{E}_x$ and $\vec{E}_y$ may occur along different paths in the xy plane [5]. Namely, depending on the eigenfrequency difference $\nu_x - \nu_y = c/2L \times \Delta\Phi_{xy}/\pi$, where c is the velocity of light in vacuum and L is the optical length of the cavity, the two different mechanisms schematized in Figs. 1(b) and 1(c) are possible. The last requirement is

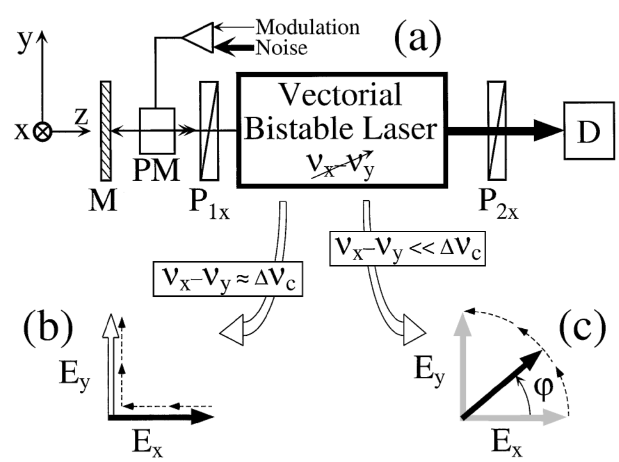

FIG. 1. (a) Experimental setup demonstrating all-optical control of a vectorial bistable laser (^{3}He-^{20}Ne laser emitting 7 μW at 3.39 μm) via feedback of vectorial noise and modulation. PM: phase modulator; P_{1x}, P_{2x}: polarizers; D: detector; M: reinjection mirror. Laser cavity bandwidth: $\Delta\nu_c \approx 30$ MHz fixed by the reflectivities $R_1 = R_2 = 0.7$ of the two laser cavity mirrors (not shown in the figure) and the cavity length $L = 50$ cm. (b) $\nu_x - \nu_y = 16$ MHz: inhibitional competition of eigenstates. During the flip, $\vec{E}_x$ diminishes along the x axis while simultaneously $\vec{E}_y$ appears along the y axis, with a discrete frequency jump from ν_x to ν_y. (c) $\nu_x - \nu_y = 0.8$ MHz: rotational competition of eigenstates. During the flip, the polarization vector rotates from the x axis to the y axis in the transverse plane with a continuous frequency drift from ν_x to ν_y.

obtained by using an external feedback which is linearly polarized and whose amplitude can be electro-optically modulated. Vectorial optical modulation and noise may then be applied to the laser via the electro-optic crystal.

When the magnitude of $\Delta\Phi_{xy}$ is such that $\nu_x - \nu_y$ is of the same order as the cavity bandwidth $\Delta\nu_c$ ($\Delta\nu_c \approx$ 30 MHz), the flip occurs through the usual inhibition mechanism [$\Delta\Phi_{xy} = 10°$; see Fig. 1(b)]. In this regime, the steady-state solution for the amplitudes of the two competing eigenstates can be associated with a potential similar to that used for structural phase transitions [4]:

$$V(E_x, E_y) = -\frac{1}{2}\alpha_x E_x^2 - \frac{1}{2}\alpha_y E_y^2 + \frac{1}{4}\beta_x E_x^4 + \frac{1}{4}\beta_y E_y^4 + \frac{1}{2}\theta_{xy} E_x^2 E_y^2, \quad (1)$$

where α_x and α_y include losses and characterize the net gain of the two eigenstates. β_x and β_y are their respective self-saturations, and θ_{xy} is their cross-saturation parameter [6,7]. External optical control of our experimental apparatus allows us to modulate this potential slightly. Indeed, the use of an external mirror M gives a simple means of reinjecting a small fraction (1%) of the emitted laser light. Moreover, the polarizer P_{1x} allows the direction of the injected light polarization to be chosen. Reinjection of light in one eigenstate only (P_{1x} is aligned along the x direction) and addition of some random noise to the same eigenstate yields a modulated term with the form $\alpha_x(t) = \alpha_{x0}[1 + A_0\cos(\Omega t) + A_1\xi(t)]$, where α_{x0} is the mean net gain for this eigenstate, Ω is the modulation angular frequency, A_0 and A_1 are constants (A_0 and $A_1 \ll 1$), and $\xi(t)$ denotes a zero-mean white Gaussian noise. A suprathreshold periodic modulation (at frequency $\Omega = 2\pi \times 1$ kHz, for instance) induces flips between $\vec{E}_x$ and $\vec{E}_y$ synchronized with the input modulation. Then, when the modulation amplitude is lowered below the threshold value, we observe that adding noise induces tunneling between $\vec{E}_x$ and $\vec{E}_y$. The periodic flips are restored at an optimal noise level, as observed using the detector D behind a polarizer P_{2x} aligned along the x direction. The experimental results obtained with zero-mean white Gaussian noise (bandwidth 100 kHz) are shown in Figs. 2(a)–2(d). The signal-to-noise ratio of the switching cycle at the modulation frequency exhibits a resonantlike behavior with increasing input noise level, as demonstrated by the right-hand curve of Fig. 3. This observation is consistent with the general theory of stochastic resonance for overdamped dynamics under the adiabatic regime, even if the experimental noise is not strictly white [8,9]. Remarkably, the all-optical signal recovery is obtained here due to the vectorial nature of the feedback. Indeed, isotropic modulation and noise would leave the system unperturbed, since the whole potential would be modulated together. We verify this by setting P_{1x} at 45° to the x direction. The laser no longer switches from one state to the other.

A careful adjustment of the system, however, so as to lower $\Delta\Phi_{xy}$ down to about 0.5° (leading to a frequency difference $\nu_x - \nu_y$ much less than $\Delta\nu_c$), allows the light vector to rotate in the xy plane [see Fig. 1(c)]. Indeed, in the case where the cavity Airy functions corresponding to the two eigenstates are close enough, the frequency can sweep continuously from one eigenfrequency to the other during the flip, still in close analogy with a structural

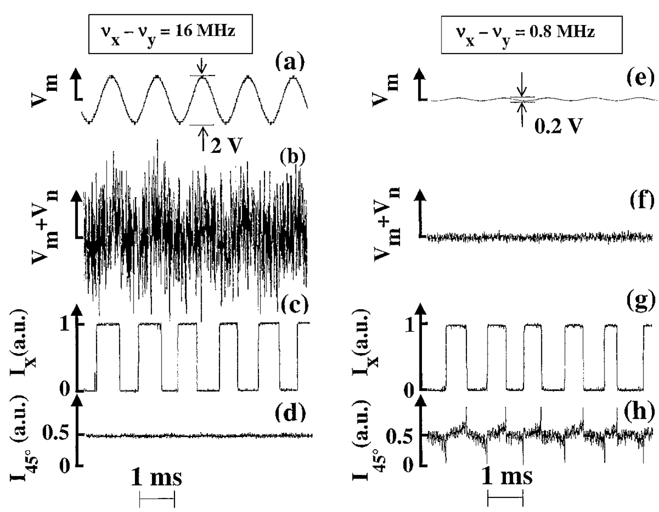

FIG. 2. (a)–(d) All-optical noise-induced synchronization in the inhibition mechanism ($\nu_x - \nu_y = 16$ MHz). (a) Subthreshold sinusoidal modulation V_m, (b) modulated optimal value of noise $V_m + V_n$, (c) restored output optical gates at optimal noise level, and (d) output intensity across the polarizer oriented at 45° with respect to the x direction. (e)–(h): same as (a)–(d) for the rotation mechanism ($\nu_x - \nu_y = 0.8$ MHz). Note the factor of 10 between the amplitudes in (e) and (f) and (a) and (b). In (h), note also the peaks and dips confirming the noise-induced vector rotation.

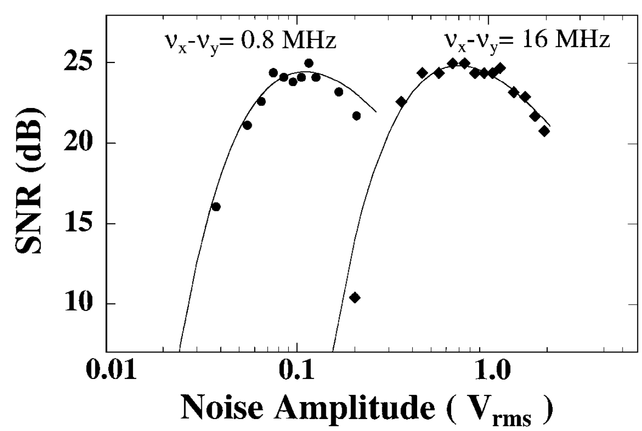

FIG. 3. Measured signal-to-noise ratio vs rms noise input. Squares: inhibitional SR ($\nu_x - \nu_y = 16$ MHz); dots: rotational SR ($\nu_x - \nu_y = 0.8$ MHz). Solid lines are curve fits given by the equation $\mathrm{SNR} = (A/D^2)\exp(-2U_0/D)$, where D is the noise amplitude, and with parameter values $A = 1100$ V^2 and $U_0 = 0.7$ V for inhibitional SR, and $A = 23$ V^2, $U_0 = 0.11$ V for rotational SR.

phase transition. One can then write $E_x = E\cos\varphi$ and $E_y = E\sin\varphi$, where φ describes the vector rotation in the xy plane, and the potential now has the following form [4]:

$$V(E, \varphi) = -\frac{1}{2}\alpha(\varphi)E^2 + \frac{1}{4}\beta E^4, \qquad (2)$$

where we have assumed that $\alpha = \alpha_x = \alpha_y$ and $\beta = \beta_x = \beta_y$. This is an unusual "Mexican-hat" potential surface with four barriers along $\varphi = \pm\pi/4$ and $\pm 3\pi/4$ in the rotation valley. In this case, the potential barriers are proportional to $\Delta\Phi_{xy}^2$, and their heights can be carefully reduced and adjusted to allow the light vector to rotate in the valley. Experimentally, the rotation of the light vector is obtained with an optical control corresponding to a modulation amplitude 10 times lower than in the previous mechanism [compare Figs. 2(a) and 2(e)]. Hence, the amount of noise needed to restore the signal when we apply a subthreshold modulation is also approximately 10 times lower than in the inhibitional tunneling case [see Figs. 2(e)–2(h)]. In Fig. 2(h), note that, when the analyzer is set at 45° to the x axis, the appearance of alternate peaks and dips detected at each barrier crossover proves the rotation of the vector in the potential valley. By varying the input noise level, a rotational stochastic resonance is obtained, having about the same signal-to-noise ratio as in the other mechanism, but with a 10 times higher sensitivity (see left curve in Fig. 3). As expected, this new type of stochastic resonance is in good agreement with the general theory of stochastic resonance [8,9]. However, the two different physics evidenced here show the importance of considering the two-dimensional character of the vectorial system. Furthermore, when the system is prepared in the rotation mechanism, one may wonder whether the laser electromagnetic field may also be sensitive to a transverse noise, i.e., polarization fluctuations in the xy plane.

In order to create a perturbation of the direction of the eigenstates, the electrical noise input is fed to a coil located around the active medium. This creates a noisy magnetic field in the z direction. Because of the small Faraday effect induced in the atomic medium, the laser field polarization then fluctuates randomly while passing through the active medium. As a result, the evolution of the vector direction is driven by a Langevin term accounting for the magnetic field-induced random fluctuations. The experimental observation of this intrawell rotation noise is shown in Figs. 4(a)–4(c) for the case where the laser oscillates on $\vec{E}_x$. In agreement with the Malus law, greater fluctuation amplitudes are detected when the output polarizer P_{2x} is oriented at 45° to the x direction than when it is aligned along x. This therefore raises the possibility of noise-induced rotational tunneling based on the internal noise of the laser. To test this idea, an optical subthreshold modulation signal is injected through the external electro-optic modulator. When the internal Faraday noise level is tuned via the electrical noise generator, a resonantlike behavior is obtained in the signal-to-noise response, as shown in Fig. 4(d). The modulation signal is restored for rms values of the magnetic field lower than 200 mG. Note that the earth's magnetic field and the residual stray magnetic fields along the axis of the laser (about 200 mG) broaden the stochastic resonance. The rotational tunneling between the two states is, hence, not only more sensitive than inhibitional tunneling, it also exhibits peculiar new features, such as the opportunity of recovering signals by adding small rotational noise to the light vector direction. It is worthwhile noting that this type of stochastic resonance is obtained by the nonlinear cooperation of a periodic optical signal with a magnetically induced atomic noise.

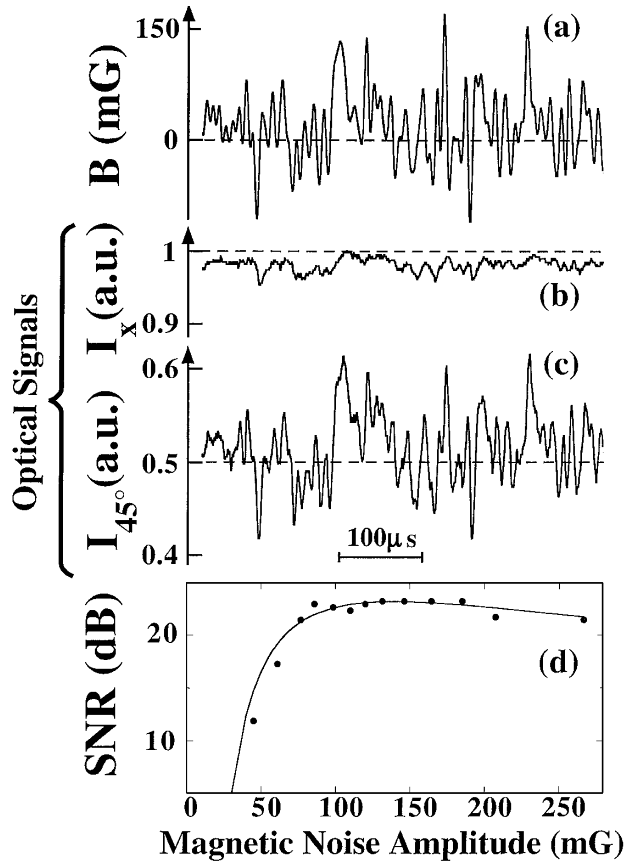

FIG. 4. Faraday noise-induced stochastic resonance. (a) Noisy axial magnetic field. (b) Corresponding induced Faraday noise with the polarizer along the x eigenstate and (c) at 45° with respect to the x-eigenstate direction. (d) Dots: experimental SNR measured vs magnetic noise. Solid curve is a fit using the same equation as in Fig. 3 with $A = 3 \times 10^7$ mG^2 and $U_0 = 140$ mG.

Finally, the inherent high sensitivity of the system gives the opportunity of exploiting pure optical noise, i.e., spontaneous emission, in order to induce stochastic resonance. To investigate the possible application of detrimental spontaneous emission to digital data transmission in noisy systems, we apply a subthreshold aperiodic binary signal via the electro-optic modulator, and we inject quantum optical noise from an optically isolated external source which is similar to the active medium of the laser. It provides amplified spontaneous emission around the laser wavelength.

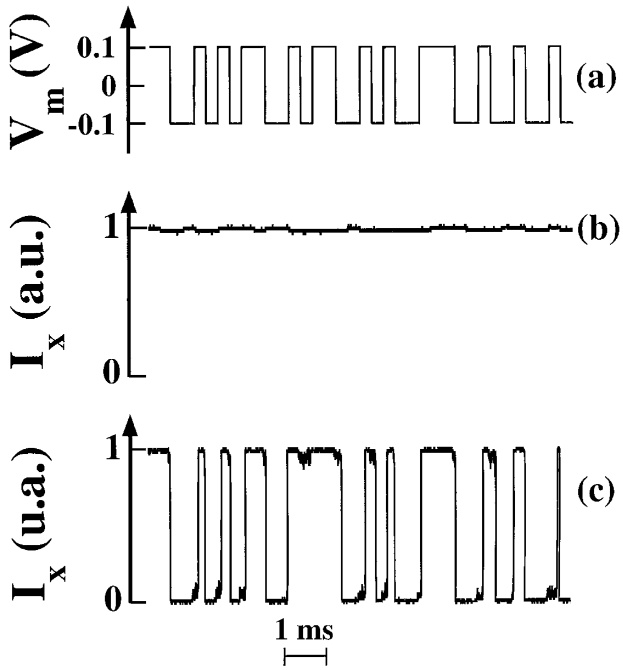

FIG. 5. Aperiodic binary gates recovered by spontaneous emission noise. (a) Input random subthreshold electrical gates. (b) Output optical signal without spontaneous emission noise with the polarizer along the x eigenstate. (c) Recovered gates with spontaneous emission.

We have verified that the power spectral density of this optical noise reflects nicely the usual Gaussian line shape of the considered atomic line. In such a case, the system may be modeled by including a pure Langevin term in the evolution equation of $\vec{E}$. We first verify that the response of the system to the command of Fig. 5(a) is flat without any noise [see Fig. 5(b)]. When the external spontaneous emission with a power as low as a few pW is fed to the laser cavity, however, the signal is recovered [Fig. 5(c)]. Although the available amount of external incoherent light does not permit the optimal noise value to be reached, we observe, for the first time, spontaneous emission-induced information recovery. This effect could be relevant to erbium-doped amplifiers used for telecommunications, where amplified spontaneous emission limits the data transmission [10].

In conclusion, we have seen that vectorial systems described by two-dimensional potentials exhibit new stochastic resonances. Because of the transverse nature of light, stochastic resonances can occur along different paths between the two stable states. They lead to inhibitional and rotational noise-induced tunnelings of different nature, which have been experimentally isolated in an optical system. In particular, the rotational tunneling is excitable by weak transverse rotational noise (Faraday noise) due to the atomic properties of the active medium itself. Moreover, in this case, an external quantum noise source leads to a spontaneous emission-induced stochastic resonance. The boost of sensitivity in this phenomenon suggests its use in all-optical systems with low photon number to improve the transmission of information. It is also worthwhile noting that the rotation mechanism seems to be a good candidate for the study of the interplay of different noises in nonlinear systems [11]. The dynamical properties demonstrated here may appear with other transverse waves, such as polarized gravitational waves, whose detection may be enhanced by the recently debated stochastic background [12,13]. Analogous features may also be found in other transverse two-dimensional systems, such as in high-critical temperature superconductor devices (SQUIDs) whose dynamics rely on spatially competing order parameters [14,15].

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