Bloch-Nordsieck Violation in Spontaneously Broken Abelian Theories

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We point out that, in a spontaneously broken U(1) gauge theory, inclusive processes, whose primary particles are mass eigenstates that do not coincide with the gauge eigenstates, are not free of infrared logarithms. The charge mixing allowed by symmetry breaking and the ensuing Bloch-Nordsieck violation are here analyzed in a few relevant cases and in particular for processes initiated by longitudinal gauge bosons. Of particular interest is the example of weak hypercharge in the standard model where, in addition, left-right mixing effects arise in transversely polarized fermion beams.

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The planning of TeV scale accelerators has brought attention to the fact that the standard model, at energies larger than the weak scale, shows enhanced double log corrections [1] of infrared origin, even in inclusive observables. Such enhancements, involving the effective coupling $(\alpha_W/4\pi)\log^2(s/M_W^2)$, signal a lack of compensation of virtual corrections with real emission in the $M_W^2 \ll s$ limit, due to the non-Abelian (weak isospin) charges of the accelerator beams. In other words, the Bloch-Nordsieck (BN) cancellation theorem [2], valid in QED, is here violated.

The key point which invalidates the BN cancellation is the fact that gauge boson emission off one incoming beam state changes it into another state of the same gauge multiplet (e.g., a neutrino for an incoming electron) and the latter happens to have a different cross section off the other beam. As a consequence, virtual corrections are unable to cancel this contribution except on the average, i.e., by summing over all possible beams in the multiplet.

It is usually thought that such a phenomenon cannot occur in the Abelian case, because initial states (the mass eigenstates) are charge eigenstates, which do not change during the (neutral) gauge boson emission, so that the real-virtual cancellation is valid.

In this Letter we point out that, in the case of spontaneous symmetry breaking, the BN theorem is also violated in *Abelian theories.* The point is that, in a broken theory, mass eigenstates can be mixed charge states, so that soft boson emission is off diagonal. For instance, if a normal Higgs mechanism [3] is assumed, longitudinal gauge bosons can occur as (massive) initial states which act as mixed charge states and interact with the (similarly mixed) Higgs boson. As a consequence, longitudinal and Higgs bosons are interchanged during soft emission, and the basic noncancellation mechanism is again at work, as in the non-Abelian case illustrated previously.

In order to understand this point, let us recall the structure of soft interactions accompanying a hard process of the type $\{\alpha_I p_I\} \to \{\alpha_F p_F\}$, where $I = 1, 2$, $F = 1, 2, \ldots, n$, and p's and α's denote momenta and charge states, respectively, of initial and final asymptotic states, which are mass eigenstates. The corresponding S matrix is an operator in the soft Hilbert space and a matrix in the hard labels, of the form [4,5]

$$S = \mathcal{U}^F_{\alpha_F \alpha'_F}(a_s, a_s^\dagger),\ S^H_{\alpha'_F \alpha'_I}(p_F, p_I),\ \mathcal{U}^I_{\alpha'_I \alpha_I}(a_s, a_s^\dagger), \quad (1)$$

where $\mathcal{U}^F$ and $\mathcal{U}^I$ are unitary coherent state operators, functionals of the soft emission operators $a_s, a_s^\dagger$. They take in the Abelian case a simple eikonal form [4] and are *diagonal* with respect to charge eigenstates, i.e., they have a well-defined form for each energetic particle of well-defined charge.

An inclusive observable is obtained by squaring and summing Eq. (1) over soft final states. In this procedure, the coherent state $\mathcal{U}^F$ cancels out by unitarity, and we are left with the overlap matrix

$$\mathcal{O}_{\beta_I \alpha_I} = {}_S\langle 0 | \mathcal{U}^{I\dagger}_{\beta_I \beta'_I} (S^{H\dagger} S^H)_{\beta'_I \alpha'_I} \mathcal{U}^I_{\alpha'_I \alpha_I} | 0 \rangle_S\,, \quad (2)$$

where an average over the state with no soft quanta is made in the initial state. We also refer to $\mathcal{O}^H = S^{H\dagger} S^H$ as the hard overlap matrix, and we allow, in general, $\beta_I \neq \alpha_I$, even if a cross section with initial charge state α_I is diagonal, i.e., $\sigma_{\alpha_I} = \mathcal{O}_{\alpha_I \alpha_I}$ (no sum over α_I).

The Abelian Bloch-Nordsieck cancellation theorem is valid if the initial mass eigenstates are also charge eigenstates. In fact, in such a case, $\mathcal{U}^I$ is diagonal with respect to the labels $\alpha_I = (\alpha_1, \alpha_2)$ which represent definite charges, i.e.,

$$\begin{aligned} \mathcal{U}^I_{\alpha'_I \alpha_I} &= \delta_{\alpha'_I \alpha_I} \mathcal{U}^{\alpha_I p_I}, \\ \mathcal{U}^{\alpha_I p_I} &\equiv \Pi_{i=1,2} \mathcal{U}^{\alpha_i p_i}(a_s, a_s^\dagger)\,. \end{aligned} \quad (3)$$

Therefore, the inclusive cross section becomes, by Eq. (2),

$$\sigma_{\alpha_I} = \mathcal{O}_{\alpha_I\alpha_I} = {}_S\langle 0|\mathcal{U}^{\alpha_I p_I \dagger}\mathcal{O}^H_{\alpha_I\alpha_I}\mathcal{U}^{\alpha_I p_I}|0\rangle_S, \quad (4)$$

where α_I, in both $\mathcal{U}$ and $\mathcal{U}^\dagger$, is now the same set of labels (with no sum). Since soft operators occur only in the $\mathcal{U}$'s, the latter commute with $\mathcal{O}^H$, and soft enhancements cancel out by unitarity, in a trivial way.

The above reasoning fails in the non-Abelian case, because both $\mathcal{U}$ and $\mathcal{O}^H$ are (noncommuting) matrices in a non-Abelian charge multiplet, and the unitarity sum cannot be used. But it fails in the Abelian case too, if the initial states *are not* charge eigenstates, as allowed by symmetry breaking. In such a case, the coherent states are not diagonal in the initial labels α_I, and normally do not commute with the hard overlap matrix O^H. More precisely, by introducing the mixing matrix $\mathcal{M}_{A\alpha}$ and the overlap matrix $\mathcal{O}_{AB}$ in the charge eigenstates basis $\{A\}$, we obtain

$$\sigma_{\alpha_I} = \mathcal{O}_{\alpha_I\alpha_I} = \sum_{A,B}\mathcal{M}^\dagger_{\alpha_I B}\mathcal{O}_{BA}\mathcal{M}_{A\alpha_I}. \quad (5)$$

While soft enhancements cancel out by Eq. (4) in the diagonal terms $\mathcal{O}_{AA}$ of the sum (5), they are nonvanishing in the off-diagonal terms $\mathcal{O}_{AB}$ $(A \neq B)$, which are induced by the mixing, so that the BN theorem is violated.

We will illustrate the above features in the example of the longitudinal sector of the U(1) Higgs model [3].

The Lagrangian in a 't Hooft gauge is

$$\mathcal{L} = (D_\mu\Phi)^+D^\mu\Phi - V[\Phi^+\Phi] - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\zeta}(\partial_\mu A^\mu - \zeta M\Phi)^2 + \bar{\Psi}(i\not{D} - m)\Psi, \quad (6)$$

where $\Phi - v/\sqrt{2} = H = (h + i\phi)/\sqrt{2}$ is the Higgs field, V is the potential, M is the gauge boson mass, ζ is the gauge parameter, and charged fermions of mass m have been introduced. We also take $M_h \simeq M$ as the h field mass, the case $M_h \gg M$ being discussed in [6].

The states we consider are the Higgs boson h and the longitudinal boson $A^L_\mu \equiv L$ as prepared, for instance, by coupling to initial charges in a boson fusion process. Amplitudes with external longitudinal bosons are related, at high energies $E \gg M$, to the Goldstone boson amplitudes by the equivalence theorem [7]

$$\epsilon^L_\mu(p) = \frac{p_\mu}{M} + O\left(\frac{M}{E}\right)$$

$$\epsilon^L_\mu(p)\mathcal{M}_\mu(p;\ldots) \approx \frac{p^\mu}{M}\mathcal{M}_\mu(p;\ldots) = i\mathcal{M}[\phi(p);\ldots], \quad (p^2 \simeq M^2), \quad (7)$$

where the remaining amplitude labels are understood. For this reason, the soft emission properties in the L/h sector are determined by the current $h(x)\overleftrightarrow{\partial}_\mu\phi(x)$ of the scalar sector.

At leading double log level, the emission of a soft gauge boson off an energetic longitudinal boson changes it into a Higgs boson, and all subsequent interactions are described by the eikonal current

$$J^{\mu i}_{\alpha\beta}(k) = e\frac{p^\mu}{pk}q^i_{\alpha\beta} \qquad (\alpha, \beta = \phi, h), \quad (8)$$

where $q^i = \tau_2$ is just a Pauli matrix connecting the L/ϕ and h indices and acting on the charge index of the ith leg $(i = 1, 1', 2, 2')$ (see also [6], where the relation between longitudinal bosons and scalars in eikonal approximation is worked out in detail). The peculiarity of Eq. (8) is that it is off diagonal, as expected from the fact that mass eigenstates are the mixed charge states $h = 1/\sqrt{2}(H + H^\dagger)$, $\phi = -i/\sqrt{2}(H - H^\dagger)$.

Furthermore, in the 't Hooft–Feynman gauge we work with, the leading double log corrections come from interference terms in which a vector boson is emitted off leg $i_1 = 1, 1'$ (with momentum p_1) and absorbed on leg $i_2 = 2, 2'$ (with momentum p_2), the squared contributions being power suppressed at high energies. We then obtain the total eikonal factor

$$e^2\frac{p_1p_2}{(p_1k)(p_2k)}(q_1 - q_{1'})(q_2 - q_{2'}) \quad (s \approx 2p_1p_2 \gg M^2). \quad (9)$$

The actual evaluation of double logs from Eq. (9) is simplified by the remark that the total eikonal current $J_\mu(k) = \sum_i J^\mu_i u(k)$ is conserverd in the fixed-angle, high-energy regime $s \gg M^2$ that we are investigating. This means that

$$k^\mu J_\mu(k)\mathcal{O} = \sum_i q_i\mathcal{O} = (q_1 + q_2 - q_{1'} - q_{2'})\mathcal{O} = 0, \quad (10)$$

so that the total t-channel charge $Q = q_1 - q_{1'} = q_{2'} - q_2$ is conserved and the eikonal current takes the form $J^\mu = e\sum_i q_i\frac{p^\mu_i}{p_ik} = eQ(\frac{p^\mu_1}{p_1k} - \frac{p^\mu_2}{p_2k})$. The eikonal radiation factor in Eq. (9) can thus be written, by including phase space, as

$$-Q^2\frac{e^2}{8\pi^3}\int\frac{d^3k}{2\omega_k}\frac{p_1p_2}{p_1kp_2k} \equiv -Q^2\mathcal{L}, \quad (11)$$

where $\mathcal{L} = (\alpha/4\pi)\log^2(s/M^2)$ is the effective double log coupling mentioned above. The structure of radiative corrections is then the one depicted in Fig. 1(a), where for each power of α the operator $-Q^2 = -(q_1 - q'_1)^2 = -2(1 - q_1q'_1)$ is applied. Notice that the term $q_1q'_1$ exchanges the h and ϕ indices on both legs, as anticipated earlier. If we fix $\alpha_2 = \beta_2 = L$, and define $\sigma_\alpha = \sigma_{\alpha L}$ $(\alpha = L, h = \phi, h)$, the action of $q_1q'_1$ on the α indices is that of a τ_1 Pauli matrix. Therefore, by restoring the full radiation factor, we find

$$\sigma_\alpha = (e^{-2\mathcal{L}(1-\tau_1)})_{\alpha\beta}\sigma^H_\beta, \quad (12)$$

where the σ^H's are the hard (tree-level) cross sections. The final result (12) is easily recast in the diagonal form

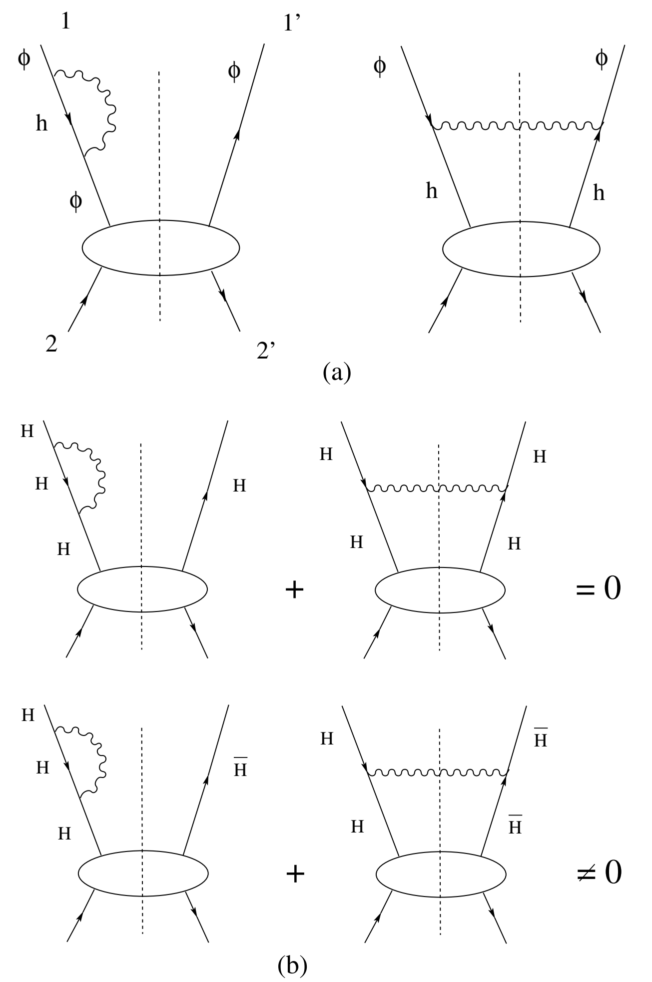

FIG. 1. Picture of radiative corrections to the overlap matrix in (a) the mass eigenstate basis and (b) the charge eigenstate basis, where off-diagonal matrix elements occur. Each figure represents the sum of diagrams in Eq. (11), and effective eikonal vertices $\sim(\frac{p_1^\mu u}{p_1 k} - \frac{p_2^\mu u}{p_2 k})$ are understood.

$$\sigma_{LL} + \sigma_{hL} = \sigma_{LL}^H + \sigma_{hL}^H,$$
$$\sigma_{LL} - \sigma_{hL} = (\sigma_{LL}^H - \sigma_{hL}^H)e^{-4\mathcal{L}}. \tag{13}$$

This means that the average cross section has no radiative corrections, while the difference is suppressed by the form factor corresponding to t-channel charge $Q^2 = 4$. Therefore, at infinite energy, radiative corrections equalize the longitudinal and Higgs cross sections.

The occurrence of the t-channel charge $Q^2 = 4$ is related to the basic fact that h and L are not charge eigenstates, because of symmetry breaking at low energies. In fact, by rewriting the cross sections in terms of the charge eigenstates H and $H^\dagger$ and by using charge conjugation invariance, we find

$$\sigma_{LL} = \sigma_{hh} = \tfrac{1}{2}[\sigma_{HH} + \sigma_{H\bar{H}} + \mathrm{Re}\mathcal{O}(H\bar{H} \to \bar{H}H)], \tag{14}$$
$$\sigma_{Lh} = \tfrac{1}{2}[\sigma_{HH} + \sigma_{H\bar{H}} - \mathrm{Re}\mathcal{O}(H\bar{H} \to \bar{H}H)],$$

where, as in Eq. (5), we notice the occurrence of the off-diagonal overlap matrix elements $\mathcal{O}$ and $O^\dagger$, corresponding to the values $Q_{\mathrm{tot}} = q_1 - q_1' = \pm 2$ of the total charge in the t-channel [Fig. 1(b)]. While the diagonal terms σ_{HH} and $\sigma_{H\bar{H}}$ correspond to $Q = 0$ and have no form factor, the off-diagonal terms are suppressed by the form factor with $Q^2 = 4$ found previously. Therefore, from Eq. (14), we find the expressions

$$\sigma_{LL}(s) = \sigma_{hh}(s) = \tfrac{1}{2}(\sigma_{LL}^H + \sigma_{Lh}^H) + \tfrac{1}{2}(\sigma_{LL}^H - \sigma_{Lh}^H)e^{-4\mathcal{L}}, \tag{15}$$
$$\sigma_{Lh}(s) = \tfrac{1}{2}(\sigma_{LL}^H + \sigma_{Lh}^H) - \tfrac{1}{2}(\sigma_{LL}^H - \sigma_{Lh}^H)e^{-4\mathcal{L}},$$

which are equivalent to Eq. (13). The derivation based on Eq. (14) makes it clear that this phenomenon is not limited to longitudinal and Higgs states, but applies to any mixed charge states which are allowed by symmetry breaking.

Our final comment is about longitudinal couplings to external charges, e.g., fermions of mass m, which make the above effect observable. It is known that, by fermion current conservation, longitudinal polarizations are suppressed by a factor M^2/k_T^2 with respect to transverse polarizations, where k_T^2 denotes the boson transverse momentum, related to its virtuality. However, if $M \gg m$, then the longitudinal k_T^2 distribution is dominated by $k_T^2 = O(M^2)$, yielding a cross section of the same order as the transverse one [8]. The situation changes in the limit of a vanishing symmetry breaking parameter. In fact if $M \ll m$, the longitudinal k_T^2 distribution is cut off by m^2, rather than M^2, thus yielding a cross section of relative order M^2/m^2, which vanishes eventually. Therefore, in the vanishing M/m limit, gauge symmetry and BN theorem are recovered at the same time.

The U(1) Higgs model just discussed is a prototype. A slightly more complicated example, which is relevant to planned accelerators, is electroweak theory itself. Here the gauge group is $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$, and important BN violating corrections are found in the longitudinal sector [6] of both non-Abelian and *Abelian* types. The latter survives in the formal limit of vanishing isospin coupling and has a structure similar to the one illustrated here.

An additional peculiarity of the standard model is that, because of the chiral nature of the gauge group, massive *fermions* of mass m are themselves a superposition of left and right states of different weak hypercharge (and isospin). Therefore, by the general argument of Eqs. (5) and (15), *Abelian* double logs are also expected for fermion beams. If initial beams are longitudinally polarized, the left-right mixing is small at high energies, so that the corresponding off-diagonal overlap is suppressed by a factor m^2/s, and was not explicitly considered before [1]. *Transverse* polarizations, however, are a superposition of left and right states with comparable weights: mixing is therefore maximal, as in the longitudinal boson case considered so far. The corresponding off-diagonal overlap provides the azimuthal dependence [9] of the inclusive cross section at tree level, and is then affected at higher orders by the appropriate double log form factor (carrying t-channel quantum numbers $Y = t_L = 1/2$ in the present case). Polarized beam effects thus provide another instance in which infrared enhancements related to mixing are to be investigated.

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