Effective Charge of the $\pi h_{11/2}$ Orbital and the Electric Field Gradient of Hg from the Yrast Structure of ^{206}Hg

B. Fornal, R. Broda, K.H. Maier,* and J. Wrzesiński
Niewodniczański Institute of Nuclear Physics, PL-31342 Cracow, Poland

G.J. Lane, M. Cromaz, A.O. Macchiavelli, R.M. Clark, and K. Vetter
Lawrence Berkeley National Laboratory, Berkeley, California 94720

A.P. Byrne† and G.D. Dracoulis
Department of Nuclear Physics, Australian National University, Canberra ACT 0200, Australia

M.P. Carpenter, R.V.F. Janssens, and I. Wiedenhoever
Physics Division, Argonne National Laboratory, Argonne, Illinois 60439

M. Rejmund
Dapnia/SPhN, CEA Saclay, F91191 Gif-sur-Yvette Cedex, France

J. Blomqvist
Department of Physics Frescati, Royal Institute of Technology, S-10405 Stockholm, Sweden

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The γ-ray decay of excited states of the two-proton hole nucleus, ^{206}Hg, has been identified using Gammasphere and ^{208}Pb + ^{238}U collisions. The yrast states found include a $T_{1/2} = 92(8)$ ns 10^+ isomer located above the known 5^- isomer. The $B(E2; 10^+ \rightarrow 8^+)$ strength is used to derive the quadrupole polarization charge induced by the $h_{11/2}$ proton hole. Also, the implied quadrupole moment has been used to provide an absolute scale for the electric field gradient of Hg in Hg metal.

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Investigations of the primary particle excitations near closed-shell nuclei are of special importance since they allow direct tests of the purity of nuclear-model wave functions, and provide the building blocks for calculating more complex configurations. Excitations of the form j^2, where a pair of identical valence nucleons (particles or holes) occupy the same orbit, are particularly valuable. First, their energy spectra give information about two nucleon residual interactions, which can be used for calculations of multiparticle configurations. Second, the rates of electromagnetic transitions connecting members of the j^2 multiplets, which are particularly pure for high j orbitals, may serve as a sensitive probe of the interplay between collective properties of the doubly magic core and the single-particle degrees of freedom. The maximally aligned states of a given multiplet for high j are usually isomeric (they are called seniority isomers) and are, therefore, both readily identifiable and accessible for lifetime measurements which give direct determinations of transition rates. These rates can be used for a straightforward evaluation of proton- or neutron-effective quadrupole charge e_{eff} (or polarization charge e_{pol}), which reflects the electric quadrupole polarizability of the core due to a particle located in a particular orbital.

A unique possibility for studying the polarization of a doubly magic core is offered by nuclei in the neighborhood of ^{208}Pb which has $Z = 82$ protons and $N = 126$ neutrons. Of the four, two-valence-particle (hole) nuclei in which high-spin j^2 isomers are expected, ^{206}Hg, ^{206}Pb [1], ^{210}Pb [2], and ^{210}Po [3], the two-proton-hole case, ^{206}Hg, is the only one for which such experimental information is missing, largely because of the difficulty in populating high-spin states in neutron-rich nuclei. We have successfully accessed such states using deep-inelastic heavy-ion reactions. Preliminary results of this study have been presented in Ref. [4].

As well as the new information on specific orbitals in the ^{208}Pb region and on polarization charges in general, this case provides a rare opportunity to specify a nuclear structure quantity which can be used as an independent basis for extracting quadrupole moments and, in turn, electric field gradients sensed by mercury nuclei in different environments.

In ^{206}Hg, configurations involving the $h_{11/2}$ proton orbital should dominate the low-lying yrast excitations. Only two excited states in ^{206}Hg were known prior to this study: the 2^+ state at 1068 keV and the 5^- isomeric state at 2102 keV with $T_{1/2} = 2.1$ μs, which arises predominantly from the $\pi s_{1/2}^{-1} h_{11/2}^{-1}$ configuration. The quadrupole moment of the 5^- excitation was measured in Ref. [5] and the effective charge for the $h_{11/2}$ proton hole was deduced, but with a rather large uncertainty. A more straightforward determination of the $\langle h_{11/2}^{-1} || M(E2) || h_{11/2}^{-1} \rangle$ matrix element and, from that, the $\pi h_{11/2}^{-1}$ quadrupole moment and effective charge should come from the decay properties of the expected $(h_{11/2}^{-2})10^+$ isomeric state.

We have used heavy-ion multinucleon transfer processes and the $(\gamma$-$\gamma)$-time technique to search for yrast states in ^{206}Hg. In several earlier experiments using ^{64}Ni, ^{76}Ge, and ^{48}Ca beams on a ^{208}Pb target [6–9], the ^{206}Hg isotope was clearly populated, but the low cross section did not allow observation of states beyond the known 2^+ and 5^- excitations. The use of a very heavy combination (at about 10% above Coulomb barrier), such as $^{208}\text{Pb} + {}^{238}\text{U}$, offered more prospect of populating high-spin states. The experiment was performed at the Argonne National Laboratory using Gammasphere [10] and a 1360 MeV ^{208}Pb beam from the ATLAS accelerator focused on a 50 mg/cm^2 ^{238}U target. $(\gamma$-$\gamma)$-time coincidence data were collected with a composite trigger, requiring three or more Compton-suppressed γ rays to be in coincidence for the in-beam events, and two or more such transitions in coincidence for the out-of-beam events. A total of 2.3×10^9 events were recorded. The beam, coming in bursts with an $\sim$0.3 ns time width, was pulsed with an $\sim$1.6 μs repetition time, providing clean separation of prompt and isomeric events, which simplified observation of γ-γ correlations across microsecond isomers.

In the thick target, reaction products stop within a few picoseconds and most of the γ rays are emitted from nuclei at rest. Because of the variety of excited reaction products, the γ-ray spectra were very complicated, but it was possible to substantially clean these spectra by using the timing information and/or multiple coincidence conditions with the help of the computer analysis code BLUE [11].

In ^{206}Hg, the $h_{11/2}$, $s_{1/2}$, and $d_{3/2}$ proton orbitals are close together, and configurations involving $h_{11/2}$ proton holes should be yrast. Above the known 5^- isomer from the $\pi s_{1/2}^{-1} h_{11/2}^{-1}$ configuration, one expects a 6^- state from the same configuration but it is likely to be non-yrast. Instead, a spectrum similar to the two-neutron-hole sequence in ^{130}Sn [12], with a 7^- state mostly of $\pi d_{3/2}^{-1} h_{11/2}^{-1}$ character and the two highest spin members of the $\pi h_{11/2}^{-2}$ multiplet, i.e., 8^+ and 10^+ states, should occur. We searched for γ rays or γ-ray cascades which proceed through the 5^- isomer in ^{206}Hg by selecting transitions preceding in time each of the 1034 and 1068 keV γ rays. Both gates showed a group of transitions with energies 364, 424, 656, 883, 1038, 1157, and 1257 keV (see Fig. 1a for the 1034 keV gate), which is therefore placed in ^{206}Hg, above the 5^- isomer. Of these newly identified γ rays the 364, 1157, and 1257 keV transitions were found to be delayed with respect to the others. Further analysis showed that they deexcite another isomer, a candidate for the expected 10^+ state, as shown in Fig. 2. The most intense of these transitions, at 364 keV, feeds the 5^- isomer directly, and the 1157 and 1257 keV γ rays, observed in prompt coincidence with it, are in parallel and deexcite levels at 3623 and 3723 keV, respectively, of which the higher one is isomeric. Although only traces of the 100 keV transition which connects these two levels could be observed, the time analysis showed that the 1157 and 1257 keV transitions follow the decay of the same isomer, for which a half-life of 92(8) ns was obtained.

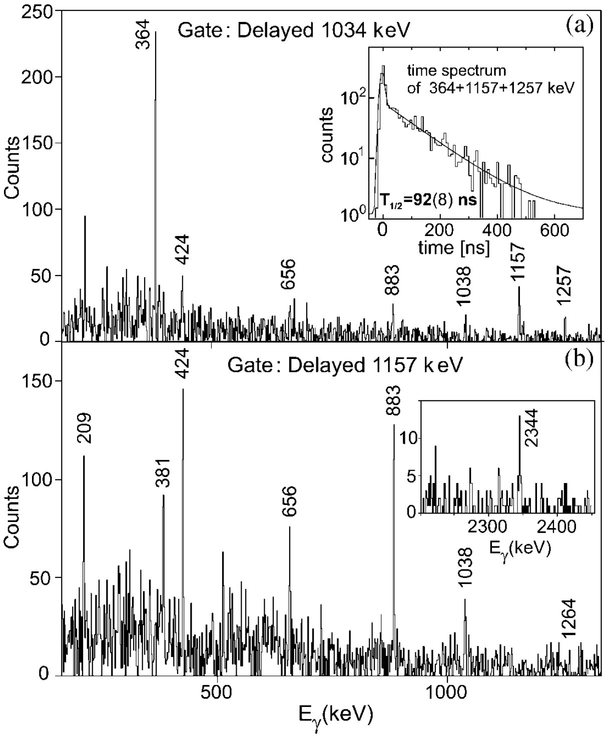

FIG. 1. γ-ray coincidence spectra for ^{206}Hg. (a) γ rays preceding the 1034 keV transition. (b) γ rays preceding the 1157 keV transition. The inset in the upper panel shows an example of the time spectrum for the 10^+ isomer.

Detailed analysis of γ rays preceding the newly located isomer, as illustrated by the spectrum shown in Fig. 1b, identified a γ-ray cascade feeding that isomer and consisting of 883, 381, 656, and 424 keV transitions with 1264 and 1038 keV crossovers. Another significant finding is a 2344 keV γ ray deexciting the level at 6067 keV and populating directly the 92 ns isomer. The final ^{206}Hg level scheme, including all new results, is shown in Fig. 2.

Given the small number of high-spin configurations available in ^{206}Hg, it is natural to associate the expected 7^-, 8^+, and 10^+ states discussed earlier with the observed states at 2466, 3623, and 3723 keV. This assumption is strongly supported by shell-model calculations, which predict energies of 2360, 3620, and 3657 keV, respectively, very close to the experimental sequence. (The interaction between proton holes in these calculations was taken from Ref. [13].)

The levels above the 10^+ isomer in ^{206}Hg must involve core excitations across the neutron shell gap. The two lowest, at 4606 and 4987 keV, are probably one-particle, one-hole 10^+ and 11^+ states with the $\nu g_{9/2} i_{13/2}^{-1}$ configuration as the main component, similar to the 10^+ state at 4895 keV and the 11^+ state at 5236 keV in ^{208}Pb [6,9,14]. The state at 5643 keV is a candidate for a 12^+ excitation of $(\pi d_{3/2}^{-1} h_{11/2}^{-1})7^- \otimes (\nu g_{9/2} p_{1/2}^{-1})5^-$ character. Since the excitation energy of the $(\nu g_{9/2} p_{1/2}^{-1})5^-$ level

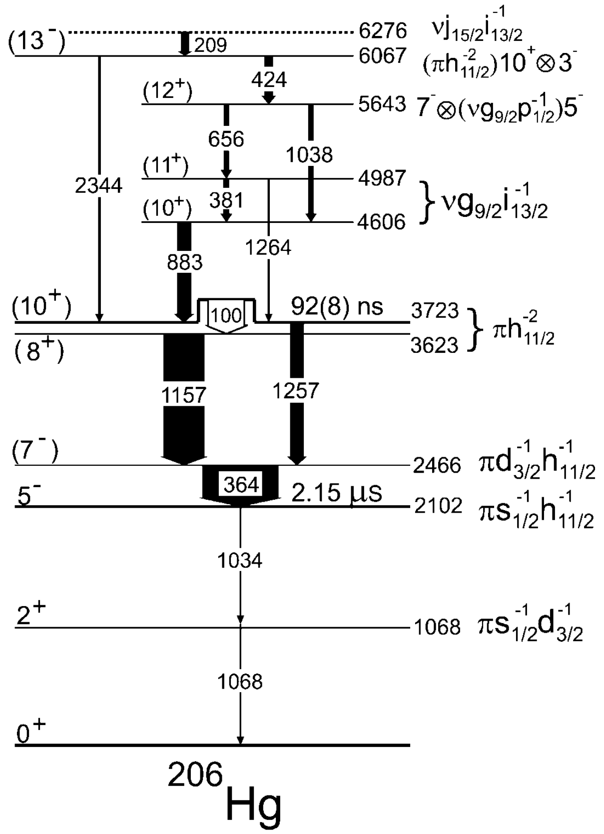

FIG. 2. Level scheme established for ^{206}Hg. Arrow widths denote the relative γ-ray intensities observed in early-delayed coincidence with the 1034 and 1068 keV transitions. Dominant shell-model configurations are indicated.

in ^{208}Pb is 3198 keV, one would expect such a state at $2466 + 3198 = 5664$ keV, very close to the experimental energy. The alternative $(\nu i_{11/2} i_{13/2}^{-1})12^+$ excitation is expected to be about 458 keV higher.

Similarly, there are two alternative ways to account for the 13^- state: $((\pi h_{11/2}^{-2})10^+ \times 3^-)13^-$ and $(\nu j_{15/2} i_{13/2}^{-1})13^-$. From the energy shift of the octupole 3^- state built on the $h_{11/2}$ orbital in ^{207}Tl, one calculates the energy for the $((\pi h_{11/2}^{-2})10^+ \times 3^-)13^-$ excitation as $3723 + 2614 - 244 = 6093$ keV, very close to the experimental energy of 6067 keV. (This is completely analogous to the calculation for ^{206}Pb [15].) The second state of $1p1h$ type should lie at about 6200 keV and the level observed at 6276 keV might be of that type.

From the measured half-life of the 10^+ state and total branching ratio $I(100)/I(1257) = 3.2(3)$ [determined from the $I(1157)/I(1257)$ ratio], the partial transition rate for the $(10^+ \rightarrow 8^+, E2)$ 100 keV γ ray is calculated to be $\lambda = 8.6 \times 10^5\ \mathrm{s}^{-1}$, with the conversion coefficient $\alpha_{\mathrm{tot}} = 5.66$. Since the 8^+ and 10^+ states are necessarily pure $\pi h_{11/2}^{-2}$ excitations, the transition rate is proportional to the square of the reduced $E2$ matrix element that can now be extracted [16] as $\langle h_{11/2} \| M(E2) \| h_{11/2} \rangle = 67(3)\ e\mathrm{fm}^2$. Using the value of 42 $e\mathrm{fm}^2$ for $\langle h_{11/2} \| M(E2) \| h_{11/2} \rangle$ given by the shell model [17], one obtains an effective charge for the $h_{11/2}$ proton hole orbital of $e_{\mathrm{eff}} = 1.60(7)e$, corresponding to a polarization charge $e_{\mathrm{pol}} = 0.60(7)e$.

The 1257 keV branch, according to the proposed configurations, is an $E3$ transition mediated by admixtures of the $h_{11/2}^{-1} d_{5/2}^{-1}$ configuration in the predominantly $h_{11/2}^{-1} d_{3/2}^{-1}$ configuration of the 7^- state, which results in a component from the allowed $h_{11/2}^{-1} \rightarrow d_{5/2}^{-1}$ $E3$ transition between the 10^+ and 7^- states. The shell-model calculations with the Kuo-Herling interactions give for this admixture the value of $(0.13)^2$. The $B(E3, h_{11/2} \rightarrow d_{5/2})$ can be estimated from an analogous transition $\nu j_{15/2} \rightarrow g_{9/2}$ [15] as 26 W.u. After accounting for the angular momentum recoupling, one calculates $B(E3, 10^+ \rightarrow 7^-) = 0.4$ W.u., in satisfactory agreement with the experimental value of 0.26(3) W.u., and in strong support of the proposed assignments.

Experimental evidence for the $E2$ polarization of the ^{208}Pb core has previously been based mainly on neutron-hole and proton-particle states. The polarization charge of the $h_{11/2}$ proton hole obtained in this paper can now be compared with the value for the $h_{9/2}$ proton particle, which is known from the ^{210}Po studies [3] to be $0.60(2)e$. To a first approximation, these two polarization charges should be equal since the radial parts of the wave functions of the $h_{9/2}$ and the $h_{11/2}$ orbitals are very similar. The agreement is very good.

In addition to the nuclei from the vicinity of ^{208}Pb, yrast isomers of the j^2 type are known near the $Z = 50$, $N = 82$ doubly closed shell of ^{132}Sn, specifically, in ^{130}Sn [12], ^{134}Sn [18], and ^{134}Te [19]. The extracted polarization charges for the $f_{7/2}$ and $g_{9/2}$ neutron as well as for the $h_{11/2}$ and $i_{13/2}$ neutron hole are very similar in magnitude and close to $0.9e$. In turn, the polarization charges for the $h_{9/2}$ and $h_{11/2}$ proton are about $0.6e$ and for the $g_{7/2}$ proton slightly larger: $0.9e$. Apparently, the proton or the neutron in a high j orbital outside the ^{208}Pb or ^{132}Sn core induces a quadrupole polarization proportional to its quadrupole moment with a similar coefficient of proportionality, even for these very different cores.

Seniority isomers are also found in ^{148}Dy—a two-proton nucleus with respect to the ^{146}Gd core, which relies on the $Z = 64$ subshell gap for its magicity. It is worthwhile to note that the e_{pol} value found for the $h_{11/2}$ orbital in ^{148}Dy is $0.52(5)e$ [20], close to the value $0.60(7)e$ found here for the same orbital in ^{206}Hg. Ideally, one would like to compare ^{206}Hg with ^{162}Hg to see whether polarization induced by the $h_{11/2}$ proton hole has the same effect on the ^{164}Pb and ^{208}Pb cores which differ by a whole major shell of $126 - 82 = 44$ neutrons. Unfortunately, ^{162}Hg lies well outside the proton drip line, and cannot be studied. Instead, the $10^+ \rightarrow 8^+$ transitions rates have been measured in lighter $N = 82$ isotones [21] with $Z = 66$, 68, 70, and 72. These rates depend predominantly on the occupation number of the $h_{11/2}$ orbital, which can be determined in this way, but it is not completely clear whether the effective charge, the other determining factor, is constant. It may therefore be more appropriate to compare only with ^{148}Dy. However, also in this nucleus some reduction of the $10^+ \rightarrow 8^+$ rate is expected from monopole pair scattering into the $h_{11/2}$ shell. In addition, low-energy 2^+ excitations of protons, ($g_{7/2}$ or $d_{5/2}$) into ($d_{3/2}$ or $s_{1/2}$), may somewhat enhance the $10^+ \rightarrow 8^+$ rate. These effects should be estimated quantitatively before one can draw any definite conclusions from the comparison of the polarization of the $N = 82$ and $N = 126$ cores.

Finally, the present evaluation of the $E2$ matrix element for the $h_{11/2}$ proton orbital can be linked to the quadrupole moment measurements which rely on the knowledge or assumption of electric field gradients acting on Hg nuclei in different environments. Mercury is one of the most widely studied metals by hyperfine spectroscopy, with isotopes in the mass range $A = 181-205$ investigated in the past, providing a canonical example of the technique [22–26]. In particular, using optical spectroscopy, the hyperfine splitting parameters (B) have been measured for all the odd $A = 185-201$ Hg nuclei in their ground and/or isomeric states and the corresponding quadrupole moments have been derived [22]. However, all these measurements rely on using the quadrupole moment of ^{201}Hg, $Q_{gs}(^{201}\mathrm{Hg}) = 38.5(4.0)\ e\mathrm{fm}^2$, as a standard which, although derived from an accurately measured interaction frequency (the product of the nuclear quadrupole moment and an electric field gradient), is itself reliant on a *calculated* value of the electric field gradient acting on a Hg nucleus in a Hg atom due to its atomic electrons [22]. Sophisticated though these calculations are, their accuracy is not better than about 10%.

The reduced matrix element $\langle h_{11/2} || M(E2) || h_{11/2} \rangle$ determined here from the $B(E2; 10^+ \rightarrow 8^+)$ value in ^{206}Hg, together with the results of various hyperfine interaction measurements, allows an independent evaluation of the electric field gradient probed by Hg nuclei in different hosts. The quadrupole moment of the 5^- isomer in ^{206}Hg with the main structure $\pi s_{1/2}^{-1} h_{11/2}^{-1}$ can be calculated directly from that matrix element, because the spherically symmetric $s_{1/2}$ proton does not contribute [5]. Use of this value of the quadrupole moment and the measured value of the interaction frequency for the ^{206}Hg 5^- isomer in Hg metal [5] allows deduction of a field gradient of $14.3(1.0)\ \mathrm{kV/nm}^2$ for Hg in Hg metal at LN$_2$ temperature. Furthermore, the existing results for the interaction frequencies of the $5/2^-$ state in ^{199}Hg in both Hg metal and HgCl$_2$ allow the subsequent determination of the field gradient for Hg in a HgCl$_2$ host. Since the interaction frequency for the ground state of ^{201}Hg has also been measured in HgCl$_2$ [24], a connecting chain is established, giving $Q_{gs}(^{201}\mathrm{Hg}) = 34.7(4.3)\ e\mathrm{fm}^2$. This value is to be compared with the "standard" value used previously, $Q_{gs}(^{201}\mathrm{Hg}) = 38.5(4.0)\ e\mathrm{fm}^2$ deduced using a calculated field gradient. That the two approaches agree within the error, which is of the order of 12%, is a satisfying result given the elaborate and demanding nature of the field gradient calculations.

In summary, we have identified excited states in ^{206}Hg including the $10^+\ h_{11/2}^{-2}$ isomer. From the lifetime of this state, the polarization quadrupole charge induced by the $h_{11/2}$ proton hole on the ^{208}Pb core has been deduced. Its value is the same as e_{pol} found for the $h_{9/2}$ proton orbital, which can be explained on the basis of similarities in the wave function spatial distributions. This result conforms with a general observation of the constancy of the polarization charges induced by high j protons and neutrons on doubly magic cores. Knowledge of the quadrupole moment of the $h_{11/2}$ orbital calculated from the $B(E2; 10^+ \rightarrow 8^+)$ transition probability has also allowed us to extract the electric field gradient of Hg in Hg, which, by independent means, provides an absolute scale for the extensive series of quadrupole hyperfine interaction results for Hg nuclei, carried out over many decades.

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*On leave from Hahn-Meitner Institute, Berlin, Germany.

†Also at Department of Physics, Australian National University, Canberra ACT 0200, Australia.

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