

Spin Currents in Magnetic Films

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Spin currents injected into magnetic thin films may noticeably change the magnetization of the films. To describe this effect, exchange coupled Landau-Lifshitz equations for the local magnetization and the spin-polarized charge carriers are combined with transport equations for charge and spin currents. For steady state transport one obtains two different instability conditions. Both conditions are supported by recent experimental data on current induced magnetization reversal and spin-wave excitations. No second ferromagnetic layer is needed for excitations due to spin transfer.

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A growing number of experiments probe magnetic elements and multilayers with high electric current densities [1–7]. These experiments indicate that the spin of the conduction electrons influences the magnetization of the elements. Since in most experiments there exists a wide range of competing electric and magnetic effects, it has proven difficult to interpret the existing data in terms of the underlying elementary processes. This work is then an attempt to combine the results of the different experiments and devise a more complete theory.

A conduction electron spin can influence the local moments in a magnetic layer due to an s - d type exchange interaction. In the presence of a current this leads to a number of effects best demonstrated by the experiments of Weber *et al.* [7]. By letting a spin-polarized electron beam produced in a photocathode pass through the magnetic film, they inject a nonequilibrium magnetization $\delta\mathbf{M}_s$ in the film. The local magnetization of the film \mathbf{M}_d experiences then a torque due to the exchange interaction with the injected nonequilibrium magnetization from the spin current [7]. For the case of two magnetic layers separated by a nonmagnetic one this has been described in terms of a nonequilibrium exchange interaction (NEXI) which is distinct from the RKKY interaction as it treats the influence of the nonequilibrium magnetization on the local moments rather than that of spin-density oscillations [8,9]. In addition, the injected spin-polarization relaxes [7]. This happens by spin-flip processes that lead, for example, to excitations from the ferromagnetic ground state of the local moments in the form of Stoner excitations which may decay into spin waves. For particular cases, however, it has been pointed out by Berger [10] and in some sense also by Slonczewski [11], as has been found experimentally by Tsoi *et al.* [1] and Rezende *et al.* [2], that a spin flip of the conduction electron can excite collective spin waves of distinct frequency above a certain threshold current.

In practice, of course, one is interested in the magnetization dynamics, i.e., how the precession created by the NEXI and different forms of relaxation mechanisms coexist. A complete treatment of the magnetic dynamics of a magnetic multilayer with a current in the perpendicular direction requires a simultaneous solution of the equation of

motion for the local magnetization of the magnetic layers, and the equation of motion of the spin-polarized charge carriers. These equations will be combined with the transport equations for the charge and spin currents in the first part of this work and later on used to generalize the treatment of Ref. [8].

The basic features of ferromagnetic metals can be described by two magnetic subsystems that are interacting with each other. There is the ferromagnetic subsystem of “core” electrons (conventionally referred to as d electrons) which in a phenomenological description can be characterized by a local magnetization \mathbf{M}_d . The individual moments are strongly coupled by a direct exchange interaction and oriented along an easy direction. Apart from the d -electron system, there exists a paramagnetic subsystem of almost free conduction electrons (conventionally referred to as s electrons) and can be characterized by a magnetization \mathbf{M}_s . Because of the exchange interaction between s and d electrons, $E_{\text{ex}} = \alpha\mathbf{M}_s \cdot \mathbf{M}_d$ in units of EL^{-3} , where α is the s - d exchange parameter, the equations of motion of the two magnetic subsystems are coupled and take approximately the form of classical Landau-Lifshitz equations similar to those given by Langreth and Wilkins [12]:

$$\frac{\partial\mathbf{M}_s}{\partial t} - \gamma\mathbf{M}_s \times \mathbf{H}_{\text{eff},s} + \vec{\nabla} \cdot \vec{\mathbf{J}}_M = \frac{\delta\mathbf{M}_d}{\tau_{ds}} - \frac{\delta\mathbf{M}_s}{\tau_s}, \quad (1a)$$

$$\frac{\partial\mathbf{M}_d}{\partial t} - \gamma\mathbf{M}_d \times \mathbf{H}_{\text{eff},d} = \frac{\delta\mathbf{M}_s}{\tau_{sd}} - \frac{\delta\mathbf{M}_d}{\tau_d}, \quad (1b)$$

where γ is the gyromagnetic factor [13]. Linked with the nonlocalized nature of the conduction electrons is the alteration of \mathbf{M}_s because of “spin diffusion” described by the divergence of the spin current $\vec{\mathbf{J}}_M$ [14]. The effective fields describing the precession of the magnetic moments on the left-hand side (lhs) of Eq. (1) are defined as

$$\mathbf{H}_{\text{eff},s(d)} = \mathbf{H}_{\text{eff},s(d)}^0 + \alpha\mathbf{M}_{d(s)}, \quad (2)$$

where $\mathbf{H}_{\text{eff},s}^0 = \mathbf{H}_e$, $\mathbf{H}_{\text{eff},d}^0 = \mathbf{H}_e + \mathbf{H}_{\text{ind}} + \mathbf{H}_{\text{dip}} + \mathbf{H}_{\text{an}} + \mathbf{H}_{\text{dd}}$. \mathbf{H}_e is the externally applied field and \mathbf{H}_{ind} the Oersted field on the d electrons induced by the flow of the

conduction electrons. The influence of the dipole-dipole, anisotropy, and direct exchange energies between the d electrons are expressed by \mathbf{H}_{dip} , \mathbf{H}_{an} , and \mathbf{H}_{dd} , respectively. Similar terms can be ignored for the s electrons, because \mathbf{M}_s describes the paramagnetic subsystem of almost free conduction electrons. In the spirit of the phenomenological approach \mathbf{M}_s is looked upon as caused primarily by the molecular field $\alpha\mathbf{M}_d$ and the external field \mathbf{H}_e .

The right-hand side (rhs) of Eq. (1) determines the relaxation in the system, where $\tau_s^{-1} = \tau_{sd}^{-1} + \tau_{sl}^{-1}$ and $\tau_d^{-1} = \tau_{ds}^{-1} + \tau_{dl}^{-1}$. When, for example, the relaxation mechanism is one in which spin angular momentum is transferred from the s - to the d -electron system, the order of subscripts is sd ($s \rightarrow d$) [15]. The subscript l —lattice—is used to denote spin-orbit and other scattering processes relaxing spin-angular momentum to a heat bath. The system relaxes to its instantaneous or local equilibrium value of magnetization so that the nonequilibrium magnetization takes the form

$$\delta\mathbf{M}_{s(d)} = \mathbf{M}_{s(d)} - \chi_{s(d)}\mathbf{H}_{\text{eff},s(d)}, \quad (3)$$

where for a free electron system $\chi_s = (\hbar\gamma)^2 N(\varepsilon_F)/4$ is the Pauli susceptibility at the Fermi energy, and $N(\varepsilon_F)$ the density of states. However, the susceptibility χ_d is not constant in a ferromagnet and depends on the applied magnetic field which is the essential difference to the system treated in Ref. [12]. To remove this ambiguity, one can make use of the condition that far below the Curie temperature M_d has to be constant. Taking the scalar product of Eq. (1b) with \mathbf{M}_d , one obtains the following replacement:

$$\chi_d = \frac{\tau_d}{\mathbf{H}_{\text{eff},d} \cdot \mathbf{M}_d} \left(\frac{M_d^2}{\tau_d} - \frac{\delta\mathbf{M}_s \cdot \mathbf{M}_d}{\tau_{sd}} \right) \quad (4)$$

that shall be assumed implicitly in the following when writing χ_d .

The scattering rates between the s - and the d -electron system are exactly balanced to their lowest order; they conserve the total spin-angular momentum $\mathbf{M} = \mathbf{M}_s + \mathbf{M}_d$ in the system. This is due to the fact that the s - d Hamiltonian commutes with \mathbf{M} [12]. In the absence of external forces, i.e., without spin currents, external driving fields, and relaxation to the lattice, it can be seen by adding Eqs. (1a) and (1b) that from the conservation of the total spin-angular momentum \mathbf{M} , it follows that the total torque is also conserved so that $\partial\mathbf{M}_d/\partial t = -\partial\mathbf{M}_s/\partial t$. The latter condition is more or less fulfilled in Ref. [7] as soon as the precession sets in due to the exchange coupling with the femtosecond pulse of the spin current up until significant relaxation to the lattice occurs, i.e., up to a fraction of a nanosecond. Another way of understanding the detailed balance in the s - d scattering is obtained by taking the functional derivative with respect to $\mathbf{H}_s^* = -\delta\mathbf{M}_s/\chi_s$ in Eq. (1b), for example,

$$\frac{\delta}{\delta H_s^{*j}} \left(\frac{\partial M_d^i}{\partial t} \right) = -\frac{\chi_s}{\tau_{sd}} \delta_{ij} \equiv \mathcal{L}_{ds}^{ij}, \quad (5)$$

where $i, j = x, y, z$ and \mathcal{L}_{ds}^{ij} is the tensor of kinetic coefficients which in the approximation of irreversible thermodynamics relate the components of $\partial\mathbf{M}_d/\partial t$ to the components of the “additional field” \mathbf{H}_s^* [16,17]. Now, the kinetic coefficients for the spin transfer between the two subsystems have to fulfill the Onsager relations $\mathcal{L}_{sd}^{ij} = \mathcal{L}_{ds}^{ji}$. Together with the relation for which $s \leftrightarrow d$ and Eq. (1), one finds a connection between scattering rates and susceptibilities,

$$\chi_d \tau_{sd} = \chi_s \tau_{ds}. \quad (6)$$

The nonlocal character of the conduction electrons gave rise to $\vec{\nabla} \cdot \vec{\mathbf{J}}_M$ in Eq. (1) wherein the spin current has to take into account the force on the conduction electron produced by the action of the field gradients on its electric and magnetic moment. Assuming for now that the conduction electrons are quantized in z direction, the electric field and the Stern-Gerlach force are given by

$$\vec{F}_\sigma = -e\vec{\nabla}V \pm \mu_B \vec{\nabla}H_{\text{eff},s}^z, \quad (7)$$

where $\sigma = \uparrow, \downarrow$ denotes the different spin orientations. Using the concepts of drift and diffusion current for spin-up and spin-down electrons,

$$\vec{J}_\sigma = n_\sigma \vec{v}_\sigma - D\vec{\nabla}n_\sigma, \quad (8)$$

where n_σ is the carrier density and D the diffusion constant, the drift velocity \vec{v}_σ is proportional to the force (7), i.e., $\vec{v}_\sigma = \mu_s \vec{F}_\sigma/e$, where μ_s is the electron mobility. The latter and D are, in principle, also spin dependent but this shall be ignored here for simplicity. When one rewrites Eq. (8) with the help of (7), the z component of the spin current $\vec{J}_M^z = \mu_B(\vec{J}_\uparrow - \vec{J}_\downarrow)$ takes the form

$$\vec{J}_M^z = -\mu_s M_s^z \vec{\nabla}V - D\vec{\nabla}M_s^z + D\chi_s \vec{\nabla}H_{\text{eff},s}^z, \quad (9)$$

where $M_s^z = \mu_B(n_\uparrow - n_\downarrow)$. The total number of electrons is given by $n = n_\uparrow + n_\downarrow$ and does not depend on the axes of spin quantization. The x and y components can be obtained in a similar manner as well as the electric current $\vec{J}_e = e(\vec{J}_\uparrow + \vec{J}_\downarrow)$. The linearized transport equations, where \mathbf{M}_s is replaced by its equilibrium value \mathbf{M}_s^0 and $\vec{\nabla}(n_\uparrow n_\downarrow) \approx 0$, the latter arises as a small “Boltzmann correction” term also in Ref. [18], determine a complete set of equations for the spin current in Eq. (1) [19]:

$$\vec{J}_e = -\sigma \vec{\nabla}V - \frac{\mu_s}{\chi_s} \mathbf{M}_s^0 \cdot \vec{\nabla} \delta\mathbf{M}_s, \quad (10a)$$

$$\vec{J}_M = -\mu_s \mathbf{M}_s^0 \vec{\nabla}V - D\vec{\nabla} \delta\mathbf{M}_s. \quad (10b)$$

At steady state, the transport coefficients take the following form: the conductivity $\sigma = ne^2\tau_{\text{tr}}/m^*$, the diffusion constant $D = \zeta\tau_{\text{tr}}v_F^2/3$, and the mobility $\mu_s = e\tau_{\text{tr}}/m^*$. The transport scattering time τ_{tr} has contributions from spin-independent impurity scattering and spin-dependent scattering $\tau_{\text{tr}}^{-1} = \tau_i^{-1} + \tau_s^{-1}$; m^* is the effective mass, and v_F the Fermi velocity. Equations (1) together with (10) present the general equations of motion for current

driven magnetic multilayers [20]. Only their combination allows one to study the effects of a spin current on a multilayer.

Thus, to obtain the distribution of the nonequilibrium magnetization $\delta\mathbf{M}_s$ in a multilayer, one needs to replace in Eq. (10b) $\vec{\nabla}V$ with the help of Eq. (10a) and insert it into Eq. (1a). At steady state Eq. (1a) takes the form of a simple diffusion equation to lowest order in the spin polarization $\eta = M_s^0/(n\mu_B)$ and for $\tau_{sl} \sim 10^{-12}$ s \ll $\tau_{dl} \sim 10^{-10}$ s, which is satisfied in most metals:

$$\nabla^2 \delta\mathbf{M}_s = \lambda_{sf}^{-2} \delta\mathbf{M}_s, \quad (11)$$

where $\lambda_{sf} = \sqrt{D\tau_{sl}}$ is the spin diffusion length. The contributions from s - d scattering have canceled. This equation describes the effect of spin accumulation [18,21]. The term $\delta\mathbf{M}_s \times \mathbf{H}_{\text{eff},s}^0$, arising from the precession term on the rhs of Eq. (1a), was neglected in Eq. (11) since its associated precession frequency is much lower than τ_{sl}^{-1} [22]. If the local magnetization is constant throughout each layer, Eq. (11) can be solved in a multilayer for each layer separately by setting its rhs to zero and making the following ansatz:

$$\delta\mathbf{M}_s^n(x) = \mathbf{m}_1^n e^{-x/\lambda_{sf}^n} + \mathbf{m}_2^n e^{x/\lambda_{sf}^n}, \quad (12)$$

where $\mathbf{m}_{1(2)}^n$ are constant vectors for layer n . These constants need to be determined by appropriate boundary conditions along the x direction. At each interface both nonequilibrium magnetization $\delta\mathbf{M}_s$ and spin current $\vec{\mathbf{J}}_M$ are continuous when neglecting inelastic interface scattering [18]. For two magnetic layers, for example, the nonequilibrium magnetization can be written as a superposition of the local moment vectors of the left and right magnetic layers, i.e., $\delta\mathbf{M}_s(x) = \mathbf{m}_L(x) + \mathbf{m}_R(x)$, where $\mathbf{m}_{L(R)} \parallel \mathbf{M}_{s,L(R)}^0 \parallel \mathbf{M}_{d,L(R)} \equiv \mathbf{M}_{L(R)}$.

In the time-dependent case this solution still holds approximately, if the time dependence of the conduction electrons is satisfactorily determined by the rotational motion of the local moments only; an assumption that Silsbee *et al.* could show is reasonably well satisfied in conduction electron spin resonance experiments on ferromagnetic/nonmagnetic metal bilayers [23]. This implies that the effect of the time dependence of the conduction electron on the rotation of the local moments can also be neglected since $M_d \gg M_s$ so that $\delta M_d \gg |\mathbf{M}_s^0 - \mathbf{M}_s(t)|$. For strong currents, however, the nonequilibrium magnetization, as occurs in Eq. (11), is of the order of the time-dependent fluctuations δM_d . To obtain the equation of motion for the magnetizations $\mathbf{M}_{L(R)}$, one has to make sure that the contributions of $\delta\mathbf{M}_s$, which enter the effective field $\mathbf{H}_{L(R)}$ in each layer, can be derived from the same energy functional, i.e., the NEXI:

$$E_{\text{nexi}} = -\alpha_{\text{eff}} \mathbf{M}_L \cdot \mathbf{M}_R, \quad (13)$$

$$\frac{\alpha_{\text{eff}}}{\alpha} = \frac{\nu_{2D}}{V_0} \int_{-\infty}^{\infty} dx \left(\frac{m_L(x_R)}{M_L} + \frac{m_R(x_L)}{M_R} \right),$$

where ν_{2D} is the lateral dimension of the multilayer, V_0 the volume of a unit cell, and $x_{L(R)}$ means that $m_{L(R)}(x)$ is to be integrated only in the left or right layer, respectively. In this way, the nonlocal effects of the spin current are taken properly into account in the effective field, i.e., $\mathbf{H}_R = \mathbf{H}_{\text{eff},R}^0 + \alpha_{\text{eff}} \mathbf{M}_L + \alpha(\mathbf{M}_{s,R}^0 + \mathbf{m}_R)$. The equations of motion are

$$\frac{\partial \mathbf{M}_R}{\partial t} - \gamma \mathbf{M}_R \times \mathbf{H}_R = -\frac{\mathbf{M}_R - \chi_R(\mathbf{H}_R + \delta\mathbf{H})}{\tau_d}, \quad (14)$$

and for the left layer accordingly, where $\delta\mathbf{H} = (\mathbf{m}_L + \mathbf{m}_R)\tau_d/(\tau_{sd}\chi_R)$ is the effective field due to spin transfer. Instead of $\alpha_{\text{eff}}\mathbf{M}_L$ one can introduce the effective nonequilibrium coupling field $\mathbf{H}_R^{\text{nexi}} = \alpha_{\text{eff}}\mathbf{M}_L$ between the layers. For $\mathbf{H}_R^{\text{nexi}}$ to give a significant contribution, there needs to be an asymmetry in the spin transport of the multilayer; i.e., one layer strongly polarizes the current, whereas the other only weakly polarizes it. Otherwise, there will be a cancellation of the torque generated by the first layer from the torque generated by the second [8].

Equation (14) is a generalization of the one given in Ref. [8] in that the rhs includes not only damping terms produced by $\mathbf{H}_{\text{eff},R}^0 + \mathbf{H}_R^{\text{nexi}}$ but also the effects of local spin accumulation as well as spin transfer terms proportional to τ_{sd}^{-1} . Since spin accumulation is an interface effect, the excess of nonequilibrium magnetization will be largest at the interfaces and, thus, the spin transfer between s and d electrons [10]. One notices from the form of $\delta\mathbf{H}$ that there also exists a cancellation in the spin transfer if both layers produce nearly identical polarizations. However, this cancellation occurs only for a close to parallel alignment of \mathbf{M}_L and \mathbf{M}_R where $m_L \approx -m_R$ [18].

Equation (14) shows two forms of instabilities driven by the spin current: one due to the precession on the lhs, and another one due to spin transfer on the rhs. In the cases outlined below, the instabilities occur when $\mathbf{H}_R^{\text{nexi}}$ is equal and opposite to $\mathbf{H}_{\text{eff},R}^0$ or when $\delta\mathbf{H}$ is equal and opposite to \mathbf{H}_R . As a simple example of the first instability is the case similar to the experiments of Refs. [3,4]: two Co films separated by a Cu layer. One Co film is considered to be fixed and thicker than the spin diffusion length λ_{sf} and the other one free and much thinner than λ_{sf} . Then one can solely focus on the right (free) layer which has to fulfill the steady state condition $\mathbf{H}_R \times \mathbf{M}_R = 0$ as follows from (14). In the presence of a current this also implies that $\mathbf{H}_R^{\text{nexi}} \times \mathbf{M}_R = 0$. Unless this condition is met, the NEXI induces a torque on the right layer, which will lead to a switching of the right layer if the torque is stronger than the coercive field. For uniaxial anisotropy this corresponds to a critical field $H_c^{\text{nexi}} = \pm 2K/M_R$ where K is the uniaxial anisotropy constant. The required current densities are of the order of 10^6 – 10^7 A/cm² [8] and similar to those in the switching experiments [3,4].

The second instability occurs when the magnetization is pushed away from the ‘‘frozen,’’ instantaneous field \mathbf{H}_R .

To illustrate this, the instantaneous field for an infinitely thin plate magnetized in the perpendicular direction is approximately $H_R^z \approx H_e - 4\pi M_R \equiv -\omega_H/\gamma$. From Eq. (14) together with the identity (6) one obtains [15]

$$\hbar\omega_H + \Delta\mu(x_R) \left(\frac{\tau_{dl}}{\tau_{dl} + \tau_{ds}} \right) < 0, \quad (15)$$

as the condition for the instability, where $\Delta\mu(x) = -\hbar\gamma[m_L(x) + m_R(x)]/\chi_s$ is the difference in “chemical potentials” for spin up and spin down electrons [18,21]. The onset of the instability is usually associated with the uniform mode, i.e., the lowest lying excitation in the spectrum, before for stronger currents other modes are excited [1]. What is interesting to note is that in contrast to the treatments by Berger, Slonczewski, and Bazaliy *et al.* [10,11,24] the presence of a second ferromagnetic layer is not necessary to predict current induced excitations in (14), as has been observed experimentally [1–3,25]. For a single Co/Cu interface and uniform mode at 50 GHz the condition (15) is met at current densities of the order of 10^8 – 10^9 A/cm² [1,21].

Both forms of instabilities can lead to a spin current induced magnetization reversal [3,4,6]. The distinct way in how the reversal occurs largely depends on the geometry of the experimental setup and will be treated elsewhere. In this respect, Eq. (14) is universal for multilayer systems in a steady state current since the nonequilibrium magnetization can be obtained independently from transport calculations. This allows one to treat the current flow in the ballistic, diffusive, or even tunneling regime by using another set of transport equations than (10), yet Eq. (14) governing the magnetization dynamics stays the same. For systems where the current flow can no longer be approximated by the steady state, which is generally the case above the threshold (15), one has to study the more complex problem of solving simultaneously Eqs. (1) and (10) or an equivalent formulation of the transport equations (10).

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