

## Supersymmetric SO(10) Grand Unified Models with Yukawa Unification and a Positive $\mu$ Term

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Supersymmetric grand unified models based on SO(10) gauge symmetry have many desirable features, including the unification of Yukawa couplings. Including  $D$ -term contributions to scalar masses arising from the breakdown of SO(10), Yukawa coupling unification only to 30% can be achieved in models with a positive superpotential Higgs mass. The superparticle mass spectrum is highly constrained and yields relatively light top squarks and charginos. Surprisingly, the pattern of grand unified theory scale soft supersymmetry breaking masses are close to those found in the context of inverted hierarchy models.

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Supersymmetric grand unified theories (SUSYGUTS) based on the gauge group SO(10) are especially attractive [1]. Not only do they unify the three forces of the standard model (SM), but they unify the matter content of each generation into a single 16 dimensional irreducible representation of SO(10):  $\hat{\psi}(\mathbf{16})$ . The  $\hat{\psi}(\mathbf{16})$  includes not only the matter superfields of the minimal supersymmetric standard model (MSSM), but also a gauge singlet superfield  $\hat{N}^c$  which includes a right-handed neutrino. The gauge singlet superpotential mass term  $M_N$  can be of order  $M_{\text{GUT}}$  and leads to sub-eV scale masses for left-handed neutrinos (in accord with SuperK results on atmospheric neutrino oscillations), while the right-handed neutrinos decouple via the well-known seesaw mechanism [2]. In the simplest models, the two Higgs doublet superfields of the MSSM reside in a single ten dimensional representation  $\hat{\phi}(\mathbf{10})$ . Then SO(10) SUSYGUT models contain a superpotential interaction term

$$\hat{f} \ni f \hat{\psi} \hat{\psi} \phi + \dots, \quad (1)$$

where  $f$  is the Yukawa coupling which leads to masses for quarks and leptons (at this stage we neglect intergenerational mixing effects). Thus, the Yukawa couplings of each generation are assumed to be unified above the GUT scale. More sophisticated treatments of the Yukawa matrices can allow for predictions of all SM fermion masses and mixing angles in terms of just a few parameters [3]. Here we focus only on third generation Yukawa couplings, since they will be large, and can have a substantial impact on the spectrum of superpartners.

In much the same way that the three gauge couplings of the MSSM can be extrapolated from their weak scale values to their GUT scale values using renormalization group (RG) evolution, so too can the Yukawa couplings be evolved from the weak to the GUT scale to test models with Yukawa coupling unification. In the bottom-up approach used in ISAJET v7.58 [4], we begin with the weak scale values of  $m_b$ ,  $m_t$ , and  $m_\tau$  in the  $\overline{DR}$  regularization scheme:  $m_\tau(M_Z) = 1.7463$  GeV,  $m_b(M_Z) = 2.92$  GeV, and  $m_t(m_t) = 163.4$  GeV. We calculate the corresponding Yukawa couplings and evolve both Yukawa and gauge

couplings to higher energies using two-loop RG equations (RGEs) [5]. Once  $M_{\text{GUT}}$  is determined [by the point at which the SU(2) and U(1) gauge couplings meet], we evolve gauge couplings, Yukawa couplings, and soft SUSY breaking (SSB) mass terms to the weak scale  $M_{\text{weak}}$ , where electroweak symmetry is broken radiatively (REWSB), and where the entire SUSY particle mass spectrum can be calculated. At this stage, the Yukawa couplings can be updated to include SM and MSSM loop corrections [6,7], and the RGE process is iterated until a convergent spectrum of SUSY particle masses is obtained. In this way, the GUT scale values of the Yukawa couplings depend on the SUSY particle mass spectrum.

In previous papers [8–10], unification of Yukawa couplings was investigated within the context of SO(10) SUSYGUT models. It is well known that in the mSUGRA model, with universal SSB masses at the GUT scale, a high degree of Yukawa coupling unification occurs only for negative values of the superpotential Higgs mass term  $\mu$ , and for values of the ratio of Higgs field vacuum expectation values  $\tan\beta \equiv \frac{v_u}{v_d} \sim 50$  [7,9]. For such high values of  $\tan\beta$ , the scalar potential no longer has the appropriate form at the weak scale to accommodate REWSB. Generally, the SSB down Higgs mass squared  $m_{H_d}^2$  gets driven to negative values before the up Higgs mass squared  $m_{H_u}^2$ . However, even if scalar masses are universal above the GUT scale, they will receive  $D$ -term mass contributions at the GUT scale arising from the breakdown of SO(10) gauge symmetry. Thus, scalar masses are shifted by an amount

$$\begin{aligned} m_Q^2 &= m_E^2 = m_U^2 = m_{16}^2 + M_D^2, \\ m_D^2 &= m_L^2 = m_{16}^2 - 3M_D^2, \\ m_N^2 &= m_{16}^2 + 5M_D^2, \\ m_{H_{u,d}}^2 &= m_{10}^2 \mp 2M_D^2, \end{aligned}$$

where  $M_D^2$  parametrizes our ignorance of the exact breakdown mechanism for SO(10) and can have either positive or negative values of order  $M_{\text{weak}}^2$ . Thus, the model is characterized by the following free parameters:

$$m_{16}, m_{10}, M_D^2, m_{1/2}, A_0, \tan\beta \text{ and } \text{sgn}(\mu). \quad (2)$$

The value of  $\tan\beta$  will be restricted by the requirement of Yukawa coupling unification to be close to  $\sim 50$ . In this model, for positive values of  $M_D^2$ , the GUT scale values of  $m_{H_u}^2$  and  $m_{H_d}^2$  are split, and  $m_{H_u}^2$  gets a head start on running towards negative values. For a sufficiently large value of  $M_D^2$ , REWSB can be recovered, and viable supersymmetric models with a high degree of third generation Yukawa coupling can be generated [9].

In previous works, model parameter space was mapped out under the restriction of GUT scale Yukawa unification to 5% [9], and implications of the model for cosmological neutralino relic density, direct detection of dark matter, radiative decays  $b \rightarrow s\gamma$ , and collider searches were determined [10]. A favorable relic density was obtained over much of model parameter space owing to  $s$ -channel neutralino pair annihilation via the very wide  $A$  and  $H$  Higgs poles at large  $\tan\beta$  [11]. However, the decay width for  $b \rightarrow s\gamma$  was found to be very large and generally in disagreement with experimental limits unless one entered the decoupling regime, where model parameters began becoming unnatural. The large  $b \rightarrow s\gamma$  branching fraction at large  $\tan\beta$  and  $\mu < 0$  is well known [12]. However, for  $\mu > 0$ , the  $b \rightarrow s\gamma$  branching ratio can be in accord with experimental limits at large  $\tan\beta$ .

In addition, the recent measurement by the E821 experiment [13] of the muon anomalous magnetic moment  $a_\mu = (g - 2)/2$  was found to deviate from SM predictions [14] by  $2.6\sigma$ . If the deviation is interpreted in terms of supersymmetric models, then it disfavors models with  $\mu < 0$ , assuming positive SSB gaugino masses [15]. For these reasons, it seemed prudent to reexamine Yukawa unification for positive values of  $\mu$ , relaxing the *ad hoc* 5% criteria used in Refs. [9,10].

Our procedure is as follows. We generate random samples of model parameters

$$\begin{aligned} 0 < m_{16} < 2000 \text{ GeV}, \\ 0 < m_{10} < 2500 \text{ GeV}, \\ 0 < m_{1/2} < 1000 \text{ GeV}, \\ -1000^2 < M_D^2 < +1000^2 \text{ GeV}^2, \\ 45 < \tan\beta < 55, \\ -5000 < A_0 < 5000 \text{ GeV} \quad \text{and} \\ \mu > 0. \end{aligned}$$

We then calculate the nonuniversal scalar masses according to formulas given above, and enter the parameters into the computer program ISASUGRA. ISASUGRA is a part of the ISAJET package [4] which calculates an iterative solution to the 26 coupled RGEs of the MSSM. We required unification of third generation Yukawa couplings at the GUT scale to 45%. Our requirement of 45% unification is determined by defining the variables  $r_{b\tau}$ ,  $r_{ib}$ , and  $r_{i\tau}$ , where, for instance,  $r_{b\tau} = \max(f_b/f_\tau, f_\tau/f_b)$ . We then require  $R = \max(r_{b\tau}, r_{ib}, r_{i\tau}) < 1.45$ . We were

able to find many solutions fulfilling the above criteria, with the best overall unification achieved for  $R = 1.28$ , or Yukawa coupling unification to 28%. The top and tau Yukawa couplings could frequently unify to very high precision, while  $f_b/f_i$  and  $f_b/f_\tau$  could unify to  $\sim 0.7$ – $0.8$ . The Yukawa couplings at the GUT scale,  $f_i(M_{\text{GUT}})$ , should differ from the unified Yukawa,  $f_{\text{GUT}}$ , due to threshold corrections,  $f_i(M_{\text{GUT}}) = f_{\text{GUT}}(1 + \epsilon_i)$ . It would be difficult to explain our 30%–40% deviation from perfect Yukawa unification by GUT scale threshold corrections in simple SUSYGUT models [16], where threshold corrections are expected of a few percent. More complicated models may be needed to explain the large threshold corrections needed for SO(10) SUSYGUT models with  $\mu > 0$  [17].

The parameter space regions with Yukawa coupling unification to 45% are shown in Fig. 1. Dots represent allowed solutions, while crosses denote solutions in violation of the experimental constraints  $m_{\tilde{\chi}_1^\pm} < 100$  GeV,  $m_h < 113$  GeV, and  $m_{\tilde{\tau}_1} < 73$  GeV. The values of  $\tan\beta$  generated were within the narrow range of  $\tan\beta \sim 46$ – $48$ , typically somewhat lower than results assuming  $\mu < 0$ . In Fig. 1(a), we show models in the  $m_{16}$  vs  $m_{1/2}$  plane. The values of  $m_{16}$  and  $m_{10}$  typically lie beyond 1 TeV and cluster about the line  $m_{10} = \sqrt{2}m_{16}$ .

In Fig. 1(b), we show solutions in the  $m_{16}$  vs  $M_D$  plane. In this case, we find  $M_D$  restricted to values of  $-0.4$  to  $+0.4$  TeV. Solutions can be obtained with  $M_D \approx 0$ , which brings us back to the mSUGRA model. This contrasts with the  $\mu < 0$  case where 5% unification required strictly positive  $D$  terms.

In Fig. 1(c), we show the  $m_{16}$  vs  $A_0$  parameter plane. Surprisingly, the best Yukawa unified solutions are found only for large negative values of the  $A_0$  parameter and cluster about the line  $A_0 = -2m_{16}$ . Finally, in Fig. 1(d), we

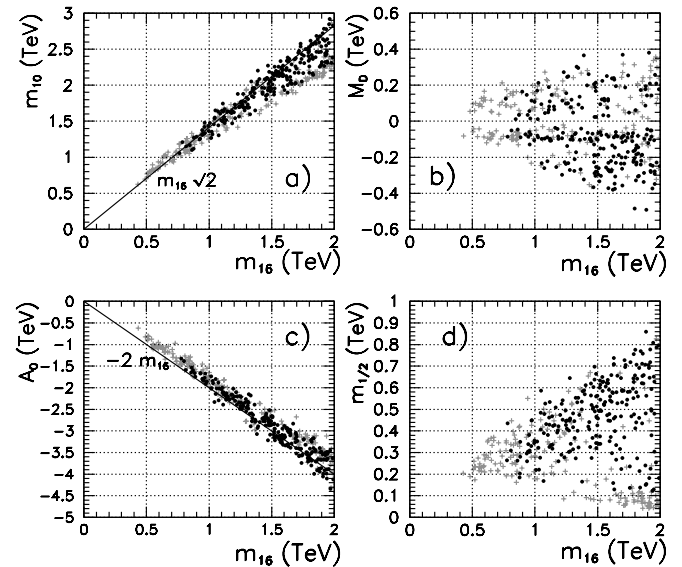


FIG. 1. Plots of regions of parameter space where valid solutions to minimal SUSY SO(10) are obtained, consistent with Yukawa coupling unification to 45% with  $\mu > 0$  and REWSB. Crosses are excluded by collider searches.

show the  $m_{16}$  vs  $m_{1/2}$  plane and find model solutions occurring for  $m_{1/2} \sim 0.1\text{--}0.9$  TeV, with  $m_{16}$  always greater than  $m_{1/2}$ .

It is particularly intriguing that model solutions with the best Yukawa coupling unification cluster about model parameters with

$$A_0^2 = 2m_{10}^2 = 4m_{16}^2. \quad (3)$$

These particular boundary conditions were found by Bagger *et al.* from a rather different approach [18] by looking analytically for fixed point behavior in third generation SSB scalar masses which would give rise to SUSY models with a radiatively driven inverted scalar mass hierarchy (RIMH). In these models, one may begin with GUT scale scalar masses beyond the TeV scale. RG evolution drives third generation and Higgs scalar masses towards zero, while scalars of the first two generations remain beyond the TeV range. In this way, multi-TeV first and second generation scalar masses act to suppress SUSY flavor and  $CP$  violating processes, while still fulfilling conditions of naturalness, which mainly apply to the sub-TeV third generation and Higgs scalar SSB masses. The RIMH mechanism is viable for SO(10) based models with Yukawa coupling unification upon implementation of the above specific set of scalar mass boundary conditions [19,20]. Our results here are obtained using a bottom-up approach and indicate that for  $\mu > 0$ , a high degree of Yukawa coupling unification can be obtained *only* using approximately the boundary conditions Eq. (3). The values of the Yukawa couplings obtained at the GUT scale are  $f_t: 0.47\text{--}0.50$ ,  $f_b: 0.35\text{--}0.37$ , and  $f_\tau: 0.47\text{--}0.52$ . Their magnitudes are sufficient to generate only a small scalar IMH [19,20]. For Yukawa unified solutions with  $\mu < 0$ , we found a weaker correlation for the soft masses around  $m_{10} = \sqrt{2}m_{16}$  [9], but we find no correlation for the trilinear parameter.

In Fig. 2, we show values of selected weak scale sparticle masses generated from the Yukawa unified model. In Fig. 2(a), we show the  $m_A$  vs  $m_h$  plane of Higgs masses. The light scalar  $h$  has a mass clustering about the region  $m_h \sim 115\text{--}130$  GeV, while  $m_A$  ranges between 100–500 GeV and is generally lower than  $m_A$  values generated in models with lower values of  $\tan\beta$ . Both the  $h$  and  $A$  (and also the heavy Higgs  $H$ ) may be accessible to Higgs searches at the Fermilab Tevatron [21], and the low values of  $m_A$  may give rise to measurable rates for  $B \rightarrow \mu^+ \mu^-$  decay [22].

In Fig. 2(b), we show the  $m_{\tilde{t}_1}$  vs  $m_{\tilde{b}_1}$  plane. We always find  $m_{\tilde{b}_1} > m_{\tilde{t}_1}$ , in contrast to Yukawa unified models with  $\mu < 0$ . The value of  $m_{\tilde{t}_1}$  ranges between 100–600 GeV for  $m_{\tilde{b}_1} < 1$  TeV.

Figure 2(c) shows the  $m_{\tilde{\chi}_1^\pm}$  vs  $m_{\tilde{\tau}_1}$  plane. We note that  $m_{\tilde{\tau}_1} \geq m_{\tilde{\chi}_1^\pm}$ , while  $m_{\tilde{\chi}_1^\pm} \lesssim 400$  GeV and is likely accessible to a linear  $e^+e^-$  collider operating with  $\sqrt{s} \sim 800$  GeV [23,24].

Finally, in Fig. 2(d), we show the  $m_{\tilde{e}_R}$  vs  $m_{\tilde{u}_R}$  plane. In this case,  $m_{\tilde{e}_R}$  and  $m_{\tilde{u}_R}$  typically lie beyond 1 TeV. Such

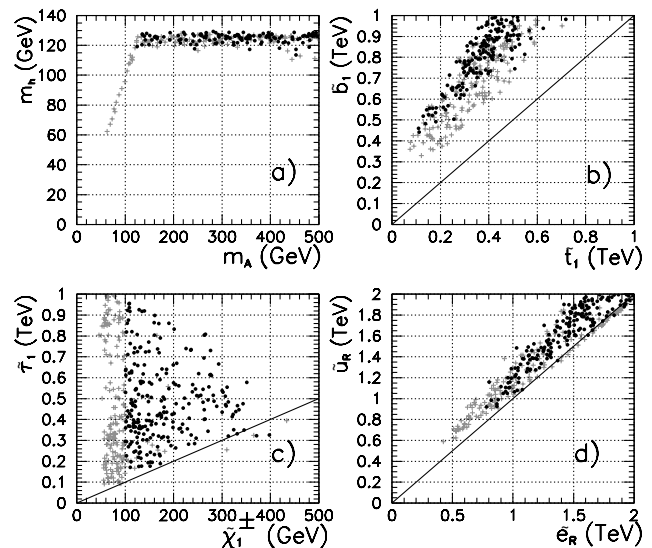


FIG. 2. The range of selected sparticle masses that are generated in minimal SUSY SO(10) models with Yukawa coupling unification to 45%,  $\mu > 0$  and REWSB. Crosses are excluded by collider searches.

high mass values for first and second generation scalars can act to suppress many  $CP$  violating processes via a decoupling solution; they are generally not sufficiently heavy to suitably suppress the most dangerous flavor violating processes, such as  $K\text{--}\bar{K}$  mixing [25]. A sample sparticle mass spectrum is shown in Table I for a Yukawa unified model with  $\mu > 0$ . In this case,  $a_\mu = 16.6 \times 10^{-10}$ , within the  $2\sigma$  limit from E821 [13]; for our other models,  $a_\mu$  typically varies around  $(10 \pm 6) \times 10^{-10}$ .

The cosmological relic density of neutralinos has been calculated in Ref. [10] for Yukawa unified models with  $\mu < 0$ . Little should change by switching to  $\mu > 0$ : the pseudoscalar  $A$  and heavy scalar Higgs  $H$  will still have large widths of order 20–50 GeV due to the large  $b$  and  $\tau$  Yukawa couplings and will be light enough that  $\tilde{\chi}_1^0 \tilde{\chi}_1^0$  annihilation can take place efficiently through  $s$ -channel annihilation. In addition, the rates for direct detection of relic neutralinos will remain large, as in the  $\mu < 0$  case [10]. The rate for  $b \rightarrow s\gamma$  can be substantially different for  $\mu > 0$  compared to the  $\mu < 0$  result, and regions of parameter space certainly exist where this decay rate falls within experimental limits. Explicit results for the RIMH model with  $\mu > 0$  have been shown in Ref. [20]. Finally, the value of  $a_\mu$  has been calculated in Ref. [15] for Yukawa unified models with  $\mu < 0$  and for RIMH models with  $\mu > 0$ . Regions of model space with an acceptable  $a_\mu$  certainly exist for the  $\mu > 0$  case. Further results along these lines will be presented in a forthcoming publication.

Finally, we note that Yukawa unified models with  $\mu > 0$  have also been recently reported by Blazek *et al.* [26]. These authors use a top-down approach and adopt independent values for  $m_{H_u}$  and  $m_{H_d}$  rather than  $D$ -term splitting among scalar masses. We verify that in this case also Yukawa unified solutions can be obtained, although

TABLE I. Weak scale sparticle masses and parameters (GeV) for an SO(10) case study.

Parameter	Value
$m_{16}$	1108.3
$m_{10}$	1497.7
$M_D$	58.1
$m_{1/2}$	325.2
$A_0$	-2108.0
$\tan\beta$	49.9
$f_t(M_{\text{GUT}})$	0.484
$f_b(M_{\text{GUT}})$	0.372
$f_\tau(M_{\text{GUT}})$	0.518
$m_{\tilde{g}}$	810.9
$m_{\tilde{u}_L}$	1267.1
$m_{\tilde{d}_R}$	1253.0
$m_{\tilde{t}_1}$	211.3
$m_{\tilde{b}_1}$	607.7
$m_{\tilde{\nu}_L}$	1120.3
$m_{\tilde{\nu}_R}$	1115.4
$m_{\tilde{\nu}_e}$	1117.5
$m_{\tilde{\tau}_1}$	308.1
$m_{\tilde{\nu}_\tau}$	829.1
$m_{\tilde{\chi}_1^\pm}$	145.5
$m_{\tilde{\chi}_2^0}$	164.6
$m_{\tilde{\chi}_1^0}$	111.6
$m_h$	121.6
$m_A$	200.1
$m_{H^\pm}$	225.6
$\mu$	165.4
$a_\mu$	$16.6 \times 10^{-10}$

in our approach these generally occur at the 35%–55% level. The solutions typically have  $m_{H_u} \sim \sqrt{2}m_{16}$ , with  $m_{H_u} < m_{H_d}$ , and  $A_0 \approx -2m_{16}$ .

It is well known that the three standard model gauge couplings approximately unify when extrapolated to the scale  $M_{\text{GUT}} \approx 2 \times 10^{16}$  GeV. This may be regarded as a coincidence, or as evidence for SUSYGUTs. Similarly, as a consequence of Yukawa unification, the clustering of the SSB parameters about an approximate SO(10) inspired fixed point, if taken seriously, can be considered as evidence for a supersymmetric SO(10) grand unified model.

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