Merging of Plasma Currents

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The merging process of current filaments in a strongly magnetized plasma is described. The evolution is calculated using a contour dynamics method, which accurately tracks piecewise constant distributions of the conserved quantities. In the interaction of two screened currents, both develop dipolar vortical flows, bringing the currents together. This is the manifestation of the Lorentz force between aligned currents. Currents will merge into single filaments. Reconnection of the magnetic field takes place, converting the magnetic topology from a figure eight to a circle.

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Turbulent ideal fluids in two dimensions are often observed to develop strongly coherent, localized structures. These vortices interact and move in the global flow field in a particlelike fashion, except for infrequent encounters when their internal structure plays a role in the interaction.

In the case of a hot and strongly magnetized plasma these structures may carry both fluid vorticity and current density. The plasma motion conserves combinations of magnetic flux, current density, and fluid vorticity. These fields are called *generalized vorticities* and are pointwise conserved by the dynamics of the system. They are direct analogs to the vorticity in incompressible hydrodynamics.

In this Letter the interaction of pure current distributions is studied. These currents are parallel to the dominant magnetic field and are represented by overlapping distributions of the generalized vorticities. Two like-signed adjacent currents attract each other by the Lorentz force and thus tend to coalesce. During this merging process the figure-eight topology of the magnetic field is converted to a circle. A contour dynamics method is used to study the merging process. It specifically uses the pointwise conservation of the generalized vorticities.

The merging of currents in a plasma is, for example, encountered in the study of the coalescence instability of magnetic islands. Reference [1] (and the references therein) discusses this instability in the magnetohydrodynamic framework. Instead of resistivity the mechanism for field line reconnection in the present case is collisionless reconnection, an effect of the finite electron inertia along the magnetic field. On scales of the electron inertial skin depth d_e, reconnection of the perpendicular field may take place. The collisionless reconnection process for the coalescence of plasma currents is discussed in Ref. [2].

We describe the plasma by an electron and an ion fluid in a strong magnetic field in the z direction. The plasma pressure is assumed to be small so that the magnetic and electric fields can be described by

$$\boldsymbol{B} = B_0\boldsymbol{e}_z + \boldsymbol{e}_z \times \nabla\psi, \qquad \boldsymbol{E} = -\nabla\varphi + \frac{\partial\psi}{\partial t}\boldsymbol{e}_z, \tag{1}$$

where $-\psi$ is the z component of the magnetic vector potential and φ is the electrostatic potential.

The electrons are described as a collisionless fluid with a homogeneous temperature. The inertia of the electrons, in the direction along the dominant magnetic field, as well as the Hall effect are taken into account. The parallel (along B_0) current is carried by the electrons only. The ion response is derived in the cold ion approximation and is coupled to the electron fluid by the quasineutrality condition. Here we focus on Alfvén phenomena and neglect electrostatic drift waves and vortices. Then, the particle density n is related to the electrostatic potential via $\tilde{n}/n_0 \approx \ln n/n_0 = \nabla^2\varphi$, where n_0 is a constant equilibrium density. The density perturbation $\tilde{n}$ is assumed to be small, but its gradient is allowed to be large [3].

The model is made (quasi) two dimensional by assuming that $\partial_z = 0$; the system is homogeneous in the z direction. There are two important length scales in the system: the ion sound gyroradius ρ_s and the skin depth d_e due to the parallel electron inertia. We normalize all lengths to ρ_s, and time is measured in units of the ion gyroperiod.

The main contribution to the dynamics is the $\boldsymbol{E} \times \boldsymbol{B} \sim \boldsymbol{e}_z \times \nabla\varphi$ flow. We start from the continuity equation and the momentum balance for the electrons, and assume that their temperature is uniform. When dissipation (resistivity, viscosity) is neglected, the dynamics are given by the advection of two conserved generalized vorticities $\omega_+(x, y)$ and $\omega_-(x, y)$, each by its own velocity field, $\boldsymbol{v}_+$ and $\boldsymbol{v}_-$, respectively [4],

$$\frac{\partial\omega_\pm}{\partial t} + \boldsymbol{v}_\pm \cdot \nabla\omega_\pm = 0\,. \tag{2}$$

The vorticities are given by

$$2d_e\omega_+ = +(\psi - d_e^2 J) + d_e\nabla^2\varphi\,, \tag{3a}$$

$$2d_e\omega_- = -(\psi - d_e^2 J) + d_e\nabla^2\varphi\,, \tag{3b}$$

where $J = \nabla^2\psi$ is the parallel current density carried by the electrons. The velocity fields are incompressible, $\boldsymbol{v}_\pm = \boldsymbol{e}_z \times \nabla\phi_\pm$, with stream functions

$$\phi_\pm = \varphi + c_\pm\psi\,, \tag{4}$$

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where $c_\pm = \pm d_e^{-1}$. The relation between potentials and vorticities is given by

$$\nabla^2 \varphi = \omega_+ + \omega_-, \tag{5}$$

$$(\nabla^2 - d_e^{-2})\psi = -(c_+\omega_+ + c_-\omega_-). \tag{6}$$

This still allows for background fields which satisfy the equations $\nabla^2\varphi = \text{const}$ and $(\nabla^2 - d_e^{-2})\psi = \text{const}$. These change the streaming potentials to $\phi_\pm + \phi_{\pm,\text{ext}}$ but leave the dynamical equations (2) in their Lagrangian form. In this paper we consider only fields which are generated by the (localized) vorticity distributions themselves, i.e., we assume the external fields $\phi_{\pm,\text{ext}}$ to be zero.

To study the interaction of localized structures we solve Eqs. (2) using the method of contour dynamics (CD). This method was originally developed to study vortex interactions in ideal hydrodynamics [5–7]. We have adapted the method for the plasma model. It is based on the fact that the evolution of a piecewise constant distribution of $\omega_\pm$ is completely determined by the evolution of the boundaries of the patches of constant generalized vorticity. This can be seen when we invert Eqs. (5) and (6), and then apply Stokes' theorem to find an expression for the velocity fields:

$$\boldsymbol{v}_\alpha(\boldsymbol{r}) = -\sum_{\beta=\pm}\sum_{m=1}^{M_\beta}\omega_{\beta,m}\oint_{\gamma_{\beta,m}} G_{\alpha\beta}(|\boldsymbol{r} - \boldsymbol{r}'|)\,d\boldsymbol{l}', \tag{7}$$

where $\alpha = \pm$, and the summation runs over all M_β contours of each vorticity type. The jump in the generalized vorticity crossing the contour $\gamma_{\beta,m}$ inward is indicated by $\omega_{\beta,m}$, and the Greens function for an unbounded domain is given by

$$G_{\alpha\beta}(r) = \frac{1}{2\pi}[\ln r + c_\alpha c_\beta K_0(r/d_e)], \tag{8}$$

where K_0 is the zeroth order modified Bessel function. The Greens function has a mixed character; it is a combination of a logarithmic term, known from incompressible hydrodynamics, and a Bessel term which is also encountered for the Charney-Hasegawa-Mima equation which describes both quasigeostrophic flows and drift vortices in plasmas. When $c_\alpha c_\beta > 1$ the Greens function has a minimum, changing from $G \sim (1 - c_\alpha c_\beta)\ln r \propto -\ln r$ for $r \ll d_e$ to $G \sim \ln r$ for large distances. This extremum in $G_{\alpha\beta}$ corresponds to a change in direction in the azimuthal velocity field (7) generated by an element of generalized vorticity. This new length scale in the interaction potential changes qualitatively the dynamics of vortex structures compared to the hydrodynamical models mentioned earlier [8].

By evaluating (7) for points on the contours themselves, we compute the motion of the contours. During the calculation, nodes are added and/or removed to ensure a smooth approximation of the contours [7]. The CD method is inviscid by nature, the absence of dissipation being crucial for the description of the motion by the contours of the patches alone. Because of the Lagrangian method no grid or filtering is needed, and the topology of the contours can be conserved exactly. In practice the number of nodes needed to smoothly approximate the contours increases strongly as the spatial complexity of the contours increases. The calculation is terminated when the number of nodes becomes too large and the area and topology of the contours can no longer be conserved.

A pure current distribution is considered by initially taking $\omega_+ + \omega_- = 0$ everywhere. Then $\nabla^2\varphi$ and φ are zero, and $\omega_\pm$ is advected by the velocity field $\boldsymbol{v}_\pm = \pm\boldsymbol{e}_z \times \nabla\psi/d_e$. The two vorticity fields are advected along the perpendicular magnetic field $\boldsymbol{e}_z \times \nabla\psi$ in opposite directions. When this flow separates the $\omega_\pm$ fields, fluid vorticity $\nabla^2\varphi = \omega_+ + \omega_-$ will emerge. To avoid confusion, we will use the term *fluid vorticity* for the vorticity $\nabla^2\varphi$ of the $\boldsymbol{E} \times \boldsymbol{B}$ flow. This generated flow field $\boldsymbol{e}_z \times \nabla\varphi$ advects both ω_+ and ω_- in the same direction. So a current distribution in the plasma will be set into motion by the perpendicular magnetic field, generated by the plasma currents themselves. An illustration of this mechanism for the case of two circular currents (four patches of generalized vorticity in total) is given in Fig. 1. It is through this mechanism that the Lorentz force between currents manifests itself in a plasma.

A circular current distribution as shown in Fig. 1 is formed by two coinciding circular contours of radius R: one of type ω_+ for which $\omega_+ = +1/2$ inside and $\omega_+ = 0$ outside the patch, and one of type ω_- with $\omega_- = -1/2$ inside and $\omega_- = 0$ outside. Both φ and $\nabla^2\varphi$ are exactly zero, while ψ and J are given by

$$\psi = \begin{cases} d_e - RK_1(R/d_e)I_0(r/d_e) & \text{for } r < R, \\ RI_1(R/d_e)K_0(r/d_e) & \text{for } r > R, \end{cases} \tag{9}$$

$$J = \psi/d_e^2 - \begin{cases} 1/d_e & \text{for } r < R, \\ 0 & \text{for } r > R, \end{cases} \tag{10}$$

where r is the distance from the center of the patches and I_n, K_n are the modified Bessel functions of the first and second kind, respectively. Because of the electron inertial skin depth d_e, the distribution of current density inside

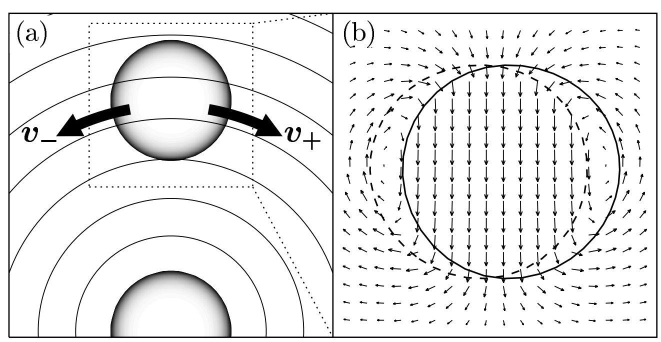

FIG. 1. (a) Sketch of a circular current in the perpendicular magnetic field of another identical current (its own field is not drawn). The fat arrows indicate the directions in which the ω_+ and ω_- patches are advected along the field. (b) The induced $\boldsymbol{E} \times \boldsymbol{B}$ velocity field for the upper current. This flow advects the plasma currents towards each other. The solid and dashed lines indicate the displaced ω_+ and ω_- contours, respectively.

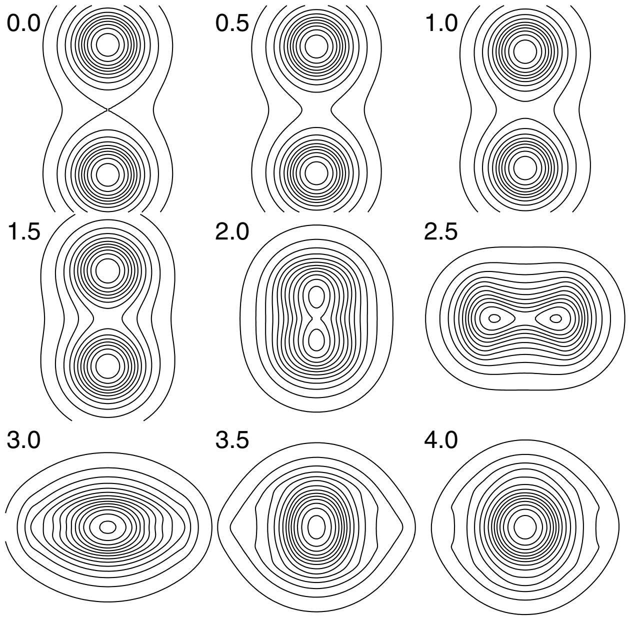

FIG. 2. The merging of two plasma currents. Snapshots for $t = 0, 0.5, \ldots, 4.0$ of the perpendicular magnetic field (isolines of ψ). For $t = 4.0$ the structure of the $\omega_{\pm}$ patches is given in Fig. 3.

the patches is hollow, i.e., the current runs mainly near the edge. Outside the patches there is a screening current running in the opposite direction, which strongly shields the magnetic field outside for distances $\gg d_e$.

When two such current "wires" are adjacent (as shown in Fig. 1), they attract each other. Via the mechanism described above they will both deform and develop a dipolar flow field, which advects them towards each other. Eventually the currents merge. The process of merging is calculated using the CD method. For the case shown here the initial patch radius is $R = 1.0d_e$ and the distance between the centers is $5.0d_e$, so that the separation between the patches is $3.0d_e$. The skin depth is $d_e = 0.3\rho_s$. The simulation is started with 40 nodes on each of the four contours. At $t = 4.0$, about 1400 nodes per contour are needed to resolve the fine structures which the patches have developed. The area of the patches is well conserved during the run. Additional calculations show that the size and separation of the currents do influence the speed of the process but do not essentially change the merging process itself. Currents of larger sizes take longer to merge, and the initial separation should be of the order of a few d_e, because the magnetic interaction between the currents is exponentially weak due to the screening.

At $t = 0$ the total perpendicular magnetic field has a figure-eight–like topology (See Fig. 2). When the currents approach each other field lines are reconnected until the current distribution is circular. Then the flow pattern in the fluid causes the current distribution to elongate perpendicularly to the direction of approach. As a result the magnetic field forms a horizontal figure eight (compare $t = 2.0$ with $t = 2.5$ in Fig. 2). The Lorentz force pulls the elongated current together again. After a few oscillations the system relaxes into a large circular current distribution with a matching circular magnetic field.

The whole process takes place in a few periods. The contours develop thin strands of generalized vorticity which wrap around the final current distribution. Figure 3 shows the four contours at $t = 4.0$, when the current distribution is almost circular. These strands carry a small amount of both current and vorticity to small scales. The ω_+ patches are colored red and carry current and a positive fluid vorticity. The blue ω_- patches carry the same current, but a negative fluid vorticity (remember that the strength of the patches is $\omega_{\pm} = \pm 1/2$). In areas where ω_+ and ω_- overlap (purple in the figure) the fluid vorticity vanishes. Contours of equal type do not reconnect. One sees that the boundaries of the patches inside the central current develop small scale structures. Also the strands which wrap around the core of the current distribution get thinner. These fine structures no longer affect the global dynamics, but do carry part of the energy in the system. This process resembles very much the phase mixing mechanism described in Ref. [9]. In a truly

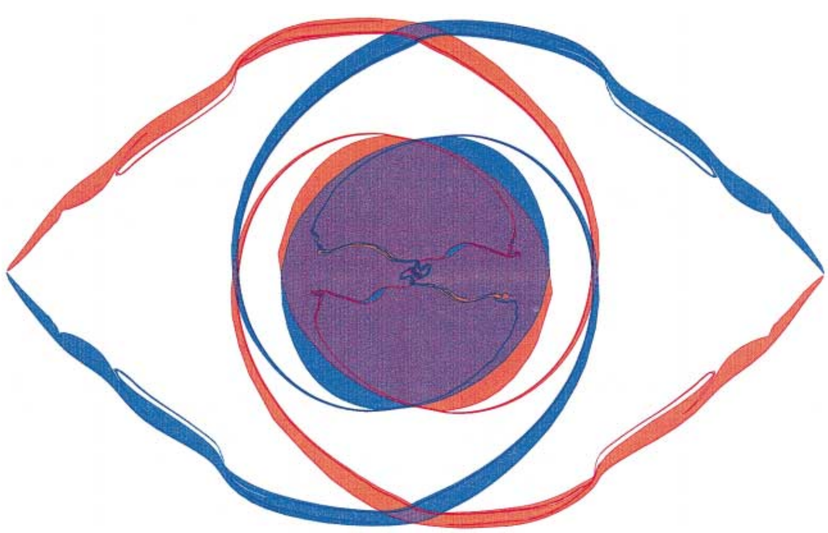

FIG. 3 (color). The ω_+ (red) and ω_- (blue) patches at $t = 4.0$, the final stage of the merging process. Overlapping areas are purple, the fluid vorticity vanishes in those areas. Nonoverlapping areas of ω_+ carry positive vorticity, and areas of ω_- carry an equal amount of negative vorticity. The thin lines are the contours of the conserved $\omega_{\pm}$ patches. Both the contours inside the central current distribution and the strands which wrap around it are evolving towards small scales.

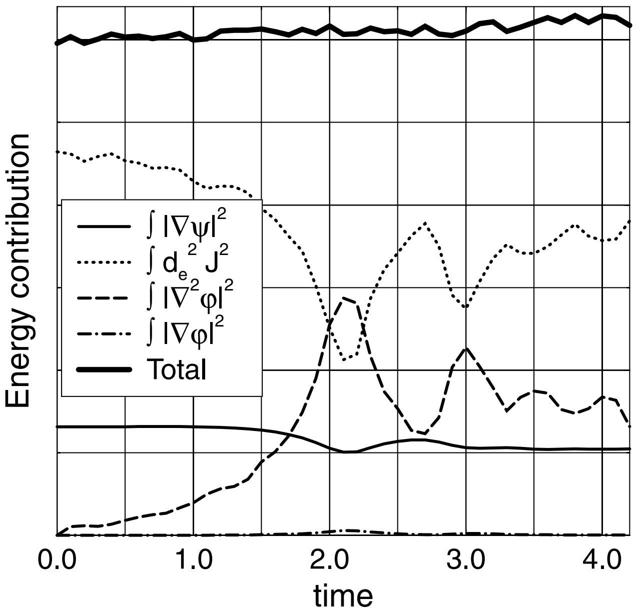

FIG. 4. The evolution of the energy contributions. Thick solid line, total energy. Thin solid line, energy of the perpendicular magnetic field. Dotted line, kinetic electron energy along the main magnetic field. Dashed line, internal energy or enstrophy. Dash-dotted line, kinetic energy of the $\boldsymbol{E} \times \boldsymbol{B}$ flow.

dissipationless system like the one considered here, it seems that such an irreversible mixing mechanism exists. When one considers the model to be an approximation to a system with small dissipation, then the small scale features, and thus the topology of the generalized vorticity fields, will eventually be affected.

The energy integral of the system is given by

$$W = \frac{1}{2} \int d^2x \, [|\nabla\psi|^2 + d_e^2 J^2 + |\nabla\varphi|^2 + |\nabla^2\varphi|^2]. \tag{11}$$

The first three terms represent the magnetic energy, the parallel kinetic energy of the electrons, and the perpendicular kinetic energy of the ions, respectively. The last term is the enstrophy of the $\boldsymbol{E} \times \boldsymbol{B}$ flow, and by $\nabla^2\varphi = \ln n$ this can be identified with the internal energy. At a given time step from the CD calculation the four energy contributions are determined by calculating the physical fields on a fixed grid encompassing the four patches using contour integrals and subsequently taking the sum over all grid points. The energy is calculated over a finite part of the (unbounded) perpendicular domain, and the small energy fluxes across the boundaries are not taken into account because their influence on the energy contributions is negligible.

The temporal evolution of the energy contributions is plotted in Fig. 4. The merging motion and relaxation are seen to be predominantly an exchange between the parallel kinetic energy of the electrons $\propto J^2$ and the internal energy, which is proportional to the $\boldsymbol{E} \times \boldsymbol{B}$ vorticity squared. The magnetic energy decreases as a result of the reconnection of the field lines. Also the kinetic energy of the $\boldsymbol{E} \times \boldsymbol{B}$ flow, which brings the currents together, is a small contribution. Note that the changes of these last two contributions are relatively small, while they are essential for the topological changes of the system.

To summarize, we have used the method of contour dynamics to study the interaction of two localized currents. When the currents are aligned, they attract each other by the $\boldsymbol{J} \times \boldsymbol{B}$ force and thus have a tendency to coalesce. Because the magnetic field of the currents is shielded by the plasma on large distances, currents will merge only when their separation is on the order of a few times d_e, the electron inertial skin depth. In this process, fluid vorticity is generated to bring the currents together, and the initial figure-eight topology of the magnetic field is changed into a circular one.

The merging mechanism is analogous to the vortex merger process in two-dimensional hydrodynamics, where it is assumed to be the main mechanism of self-organization in a turbulent fluid [10]. It is therefore expected to play an important role in a turbulent system where interactions between plasma currents dominate the dynamics.

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[1] P. L. Pritchett and C. C. Wu, Phys. Fluids **22**, 2140 (1979).
[2] D. Biskamp, E. Schwarz, and J. F. Drake, Phys. Plasmas **4**, 1002 (1997).
[3] T. J. Schep, F. Pegoraro, and B. N. Kushinov, Phys. Plasmas **1**, 2843 (1994).
[4] V. P. Lakhin, T. J. Schep, and E. Westerhof, Phys. Plasmas **5**, 3833 (1998).
[5] N. J. Zabusky, M. H. Hughes, and K. V. Roberts, J. Comput. Phys. **30**, 96 (1979).
[6] D. G. Dritschel, Comput. Phys. Rep. **10**, 77 (1989).
[7] P. W. C. Vosbeek and R. M. M. Mattheij, J. Comput. Phys. **133**, 222 (1997).
[8] J. Bergmans, B. N. Kuvshinov, V. P. Lakhin, and T. J. Schep, Phys. Plasmas **7**, 2388 (2000).
[9] D. Grasso, F. Califano, F. Pegoraro, and F. Porcelli, Phys. Rev. Lett. **86**, 5051 (2001).
[10] A. Bracco, J. C. McWilliams, G. Murante, A. Provenzale, and J. B. Weiss, Phys. Fluids **12**, 2931 (2000).