Ion Heating by Fast-Particle-Induced Alfvén Turbulence

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A novel mechanism that directly transfers energy from super-Alfvénic energetic ions to thermal ions in high-β plasmas is described. The mechanism involves the excitation of compressional Alfvén eigenmodes in the frequency range with $\omega \lesssim \omega_{ci}$. The broadband turbulence resulting from the large number of excited modes causes stochastic diffusion in velocity space, which transfers wave energy to thermal ions. This effect may be important on National Spherical Torus Experiment [M. Ono *et al.,* Nucl. Fusion **40**, 557 (2000)] and may scale up to reactor scenarios. This has important implications for low-aspect-ratio reactor concepts, since it potentially allows for increased fusion reactivity in such a device.

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Emissions in the ion cyclotron range of frequency driven by energetic alpha particles have been observed in the past on tokamaks [1,2], but were not observed to have a pronounced effect on the plasma behavior. One of the reasons for this lack of impact was that the energy available to drive the modes is determined by the fraction of energetic particles with a velocity greater than the Alfvén velocity. For the tokamak examples considered, this represents only a small fraction of the total energy available in the alpha particle distribution. If, instead, the fast particle velocity is many times the Alfvén velocity, the power available to sustain the modes can be much larger, on the order of the total fusion power.

The current generation of spherical torus experiments typically operate at very low values of the axial toroidal magnetic field (0.3–0.6 T). However, the velocity of energetic particles provided by the neutral beam heating sources is quite high. For example, on National Spherical Torus Experiment (NSTX) [3], which operates with 80 keV deuterium neutral beams and 0.3 T toroidal field, the neutral beam velocity is typically given by $v_{\rm beam} \sim 4v_A$. The existence of a large class of super-Alfvénic particles can change the regime from one of weakly interacting waves to one where the wave amplitudes become large enough to modify the thermal particle energy. The energy flow is from the fast particles to the waves by resonance, and from the waves to the thermal particles by stochastic heating.

Ion temperatures greater than the electron temperature have been measured on NSTX during neutral beam heating [4,5]. Whereas $T_i > T_e$ is not noteworthy in and of itself, the power balance for these discharges is difficult to explain in terms of collisional power balance calculations. Q_{ie} [$\equiv 3/2n\nu_i(T_i - T_e)$, the collisional power flow from ions to electrons] exceeds the total power going into the ions [5,6]. The deficit of power is large and, in some cases, on the order of the total neutral beam heating power. The discrepancy is well outside the random errors in the measurements. From this we ascertain that there may be another source of input power to the ions. Also, a series of high frequency plasma modes has been observed during neutral beam heating that has been identified as the compressional Alfvén eigenmode, mentioned above [7]. On the order of 20 individual modes were observed in the frequency range from $\omega_{ci} > \omega > 0$. Another important observation was an off-axis local maximum in the ion temperature. The off-axis peak correlates with the predicted location of the peak amplitude of the compressional Alfvén eigenmode (CAE) [8]. These observations motivate a theoretical consideration of the possible effect of multifrequency large amplitude CAE modes on ions.

The procedure followed to determine the existence of a wave-based ion heating mechanism was to enter the expected wave spectrum from a theoretical model of the CAE mode into a code that calculates particle trajectories. The model used to determine the wave number distribution of the CAE modes is described in [8]. The model calculates which modes are unstable based on a balance between the linear growth rate of the mode and damping on ions and electrons. The primary ion damping mechanism is cyclotron resonance, whereas the primary electron damping mechanism is Landau damping.

We note that these modes are largely undamped in the frequency range observed in the experiment $0.2\omega_c < \omega < 0.6\omega_c$. Finite β_e reduces Landau damping for small but finite $k_\parallel$. This can be seen by considering the usual expression for the Landau damping rate, $\gamma_e/\omega = \beta_e \zeta e^{-\zeta^2}$, where $\zeta = \omega/k_\parallel v_{\mathrm{th}_e}$. We take the limit $\zeta \ll 1$ and approximate $\omega \sim k_\perp v_A$ yielding $\sqrt{m_e/\beta_e m_i} \ll k_\parallel/k_\perp < 1$. Assuming deuterium ions and $\beta_e = 0.1$ yields $1/19 \ll k_\parallel/k_\perp < 1$. Finite $k_\parallel$ Doppler shifts the modes away from the cyclotron frequency eliminating the resonant thermal ion damping. The lack of damping may allow the modes to grow to sufficient amplitude for the onset of stochasticity.

We assume a cold plasma dielectric tensor for these modes given by

$$\overleftrightarrow{\epsilon}_j = \begin{vmatrix} \varepsilon_j & ig_j & 0 \\ -ig_j & \varepsilon_j & 0 \\ 0 & 0 & \eta_j \end{vmatrix}$$

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with

$$\varepsilon_j = \frac{c^2}{v_A^2(1 - \frac{\omega_j^2}{\omega_c^2})}, \qquad g_j = \varepsilon_j \frac{\omega_j}{\omega_c}, \qquad \text{and} \qquad \eta_j = -\frac{m_i}{Zm_e}\frac{c^2}{v_A^2}\frac{\omega_c^2}{\omega_j^2}.$$

The dispersion relation is given by

$$\omega_j^2 = \frac{k_j^2 v_A^2}{2}\left[1 + \cos^2\theta_j + \sqrt{1 + 2\left(\frac{2\omega_j^2}{\omega_c^2} - 1\right)\cos^2\theta_j + \cos^4\theta_j}\,\right],$$

and for completeness, the wave vectors are as follows:

$$\begin{aligned}\vec{k}_j &= k_{\perp j}(-\sin\phi_j\hat{x} + \cos\phi_j\hat{y}) + k_{\parallel j}\hat{z}\\ &= k_{\perp j}\hat{r}_j + k_{\parallel j}\hat{z},\end{aligned}$$

$$\vec{E}_{1_j} = E_{1_j}e^{i\varphi_j}\left[(\alpha_j\cos\phi_j - \sin\phi_j)\hat{x} + (\alpha_j\sin\phi_j + \cos\phi_j)\hat{y} - \frac{k_{\parallel j}k_{\perp j}c^2}{\omega_j^2\eta_j}\hat{z}\right]$$

$$\vec{B}_{1_j} = B_{1_j}e^{i\varphi_j}\left[\left(\gamma_j\cos\phi_j - \frac{k_{\parallel j}}{k_{\perp j}}\sin\phi_j\right)\hat{x} + \left(\gamma_j\sin\phi_j + \frac{k_{\parallel j}}{k_{\perp j}}\cos\phi_j\right)\hat{y} - \hat{z}\right]$$

with

$$\alpha_j = -i\frac{\omega_c}{\omega_j}\left[1 - \frac{k_{\parallel j}^2c^2}{\varepsilon_j\omega_j^2}\left(1 + \frac{k_{\perp j}^2c^2}{\omega_j^2\eta_j}\right)\right],$$

$$\gamma_j = \frac{k_{\parallel j}k_{\perp j}c^2}{\alpha_j\omega_j^2\eta_j} - \frac{k_{\parallel j}}{\alpha_j k_{\perp j}},$$

$$\varphi_j = \vec{k}_j \cdot \vec{x} - \omega_j t, \qquad B_{1_j} = -\frac{\alpha_j k_{\perp j}c}{\omega_j}E_{1_j},$$

$$\theta_j = \cos^{-1}\left(\frac{k_{\parallel j}}{k_j}\right), \quad \text{and} \quad \vec{B}_0 = B_0\hat{z},$$

where the subscript j indexes individual mode frequencies ω_j, and subscript 1 indicates the wave field, subscript 0 indicates the background field, B_0 is the equilibrium component of the magnetic field, $v_A = B_0/\sqrt{\mu_0\rho_i}$ is the Alfvén velocity, $\rho_i = m_i n_i$ is the plasma mass density, ω_c is the ion cyclotron frequency, m_e is the electron mass, Z is the charge of the plasma ions, $k_{\perp j}$ and $k_{\parallel j}$ are the perpendicular and parallel wave numbers for the jth mode which define the wave vector $\vec{k}_j$, ϕ_j is the propagation angle in the x-y plane, θ_j is the angle between $\vec{k}_j$ and the z axis, and r and R are the local minor and major radii of the center of curvature of the flux surface, respectively.

The wave fields calculated in this way are used as input to a code that solves the equation of motion for particles in the prescribed wave combined with a constant background magnetic field. This formulation is the simplest relevant model that can describe the behavior of particles in the presence of a CAE mode spectrum. It has been shown [9] that stochastic heating of ions is possible in the presence of a single large amplitude Alfvén wave below the cyclotron frequency given both a large wave amplitude and oblique propagation, with $k_\perp v_A\tilde{B}/B_0\omega_c \gtrsim 0.1$. In the presence of several modes the amplitude threshold can be expected to be much lower.

The actual mode amplitude will be determined by the relative energy transfer efficiencies between the fast ions and the wave versus between the wave and the thermal ions. Such a theory would require a self-consistent nonlinear treatment of the fast-particle distribution function, the wave amplitude, and the thermal-particle distribution function, which is beyond the scope of this Letter. As a first step towards understanding the effects of these modes we simply observe the onset and rate of heating, as a function of mode amplitude.

Because the modes in question tend to transfer perpendicular energy to the plasma ions, a pitch angle scattering collision operator was included in the equation of motion. This helps avoid the nonphysical result of strong anisotropy. The scattering rate was taken to be constant in time with $\nu_i = 0.01\omega_c$.

The mode spectrum was chosen to roughly match the experimental data. We used $k_\perp = m/r$ with $2 \leq m \leq 8$, and $k_\parallel = n/R$ with $1 \leq n \leq 3$ giving 21 modes. The effective radii, r and R, were chosen so that $0.2\omega_c < \omega < 0.6\omega_c$ and $0.2k_\perp < k_\parallel < 0.5k_\perp$, as indicated by mode stability theory [8]. The propagation angle in the x-y plane, ϕ, was chosen randomly for each mode. The values of $k_\parallel$ were chosen to be in the range that resonates with beam particles, since $k_\parallel$ is unknown.

The results of the calculation are shown in Figs. 1 and 2. There is a clear heating effect that has a strong amplitude dependence. The results were insensitive to most details of the mode spectrum and collision frequency, other than the actual frequency range and the ratio of $k_\perp$ to $k_\parallel$. More detailed studies of the heating rate and stochastic thresholds as functions of mode spectrum will be reported in future publications. The self-consistent energy balance of these modes will likely be quite complicated, perhaps involving bursting "fishbone-like" behavior [10].

The mode amplitudes required for the heating rates in Fig. 2 to be similar to that observed on NSTX are on the order of $\sim$2–4 G in NSTX ($dB_y/B \approx 10^{-3}$), which is plausible since the measured amplitude at the NSTX wall is $\sim$4 mG [7]. This implies approximately 6–7 e-foldings in the distance between the propagation region and the vessel wall ($\sim$0.5 m).

We stress that these calculations are only a proof-of-principle and it would be premature to claim agreement

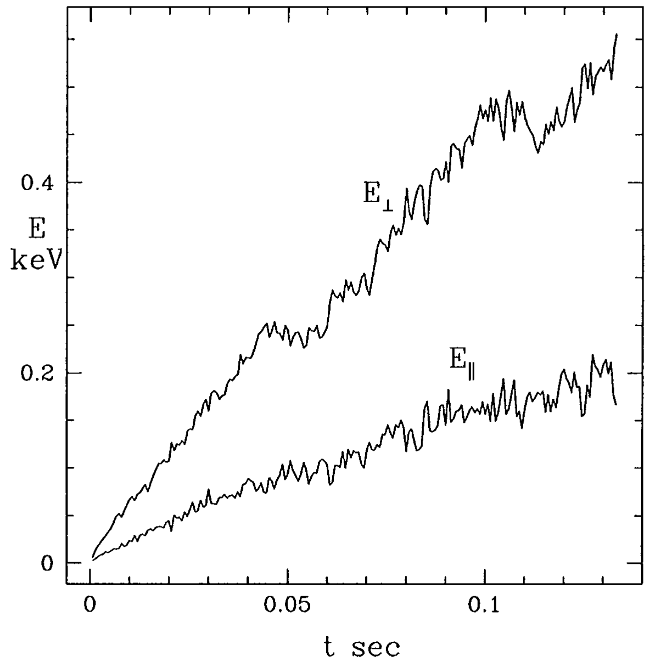

FIG. 1. Ion energy in keV vs time, for $\delta B_y/B = 6.0 \times 10^{-4}$.

with the NSTX data. In order to make a proper comparison between observations and theory, a more complete theoretical treatment is required, as well as more detailed experimental data. In particular, the calculation should be done in the correct geometry, the actual growth rates and damping should be used to ascertain the actual expected spectrum, and all modes to higher order in the cyclotron resonance should be included, with all elements configured to match the experiment.

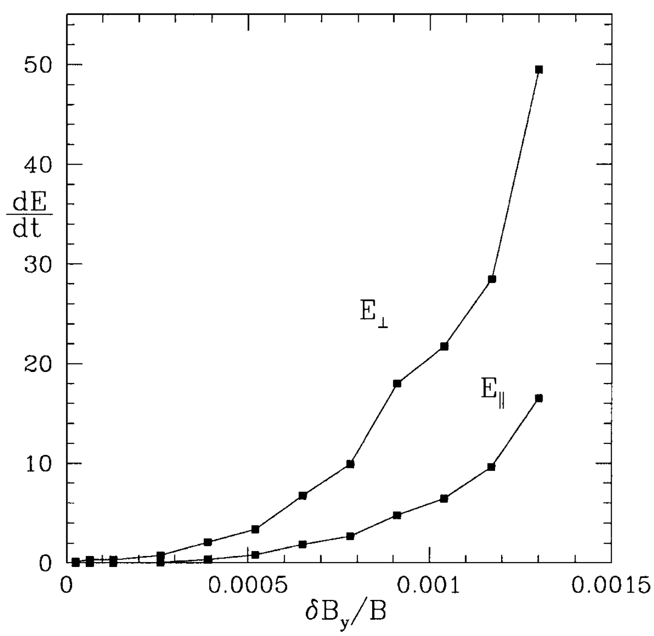

FIG. 2. Heating rate (keV/s) vs $\delta B_y/B$.

The result of this calculation is important at many levels. As mentioned above, it is important for the current generation of low-aspect-ratio toroidal confinement devices, since in these devices the velocity of particles from the neutral beams is well in excess of the Alfvén velocity. It is important to note that the conditions for these modes to grow are substantially easier to arrange at low-aspect ratio. The constraints that imply low-aspect ratio are that $v_f \gg v_A$ and $\rho_f < a$, where a is the radius of the device and v_f and ρ_f are the relevant fast-particle velocity and orbit size. Also one must avoid Landau damping for small but finite $k_\parallel$, as above.

Perhaps more importantly, the velocity of fusion alpha particles in proposed ST reactor concepts, such as ARIES-ST [11], is also well in excess of the Alfvén velocity. In particular, the ARIES-ST design has a peak toroidal field on axis of 2.14 T which makes the fusion alpha-particle velocity $v_\alpha \sim 5v_A$. Therefore the CAE modes observed on NSTX could be apparent in a larger ST device with a significant fusion α population. The remaining issue is to determine what fraction of the α energy distribution, which is more isotropic, would interact with the CAE mode. Such a calculation is beyond the scope of this Letter.

The implications of coupling fast-particle energy directly to ions are covered in detail in [12,13], and are readily summarized. The criterion for fusion heating power exceeding plasma loss power in a deuterium-tritium reaction is given by $n_i^2 T_i^2/[n(T_e + T_i)/\tau_E] \gtrsim 10^{21}$ (keV s m^{-3}). The ignition criterion assumes that the primary energy transfer mechanism from fusion α's is via collisional coupling of the α energy to electrons, which then collisionally couple to the fuel ions and that $n = n_e = n_i$. This implies $T_e \gtrsim T_i$, simplifying to the usual criterion $n_i T_i \tau_E \gtrsim 2 \times 10^{21}$ (keV s m^{-3}). If a direct ion heating effect exists, the higher ion temperature leads to an additional factor f, yielding $f n_i T_i \tau_E \gtrsim 2 \times 10^{21}$ (keV s m^{-3}), where $f = 2/(1 + T_e/T_i)$. The improvement can be larger if τ_E decreases more strongly with increasing T_e than with increasing T_i.

A concept somewhat similar to the one proposed here, popularly referred to as alpha channeling, has been proposed in the past [14]. The mechanism we propose has several advantages over the alpha-channeling mechanism. First, the modes we discuss occur naturally in the presence of super-Alfvénic particles, therefore excitation power may not be required. Second, the mode amplitude heating threshold is small, minimizing concerns about rapid fast-particle scattering losses due to large wave amplitudes. Third, the waves extract energy from the fast particles and scatter them towards passing orbits, improving the fast-particle confinement. Finally, the ion heating mechanism is robust, since the mode frequency is naturally near the ion cyclotron resonance.

A more similar, but less well-known, calculation [15] considers the tokamak case with $v \gtrsim v_A$, $\omega \sim (3\text{–}6)\omega_c$, and $k_{\|} \simeq 0$. This case shows negligible heating. In particular the $v \gtrsim v_A$ constraint limits the energy available to the modes, the high frequency degrades the coupling to the bulk ions, and the $k_{\|} \simeq 0$ condition (which is forced by the low β_e in a tokamak, see above) reduces the Doppler shift away from the cyclotron frequency, preventing the modes from growing to sufficient amplitude for stochastic heating to take place. The far-from-optimum parameter regime reduces the stochastic heating effect. The insight shown in [15] is nonetheless remarkable.

In conclusion, a novel mechanism for transferring energy directly from a fast-ion population to bulk thermal ions in a magnetically confined toroidal plasma with low-aspect ratio has been identified. The mechanism involves stochastic heating of thermal particles by compressional Alfvén waves driven by the super-Alfvénic fast particles. These waves have frequencies such that $\omega \sim \omega_c/2$. This effect is a possible candidate for explaining the higher than expected ion temperatures seen during neutral beam heating experiments on NSTX. In particular, the off-axis peak in the ion temperature which correlates with the predicted mode location, the experimental observation of the modes, and the lower than expected fast-ion loss are strongly circumstantial evidence for this effect. If confirmed on NSTX, this effect has a high probability of scaling to an ST reactor. Finally we note that, following the existing experimental observations, we have included only modes with frequencies below the cyclotron frequency. The theory of these modes also predicts resonances at Doppler shifted multiples of the cyclotron frequency. We speculate that modes with frequencies $\omega > \omega_c$ will also contribute to the heating effect, lowering the threshold and/or raising the power transfer to the plasma ions. In addition we speculate that the inclusion of multiple modes could bring the thresholds for ion heating presented in [9] in line with the amplitudes observed in the solar corona.

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