Charge Excitations in Easy-Axis and Easy-Plane Quantum Hall Ferromagnets

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(Received 21 June 2001; published 17 October 2001)

We study charge excitations in quantum Hall ferromagnets realized in a symmetric quantum well. Landau levels (LLs) with different subband and orbital indices crossing at the Fermi level act as up and down pseudospin levels. The activation energy measured as a function of the pseudospin Zeeman energy, Δ_Z, reveals easy-plane and easy-axis ferromagnetism for LL filling of $\nu = 3$ and 4, respectively, for which the crossing levels have parallel and antiparallel spin. For $\nu = 4$, we observe a sharp reduction in the gap for $\Delta_Z \to 0$, which we discuss in terms of a topological excitation in domain walls akin to Skyrmions.

DOI: 10.1103/PhysRevLett.87.196801 PACS numbers: 73.43.Lp, 71.70.Gm, 73.21.–b

Quantum Hall (QH) ferromagnetism provides a physically intuitive way to interpret the integer as well as fractional QH effects as manifestations of strong electron interactions in two dimensions. The QH system is thereby regarded as a novel quantum fluid in which ferromagnetism and incompressibility are linked by the density commensurability between electrons and flux quanta. This is best illustrated in the QH system at Landau level (LL) filling factor $\nu = 1$, where the lowest LL is fully occupied and the spins are spontaneously polarized. The ferromagnetic order in the ground state imposes a large exchange energy penalty for a spin flip, thereby producing a gap for charged excitations. A further unique feature of QH ferromagnets lies in the fact that a topological defect in the spin field couples to the charge degree of freedom, and allows for a nontrivial charged excitation which costs a lower energy than a single spin flip. A well-known example is a Skyrmion for $\nu = 1$ [1], which is a topological soliton in isotropic Heisenberg spin systems. When the Coulomb energy gain overcomes the Zeeman energy penalty, the addition or removal of an electron to or from $\nu = 1$ results in a spin texture around the quasiparticle accompanied by many spin flips. Rapid loss of the spin polarization away from $\nu = 1$ provides evidence for Skyrmions [2]. In transport, Skyrmions manifest themselves in an abrupt decrease in the activation energy upon reducing the Zeeman energy [3,4].

Pseudospin QH ferromagnets, in which two energetically close LLs with different orbital states play the role of up and down pseudospin levels, provide even richer physics because of the pseudospin dependence of Coulomb interactions. Depending on the orbital states and the spins of the LLs involved, the system can exhibit easy-axis or easy-plane anisotropy, and accordingly behaves similar to an Ising or *XY* ferromagnet [5,6]. Thus far, pseudospin QH ferromagnets have been studied in various systems including single-layer [7,8], bilayer [9], and fractional systems [10]. Both easy-axis [7,8,10] and easy-plane [9] anisotropy have been demonstrated. However, detailed studies on the charge excitations in these systems are still lacking. Activation studies as a function of the single-particle gap are useful to identify the origin of the gap [3,4]. In particular, in the limit of zero single-particle gap, the ground state possesses a broken symmetry, whereby one can expect different types of topological defects, such as vortices or domain walls (DWs), to come into play.

In this Letter, we investigate the charge excitations in QH ferromagnets realized in a symmetric quantum well (QW). LLs with different Landau orbital indices originating from the first and second subbands serve as up and down pseudospin levels when they cross at the Fermi level. Different from usual tilted-field experiments [8], in which crossings occur only between LLs with opposite spins, our system allows for crossings between LLs with the same spins as well. Hence, different types of QH ferromagnets can be realized in one sample by choosing the single-particle levels to cross. We show that easy-plane and easy-axis ferromagnetism occurs for $\nu = 3$ and 4, respectively, for which the crossing levels have the same and opposite spins. Activation energy measurements reveal a striking difference between the easy-plane and easy-axis systems in the limit of zero single-particle gap.

The sample we study is a single-quantum-well (SQW) structure consisting of a nominally 40-nm thick GaAs QW and $Al_{0.3}Ga_{0.7}As$ barriers [11]. The low-temperature electron mobility is about 400 $m^2/V\,s$. Using front and back gates, we can tune the electron density n_s from $(0.5–4.0) \times 10^{15}$ m^{-2} while keeping the QW potential symmetric [12]. This enables us to investigate the activation energy as a function of the single-particle gap without perturbing the subband wave functions. Shubnikov-de Haas oscillations reveal that the energy separation of the first and second subbands, Δ_{SAS}, is 78 K for $n_s = 2.85 \times 10^{15}$ m^{-2}, and decreases with increasing n_s. The sample is cooled in a dilution refrigerator, ^{3}He ($T = 0.3–1.4$ K), and ^{4}He ($>$1.4 K) cryostats. Magnetotransport measurements are carried out using a standard ac lock-in technique with a current of 20 nA.

Figure 1(a) shows the calculated wave functions for the first and second subbands, which we refer to as symmetric (S) and antisymmetric (A) states. When a perpendicular magnetic field B is applied, two sets of LLs originate from

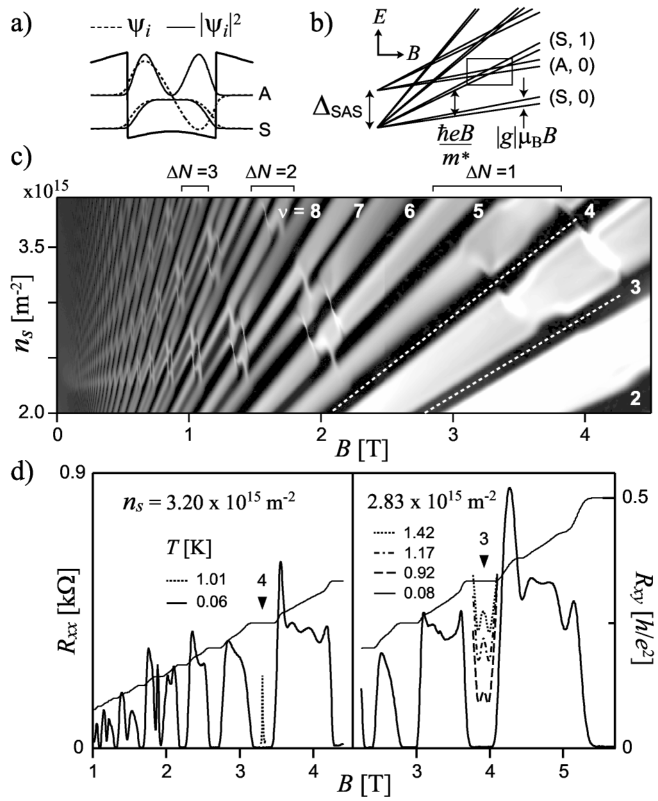

FIG. 1. (a) Calculated wave functions and charge density distributions for the SQW. (b) LL energy diagram. Level crossings at $\nu = 3$ and 4 are shown in the box. (c) Gray-scale plot of R_{xx} at 50 mK. Dark regions represent small values of R_{xx}. The overall shift of the crossings to lower fields with increasing n_s reflects the concomitant decrease in $\Delta_{\rm SAS}$. (d) R_{xx} and R_{xy} for $n_s = 3.20$ and 2.83×10^{15} m^{-2}.

the S and A subbands, which give rise to crossings between LLs with different Landau orbital indices. We label the single-particle levels (i, N, σ), where i $(= S, A)$, N, and σ $(=\uparrow, \downarrow)$ are the subband, Landau orbital, and spin indices, respectively. For LLs with N and $N' = N + \Delta N$, the crossing occurs when $(|\Delta N|\hbar e/m^* + \Delta\sigma|g|\mu_B)B = \Delta_{\rm SAS}$. Here, m^* and g are the effective mass and g factor of electrons, respectively, and other symbols have usual meanings. $\Delta\sigma = 0$ (± 1) identifies crossings between parallel (antiparallel) spins. When the Fermi level E_F lies between two adjacent single-particle levels, we take these levels as up and down pseudospins irrespective of their real spins. It can easily be seen that $\Delta\sigma = \pm 1$ and 0 hold for crossings at even and odd ν, respectively. When the crossing occurs at $B = B_c$, the energy difference between the up and down pseudospins is given by

$$\Delta_Z = \left[|\Delta N| \frac{\hbar e}{m^*} + \Delta\sigma |g| \mu_B - \left(\frac{\partial \Delta_{\rm SAS}}{\partial n_s} \right) \frac{\nu e}{h} \right] \times (B - B_c).$$

This acts as an effective Zeeman energy, or an external field in the Z direction, for the pseudospins. The last term in the square bracket is due to the n_s dependence of $\Delta_{\rm SAS}$. Although the Coulomb interactions shift the magnetic field positions of the crossings, the above expression for Δ_Z remains valid [13].

In Fig. 1(c), the magnetoresistance R_{xx} at 50 mK is shown in a gray scale as a function of B and n_s [14]. The data are obtained by simultaneously scanning the front and back-gate biases at each B so as to keep the QW potential symmetric. Two sets of Landau fans due to the S and A subbands are visible. The LL crossings enhance dissipation and appear as an increase in R_{xx}. The most striking observation here is that we do not see any increase in R_{xx} at fields where we expect the level crossings for $\nu = 3$ ($B = 3.90$ T) and $\nu = 4$ ($B = 3.31$ and 3.89 T). This is confirmed by the R_{xx} data shown in Fig. 1(d), for which n_s is adjusted so that the crossings occur at $\nu = 3$ (right) and $\nu = 4$ (left). At low temperatures, R_{xx} falls to zero and R_{xy} is quantized for both cases. When the temperature is increased, a peak shows up in R_{xx}, meaning the occurrence of the level crossings. The existence of the QH effect at complete level degeneracy is a hallmark of pseudospin ferromagnetism. Notice, however, the striking difference in the widths of the features appearing at high temperatures. The peak at $\nu = 4$, which is due to the crossing between $(S, 1, \downarrow)$ and $(A, 0, \uparrow)$, is found to be extremely sharp, in marked contrast to the broad feature at $\nu = 3$ associated with the crossing between $(S, 1, \uparrow)$ and $(A, 0, \uparrow)$. Similar even/odd asymmetry is also discernible in Fig. 1(c) for crossings at larger ν.

The sharp peak at $\nu = 4$ bears a striking resemblance to the resistance spikes recently observed in tilted-field experiments on AlAs QWs by Poortere *et al.* [8]. They explained the resistance spikes in terms of electron scattering at DWs separating regions of opposite magnetization. The domain formation results from the discrete symmetry and the long-range order specific to easy-axis systems. Consistent with their argument, we classify $\nu = 4$ in our system as an easy-axis ferromagnet. This is also confirmed in the data of Fig. 1(c). At $B = 3.3$ and 3.9 T, the $\nu = 4$ QH region shows kinks, where the center of the $\nu = 4$ region jumps vertically in gate voltage [14], indicating that the ground state changes almost discontinuously. At these points, the electrons are suddenly transferred from $(S, 1, \sigma)$ to $(A, 0, \sigma')$, or vice versa $(\sigma \neq \sigma')$. The two $\nu = 4$ phases are realized at different biases because the S and the A states have different charge distributions normal to the plane, and, hence, different capacitance couplings to the gates.

In turn, the smooth evolution of the $\nu = 3$ QH region around $B = 3.9$ T in Fig. 1(c) implies that the ground state changes continuously, where $(S, 1, \uparrow)$ and $(A, 0, \uparrow)$ are mixed. We stress that this mixing is not of a single-particle origin. Without interactions, the Hamiltonian is separable for the motions parallel and perpendicular to the plane, and allows no anticrossings between $(S, 1, \uparrow)$ and $(A, 0, \uparrow)$. The mixing of up and down pseudospin states in the presence of the external field Δ_Z implies easy-plane anisotropy. Here, the easy-plane anisotropy stems from the fact that the up and down pseudospin states have different charge distributions normal to the plane. Hence, the Hartree energy can be lowered more efficiently by mixing the up and down pseudospin states and tilting the magnetization toward the XY plane. Returning to $\nu = 4$, the up and down states have opposite spins. This makes the exchange energy highly pseudospin dependent, and leads to easy-axis anisotropy. As a result, the magnetization is aligned either up or down at the cost of Hartree energy.

As shown in Fig. 2(a), R_{xx} follows the activation behavior $R_{xx} \propto \exp(-\Delta_\nu/2T)$ even at the level degeneracy. We measure the activation energy, Δ_ν, at different magnetic fields with the filling factor fixed at $\nu = 3$ or 4, i.e., along the dotted lines in Fig. 1(c). The data compiled in Fig. 2(c) reveal a striking difference between $\nu = 3$ and 4. Δ_4 exhibits steep minima at $B = 3.31$ and 3.89 T, and this explains the observed sharp R_{xx} peak. By contrast, Δ_3 varies smoothly around the minimum. Away from the minima, Δ_ν increases almost linearly in both cases reflecting the single-particle gap $|\Delta_Z|$. The solid lines show the slopes of $|\Delta_Z|$ calculated using $m^* = 0.07m_e$, $|g| = 0.4$, and the measured n_s dependence of Δ_{SAS}. The excellent agreement with the data confirms that electron-hole excitations between the up and down pseudospin levels, shown as vertical arrows in Fig. 2(b), are responsible for the measured gaps in these regions [15].

In Fig. 3(a), we plot Δ_3 against Δ_Z, both normalized by the characteristic Coulomb energy $E_{\mathrm{C}} = e^2/4\pi\epsilon\ell_B$, where $\ell_B = \sqrt{\hbar/eB}$ and ϵ is the dielectric constant. The smooth variation of Δ_3 around $\Delta_Z = 0$ indicates the continuous evolution of the ground state as well as the quasiparticle. This rules out a first-order quantum phase transition [16] as the origin of the finite gap at $\Delta_Z = 0$, for which the ground state changes discontinuously, and the gap shows a cusp at $\Delta_Z = 0$ [17]. Instead, the gap here is due to pseudospin coherence, which causes an exchange energy penalty for a pseudospin flip. In our easy-plane system, the electrons condense in a superposition of $(S, 1, \uparrow)$ and $(A, 0, \uparrow)$ sharing the same phase $e^{i\varphi}$. Our system is interesting in that the phase angle φ can take an arbitrary value [18], so that one can expect coherence and a Goldstone mode [9] associated with the breaking of continuous symmetry. In an idealized model [6], the Hartree-Fock (HF) quasiparticle gap of an easy-plane QH ferromagnet is constant for small values of $|\Delta_Z|$, and starts to increase linearly when the magnetization is fully aligned and the coherence associated with the phase $e^{i\varphi}$ is quenched. In Fig. 3(a), Δ_3 starts to increase linearly for $|\Delta_Z|/E_{\mathrm{C}} > 0.01$, indicating that the magnetization is almost fully aligned and the quasiparticle becomes a simple electron-hole excitation between the single-particle levels. In turn, we can say that the coherence exists for $|\Delta_Z|/E_{\mathrm{C}} \leq 0.01$.

Now we discuss the origin of the sharp gap reduction for $\nu = 4$. As shown in Fig. 3(b), this reduction occurs for $|\Delta_Z|/E_{\mathrm{C}} \leq 0.005$ and reaches (60–70)% that of the extrapolated value at $\Delta_Z = 0$. Interestingly, the data are reminiscent of the $\nu = 1$ Skyrmion data for $g \to 0$ [4]. If we take $\partial\Delta_4/\partial|\Delta_Z|$ near $\Delta_Z = 0$ as a measure of the number of pseudospin flips, we find the number to be 30–50.

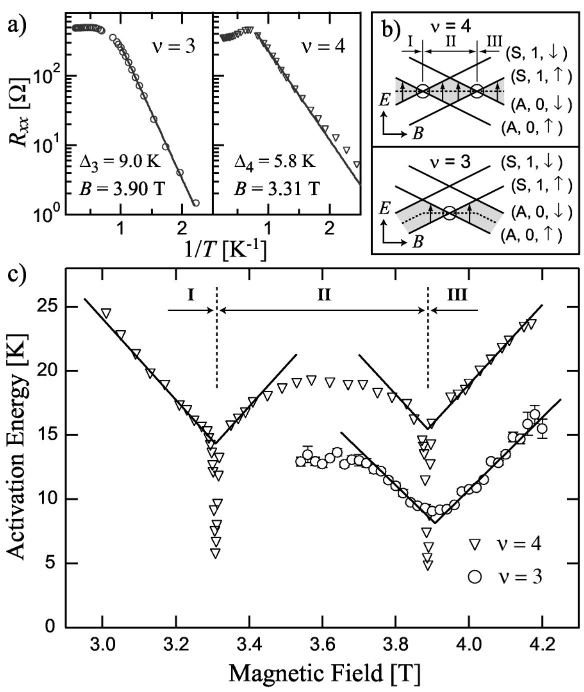

FIG. 2. (a) R_{xx} vs $1/T$ measured for $\Delta_Z = 0$. (b) Energy level diagram near the crossings for $\nu = 3$ and 4. The dotted lines represent traces of E_F for each ν. (c) Activation energies measured as a function of B.

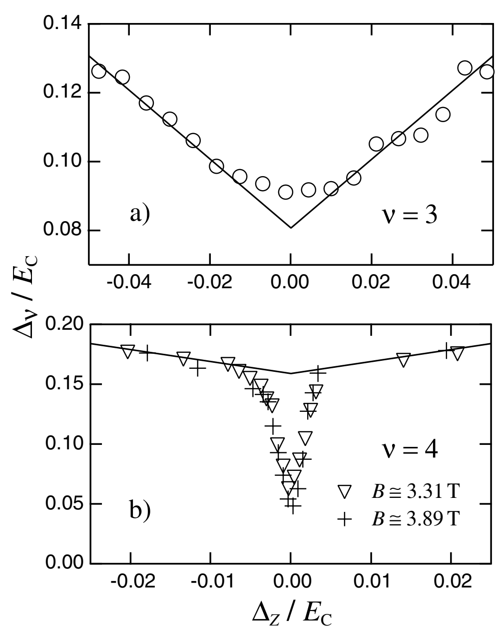

FIG. 3. Activation energy Δ_ν as a function of pseudospin Zeeman energy Δ_Z, both normalized by the characteristic Coulomb energy $E_{\mathrm{C}} = e^2/4\pi\epsilon\ell_B$. (a) $\nu = 3$; (b) $\nu = 4$.

The reduction here, however, is larger than the 50% expected for ideal 2D Skyrmions [1]. HF calculations for a similar easy-axis system show that a Skyrmion costs a larger energy than a single pseudospin flip [19]. To account for the resistance spikes in AlAs QWs, Jungwirth and MacDonald have calculated the energy of a HF quasiparticle, or a pseudospin flip, inside DWs [20]. They find that the energy is lowered to about half that of the bulk. This is plausible, since at the center of a DW the exchange contributions from regions with opposite magnetization cancel each other. We speculate that the energy may be reduced further by allowing the pseudospin to rotate smoothly around the quasiparticle in the DW. This leads to a pseudospin rotation not only *across* the DW by an angle π, but also *along* the DW by 2π in the XY plane. The resultant structure is equivalent to the topological defect discussed for a domain structure at $\nu = 1$ defined by an inhomogeneous g factor [21]. As pointed out in Ref. [21], the structure can be regarded as a Skyrmion trapped at a DW. In the limit of long DWs and in the absence of spin-orbit interactions, such a mode is predicted to become gapless [22]. In real systems, the activation energy is affected by the finite length of the DWs and weak spin-orbit interactions, and the temperature dependence of the DW length. We note, however, that the above arguments are not compelling and therefore further investigations are necessary for a more detailed understanding [23].

We also note that our system does not show clear hysteretic behavior in the magnetotransport, indicating that the DWs can move rather easily as generally expected in a clean system. Yet, we do not know how the initial nucleation takes place. A possible clue may be that the two phases have slightly different electron densities for the same bias. Hence, at least one of the two phases contains a substantial density of quasiparticles or quasiholes, which can be the source of minority domains.

Finally, we mention the behavior at high temperatures. For $\nu = 3$, R_{xx} becomes constant for $\Delta_Z = 0$ at $T > 1.9$ K [Fig. 2(a)], which is considerably lower than the measured gap of $\Delta_3 = 9.0$ K. This implies that the pseudospin correlation responsible for the gap vanishes at a finite T. Similar data have been reported for a $\nu = 1$ bilayer system and discussed in conjunction with the Kosterlitz-Thouless transition associated with the unbinding of vortices [24]. As seen in Fig. 2(a), $\nu = 4$ exhibits an even stronger anomaly. R_{xx} first increases with T and then starts to decrease above 1.5 K. This suggests a finite-temperature phase transition specific to Ising spin systems. These subjects exceed the scope of this work and deserve further investigations.

The authors are grateful to Z. F. Ezawa, A. Sawada, Y. Tokura, V. I. Fal'ko, T. Jungwirth, and A. H. MacDonald for useful discussions. Part of this work was supported by NEDO international joint research, NTDP-98.

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[14] The vertical axis for Fig. 1(c) is calculated from the Hall voltage at low B and is proportional to the gate voltages.
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