Is the Particle Current a Relevant Feature in Driven Lattice Gases?

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By performing extensive Monte Carlo simulations we show that the infinitely fast driven lattice gas (IDLG) shares its critical properties with the randomly driven lattice gas (RDLG). All the measured exponents, scaling functions, and amplitudes are the same in both cases. This strongly supports the idea that the main relevant nonequilibrium effect in driven lattice gases is the anisotropy (present in both IDLG and RDLG) and not the particle current (present only in the IDLG). This result, at odds with the predictions from the standard theory for the IDLG, supports a recently proposed alternative theory. The case of finite driving fields is also briefly discussed.

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The Ising model exhibits a prototypical equilibrium phase transition, and the associated ϕ^4 Ginzburg-Landau theory is a paradigm of continuous theory for equilibrium critical phenomena [1,2]. However, thermodynamic equilibrium is exceptional in nature where stationary states are typically away from equilibrium [3]. With the purpose of defining simple lattice models describing generic *nonequilibrium* phase transitions, different trials have been made in the last two decades. Among them, perhaps the most intriguing example is the driven lattice gas (DLG) [3–5]. (Other interesting examples are the directed percolation model [3] and the Kardar-Parisi-Zhang equation.) The DLG, being a straightforward extension of the Ising model, has, in fact, become a workbench for emergent nonequilibrium theories and field theoretical approaches.

The DLG is a d-dimensional kinetic Ising model with conserved dynamics, in which transitions in the direction (against the direction) of an externally applied field, $\vec{E}$, are favored (unfavored) [3–5], while transitions perpendicular to the field are unaffected by it. The field induces two main nonequilibrium effects: (i) the presence of a net current of particles along its direction, and (ii) *strongly* anisotropic configurations [6]. At high temperatures, the system is in a disordered phase while, for half-filled lattices (the only case we refer to in what follows) there is a second-order critical point, below which the DLG segregates into (two) high and low density aligned-with-the-field stripes. Establishing unambiguously the DLG universality class is an important issue in the way to rationalize the behavior of nonequilibrium systems.

Continuous approaches such as Langevin and associated field theories [2] have been most useful in studying universality issues in equilibrium critical phenomena. In particular, coarse-grained approaches combined with renormalization group (RG) techniques provide a method for the classification of the different possible terms (operators) as relevant, irrelevant, or marginal. In fact, Langevin equations are more illuminating than other (even more rigorous) approaches, as they permit one to understand systematically how possible perturbations or model variations would affect critical properties. Consequently, many studies have focused on the DLG and its universality by using both nonequilibrium continuous approaches and computer simulations (unfortunately, general exact solutions are not available). Within this perspective, it is somewhat deceptive that after many computer and analytical studies, the universality class of the DLG remains a debated issue [3,5,7,8].

A phenomenological Langevin equation intended to capture the relevant physics of the DLG at criticality was proposed and renormalized more than a decade ago [9]. This equation, referred to as *driven diffusive system* (DDS), is a natural extension of the conserved ϕ^4 theory for the Ising equilibrium transition (model B [1]) and seems to capture the main symmetries and conservation laws of the discrete DLG. It includes a particle current term (which from naive power counting turns out to be the most relevant nonlinearity) as well as anisotropic coefficients. It certainly is a suitable and very reasonable candidate to be *the canonical coarse-grained model representative of the DLG universality class.* The DDS Langevin equation reads

$$\partial_t \phi(\mathbf{r},t) = \tau_\perp \nabla_\perp^2 \phi - \nabla_\perp^4 \phi + \frac{\lambda}{6} \nabla_\perp^2 \phi^3 + \tau_\parallel \nabla_\parallel^2 \phi - \alpha \nabla_\parallel \phi^2 + \zeta(\mathbf{r},t), \qquad (1)$$

where ϕ is the coarse-grained field, ζ is a conserved Gaussian noise, and the cubic term, being a dangerously irrelevant variable [2], is kept in order to ensure stability [9]. $\tau_\parallel$, $\tau_\perp$, λ, and α are model parameters. The most emblematic (exact) prediction derived from the DDS RG analysis, namely, the mean field behavior of the order parameter critical exponent, $\beta = 1/2$ [9], has eluded a large number of Monte Carlo (MC) analysis aimed at probing it [12], however. In particular, systematic deviations from the predicted scaling are observed both in $d = 2$ [3,10] and in $d = 3$ [11,12]. Indeed, different MC analysis

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(performed using a variety of aspect ratios and order parameters) lead systematically to a value of β close to ≈ 0.3 (in $d = 2$), with error bars excluding apparently the value $\beta = 1/2$ (see [3] for a critical review of simulation analysis). This is a main indication that, strikingly enough, *the DDS equation does not describe properly the infinitely fast driven DLG (IDLG) critical properties.* Moreover, there are some other hints suggesting strongly that the differences between the predictions of the standard Langevin approach and MC results are more fundamental than a simple discrepancy in the value of β. In particular, the intuition developed from MC simulations of the DLG and variants of it [3] suggests that, contrary to what the DDS equation establishes, *it is the anisotropy and not the presence of a current that is the basic ingredient controlling the critical behavior* [6]. For instance, in a modified DLG in which anisotropy is included by means other than a current [13], the scaling behavior at criticality remains unaltered upon switching on (infinite) driving (see [3,13]). Other compelling evidence supporting this hypothesis can be found in [3,14].

In an attempt to clarify this puzzling situation, and reconcile continuous approaches with numerics, different scenarios have been explored. In particular, an alternative route to build up Langevin equations starting from generic microscopic master equations was recently proposed [7]. By applying this approach to the DLG, one observes that, owing to a transition-rate saturation effect, the coefficient α of the nonlinear current term, $\nabla_\parallel \phi^2$, vanishes in the limit of infinite driving fields and, therefore, it does not appear in the final Langevin equation nor is it generated perturbatively [7]. The resulting theory [alternative to Eq. (1)] is

$$\partial_t \phi(\mathbf{r}, t) = \tau_\perp \nabla_\perp^2 \phi - \nabla_\perp^4 \phi + \frac{\lambda}{6} \nabla_\perp^2 \phi^3 + \tau_\parallel \nabla_\parallel^2 \phi + \zeta \qquad (2)$$

plus higher order irrelevant contributions (note that a linear current term has been eliminated by employing a Galilean transformation [5,7]). This equation, named below *anisotropic diffusive system* (ADS), is a well known one: it coincides with the Langevin equation representing the random DLG (RDLG) [15,16] (for which the driving field takes values ∞ and $-\infty$ in a random unbiased fashion, generating anisotropy but not an overall current). This theory has been extensively studied in [15,16]; its critical dimension is $d_c = 3$ (instead of $d_c = 5$ for the DDS), and the critical exponents and finite size scaling (FSS) properties are now well known. Other systems in this universality class are the two-temperature model [17] and the anisotropic lattice gas automaton model [13]. This theory for the IDLG includes anisotropy as its basic nonequilibrium ingredient. Instead—for nonsaturating, finite, driving fields—the cancellation of the nonlinear current term does not occur, and our method recovers the standard DDS equation.

Aiming at further clarifying these issues, we report here on extensive MC simulations of the IDLG and the RDLG in $d = 2$. The main objectives are the following: (i) trying to conclude whether the IDLG and the RDLG share the same critical behavior or not; and (ii) measuring the critical exponents by performing systematic anisotropic finite-size scaling (FSS). In fact, we perform FSS analysis for both the IDLG and the RDLG by following the anisotropic FSS scheme proposed in [15] consistent with the ADS theory; this allows us to analyze systematically possible scaling differences between both models. We also report on the case of finite-driving DLG.

We consider rectangular lattices of size $L_\parallel \times L_\perp$ with periodic boundary conditions and random sequential updating [3,5]; the external field $\vec{E}$ acts in the x (parallel) direction. Particles jump to a randomly chosen nearest neighbor site (provided that it is empty) with probability: $\min(1, \exp[-\beta(\Delta H + E\Delta j)])$, where ΔH is the energy (Ising Hamiltonian) variation, and $\Delta j = (-1, 0, 1)$ for jumps along, against, and orthogonal to the direction of the field, respectively. Following [10,16] the order parameter is chosen as the structure factor $S(0, 2\pi/L_\perp)$. In order to perform a systematic anisotropic FSS we considered system sizes 20×20, 45×30, 80×40, and 125×50. These aspect ratios satisfy $L_\parallel^{\nu_\perp/\nu_\parallel} = 0.2236 \times L_\perp$, where $\nu_\perp/\nu_\parallel \approx 1/2$ consistent with ADS anisotropic spatial scaling [10,16]. The number of MC steps considered varied between 1.8×10^8 and 2.4×10^8, much larger than in any previously reported MC simulations. The total CPU time employed is about eight months in a PentiumIII 400 MHz machine. The critical temperature is determined by using the fourth (Binder) cumulant method [18]. For the IDLG, the critical temperature is found to be $T_c^I = 1.396(4) T_\mathrm{O}$ (T_O is the Onsager temperature), slightly below previously reported values [3,5], while we find $T_c^R = 1.390(4) T_\mathrm{O}$ for the RDLG (see insets of Fig. 3). These critical values were employed for the FSS analysis. In Fig. 1 we plot the order parameter, rescaled by a factor $L_\parallel^{\beta/\nu_\parallel}$, versus $\epsilon L_\parallel^{1/\nu_\parallel}$, where ϵ is the distance to the critical point, for different system sizes $L_\parallel$. A nearly perfect data

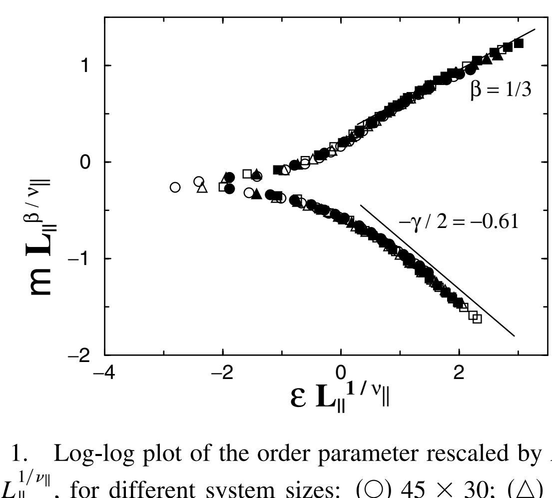

FIG. 1. Log-log plot of the order parameter rescaled by $L_\parallel^{\beta/\nu_\parallel}$ vs $\epsilon L_\parallel^{1/\nu_\parallel}$, for different system sizes: (○) 45×30; (△) 80×40; (□) 125×50. Filled (empty) symbols stand for the RDLG (IDLG); error bars are smaller than the symbols.

collapse is obtained by fixing $\nu_\parallel = 1.25$ and $\beta = 0.33$. The collapse gets worse upon slightly changing these values; more precise estimates of the associated error bars is a difficult and not essential issue in this context. Notice that we are plotting in the same graph data for the IDLG and for the RDLG, implying that the FSS scaling function is precisely the same for both models. Furthermore, the slopes of the asymptotic branches are approximately $1/3$ and -0.61, consistent both with the order parameter exponent being $\beta \approx 0.33$, and $\gamma \approx 1.22$ (see below) [19]. In general, even when the scaling functions are universal, their corresponding amplitudes are not expected to be so. For this reason, usually one has to introduce the so-called *metric factors* (varying amplitudes) [20] in order to obtain superposition of scaling functions within the same universality class. Contrary to this expectation, the magnetization scaling functions for IDLG and RDLG overlap perfectly. Therefore, it comes as a surprise that not only are the scaling functions and the β exponent universal in both models, but even the amplitudes coincide. A similar situation has recently been reported for a different type of anisotropic FSS [21].

We have also computed the system susceptibilities, defined as the relative fluctuations of the order parameter: $\chi = \frac{L_\parallel}{\sin(\pi/L_\perp)}[\langle m^2\rangle - \langle m\rangle^2]$. In Fig. 2, we plot the susceptibility times $L_\parallel^{-\gamma/\nu_\parallel}$ as a function of the rescaled distance to the critical point, $\epsilon L_\parallel^{1/\nu_\parallel}$. The best data collapse is obtained by employing the values $\gamma = 1.22$ and $\nu_\parallel = 1.25$ for both models with, again, coinciding amplitudes. It should be stressed that this is the first time a really good collapse is observed below the critical point for anisotropic scaling of the IDLG. Plotting the dimensionless Binder cumulant as a function of the rescaled distance to the critical point with $\nu_\parallel = 1.25$, again, nearly perfect data collapse is obtained for both models and all system sizes (Fig. 3). We also performed simulations in square lattices (128×128) as in some previous studies [3,14]. Monitoring $m^{1/\beta}$ as a function of T/T_O we see no appreciable systematic difference between the curves for IDLG and RDLG, which have the same slope within numerical accuracy. The best linear fit correlation is obtained for $\beta \approx 0.33$ in both cases, providing an extra consistency check for our results. Moreover, eye inspection of IDLG and RDLG configurations, for any geometry, at a fixed relative temperature, does not permit one to distinguish one from the other. In particular, the interfacial properties look identical. Let us also stress that all the obtained exponent values are compatible with previous measures for the RDLG, as well as with the exponents obtained within an ϵ expansion of the ADS theory [15,16].

In conclusion, *MC results support strongly that both the IDLG and the RDLG belong in the same universality class, and share not only critical exponents and scaling functions, but also the scaling amplitudes.* This universality class is described by the ADS equation, Eq. (2). There is absolutely no hint of any difference in the asymptotic behavior between the model with a current (IDLG) with respect to the current-less one (RDLG). *All the numerical evidence confirms that it is the anisotropy and not the net current (if any) the most relevant nonequilibrium ingredient of driven systems.* As discussed in the introduction, this is striking from a field theoretical perspective given that the nonlinear current term, $\nabla_\parallel \phi^2$, is naively a relevant perturbation at the ADS fixed point. In an alternative approach, the coefficient of such a term vanishes. In this picture, the fast drive limit corresponds to a sort of *multicritical point* in which an *a priori* relevant operator is absent due to a cancellation of its coefficient and, consequently, the usual "up-down" Ising symmetry (i.e., the three-point correlation functions vanish) is restored at criticality. In any case, it should be stressed that, from a more general perspective, field theoretical descriptions of nonequilibrium systems are much more delicate and subtle than their equilibrium counterparts, and an extremely careful inspection of the system symmetries, conservation laws, and dynamical features is required before venturing

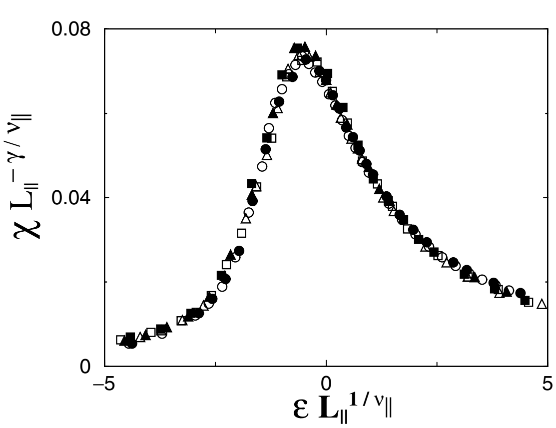

FIG. 2. Log-log plot of the susceptibility rescaled by $L_\parallel^{-\gamma/\nu_\parallel}$ vs $\epsilon L_\parallel^{1/\nu_\parallel}$. Symbols are as in Fig. 1 (larger than error bars).

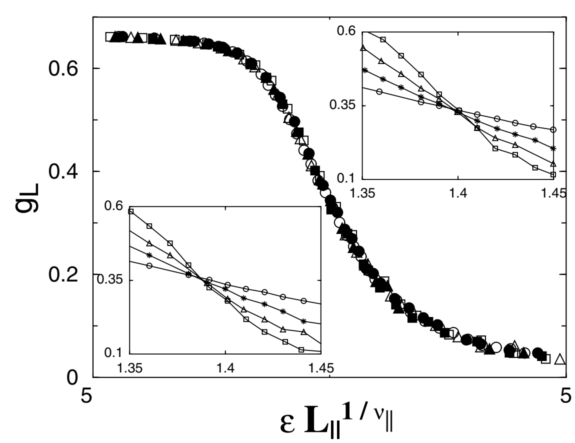

FIG. 3. Scaling plot of the fourth cumulant vs $\epsilon L_\parallel^{1/\nu_\parallel}$. Upper (lower) inset: fourth cumulant for the IDLG (RDLG) vs T/T_O. Symbols are as in Fig. 1.

to make predictions on nonequilibrium universality issues. For example, fermionic and bosonic nonequilibrium systems with the same dynamics, symmetries, and conservation laws have recently been reported to belong to different universality classes [22]. One could wonder whether the DLG hard-core interaction should be taken into account in its Langevin representation.

Elucidating the critical behavior for finite E remains a challenging and interesting objective. Both our alternative Langevin-building approach and the standard one, Eq. (1), include a relevant current term in this case and, consequently, predict $\beta = 1/2$. Obtaining clear-cut results in this case is a computationally expensive task for the following reasons: (i) As the external field appears in the argument of an exponential, even relatively small values of E generate situations close to saturation, and strong crossover effects could hide the true asymptotic regime. (ii) If the field value is taken too small, crossovers from the equilibrium regime may also burden observations. A possible scenario that could follow from MC analysis is that finite fields show mean field behavior; that would be a strong backing for our theory [7] that predicts the finite and the infinite driving cases to be qualitatively different. If, instead, scaling happens to be that of the ADS (as our preliminary MC results for $E = 3$ and $E = 0.5$ seem to indicate; for $E = 0.25$ results do not quite fit this indication), it would prove that it is for any arbitrary value of the driving field that anisotropy is the most relevant ingredient of driven systems. This scenario would uncover a new puzzling situation and would certainly call for deeper theoretical understanding. Huge and careful simulations would be required to extract neat conclusions overcoming difficulties (i) and (ii) above.

In summary, we have performed extensive MC simulations of the IDLG and the RDLG. By using anisotropic finite size scaling techniques we have shown that both models belong to the same universality class: their critical exponents, scaling functions, and amplitudes are undistinguishable and coincide with those of the ADS equation. This result supports the conclusion that it is the presence of anisotropic coefficients, and not the particle current, the most relevant ingredient in these nonequilibrium driven problems (at least in the fast drive limit). Further theoretical efforts are certainly required in order to (i) sort out if our alternative Langevin approach is correct and what are its possible limitations, and (ii) further clarify the universality issues of this quintessential nonequilibrium problem. Finally, it would also be very interesting to combine the powerful finite size methods recently introduced in this context by Caracciolo *et al.* in a nice recent work [23] with our alternative theory to verify if they lead to better data collapse than when used to test the standard DDS equation (hopefully without having to introduce strong corrections to scaling and providing good order-parameter scaling).

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[1] P. C. Hohenberg and B. I. Halperin, Rev. Mod. Phys. **49**, 435 (1977).
[2] J. Zinn-Justin, *Quantum Field Theory and Critical Phenomena* (Clarendon Press, Oxford, 1994).
[3] J. Marro and R. Dickman, *Nonequilibrium Phase Transitions in Lattice Models* (Cambridge University Press, Cambridge, U.K., 1999).
[4] S. Katz, J. L. Lebowitz, and H. Spohn, Phys. Rev. B **28**, 1655 (1983); J. Stat. Phys. **34**, 497 (1984).
[5] B. Schmittmann and R. K. P. Zia, *Statistical Mechanics of Driven Diffusive Systems,* in Phase Transitions and Critical Phenomena, Vol. 17, edited by C. Domb and J. Lebowitz (Academic, London, 1995).
[6] Anisotropy can also appear in equilibrium systems such as the Ising model with anisotropic couplings; R. Baxter, *Exactly Solved Models in Statistical Mechanics* (Academic Press, London, 1982). The term *weak anisotropy,* as opposed to *strong anisotropy,* applies to these cases, for which, owing to the fluctuation-dissipation relation, the equilibrium critical properties remain unaltered [5].
[7] P. L. Garrido, M. A. Muñoz, and F. de los Santos, Phys. Rev. E **61**, R4683 (2000).
[8] B. Schmittmann *et al.,* Phys. Rev. **61**, 5977 (2000).
[9] H. K. Janssen and B. Schmittmann, Z. Phys. B **64**, 503 (1986); K.-t. Leung and J. L. Cardy, J. Stat. Phys. **44**, 567 (1986); **45**, 1087(E) (1986).
[10] K.-t. Leung, Phys. Rev. Lett. **66**, 453 (1991); J. S. Wang, J. Stat. Phys. **82**, 1409 (1996).
[11] K.-t. Leung *et al.,* Int. J. Mod. Phys. C **10**, 853 (1999).
[12] Other predictions of the standard theory seem to be reasonably well verified by performing an adequate finite size scaling analysis including a dangerously irrelevant scaling field (see [10] and [11]), but a fully consistent scaling, including good data collapse for the order parameter has never been observed. See the criticisms to [10] and [11] in [3,13,14]. See also [23].
[13] J. Marro and A. Achahbar, J. Stat. Phys. **90**, 817 (1998).
[14] J. Marro *et al.,* Phys. Rev. **53**, 6038 (1996).
[15] B. Schmittmann and R. K. P. Zia, Phys. Rev. Lett. **66**, 357 (1991); B. Schmittmann, Europhys. Lett. **24**, 109 (1993). See also [5], Sec. 6.1.
[16] E. Praestgaard *et al.,* Europhys. Lett. **25**, 447 (1994). See also E. Praestgaard *et al.,* cond-mat/0010053.
[17] P. L. Garrido, J. L. Lebowitz, C. Maes, and H. Spohn, Phys. Rev. A **42**, 1954 (1990).
[18] K. Binder, Z. Phys. B **43**, 119 (1981).
[19] The scaling function $f(x)$ needs to obey $f(x) \sim x^{\beta}$ $[f(x) \sim x^{-(\gamma/2)}]$ for large x in the supercritical (subcritical) branch.
[20] V. Privman *et al., Phase Transitions and Critical Phenomena,* edited by C. Domb and J. L. Lebowitz (Academic, London, 1991), Vol. 14.
[21] M. Henkel and U. Schollwöck, cond-mat/0010061.
[22] S. Kwon *et al.,* Phys. Rev. Lett. **85**, 1682 (2000).
[23] S. Caracciolo *et al.,* e-print cond-mat/0106221, 2001.