Coherent Gluon Production in Very-High-Energy Heavy-Ion Collisions

Alex Krasnitz,[1] Yasushi Nara,[2] and Raju Venugopalan[2,3]

[1]CENTRA and Faculdade de Ciências e Tecnologia, Universidade do Algarve, Campus de Gambelas, P-8000 Faro, Portugal

[2]RIKEN BNL Research Center, Brookhaven National Laboratory, Upton, New York 11973

[3]Physics Department, Brookhaven National Laboratory, Upton, New York 11973

(Received 13 August 2001; published 23 October 2001)

The early stages of a relativistic heavy-ion collision are examined in the framework of an effective classical SU(3) Yang-Mills theory in the transverse plane. We compute the initial energy and number distributions, per unit rapidity, at midrapidity, of gluons produced in high-energy heavy-ion collisions. We discuss the phenomenological implications of our results in light of the recent Relativistic Heavy-Ion Collider data.

DOI: 10.1103/PhysRevLett.87.192302 PACS numbers: 25.75.–q, 24.10.–i, 24.85.+p

The Relativistic Heavy-Ion Collider (RHIC) is currently colliding beams of gold nuclei at the highest center of mass energies, per nucleon, $\sqrt{s_{NN}} = 200$ GeV. The goal of these experiments is to explore strongly interacting matter, in particular the quark gluon plasma (QGP) predicted by lattice QCD [1].

The possible formation and dynamics of the QGP depend crucially on the initial conditions, namely, the distribution of partons in each of the nuclei *before* the collision. At high energies and for large nuclei, parton distributions saturate and form a color glass condensate (CGC). (For other perturbative QCD based approaches, see Ref. [2] and references therein.)

The physics of saturated gluons in the CGC is as follows. As the energies of the colliding nuclei grow (equivalently $x_{Bj} \ll 1$), partons in the nuclear wave functions multiply until they begin to overlap in phase space. Repulsive interactions among the partons ensure that the occupation number saturates at a value proportional to $1/\alpha_s$. This phenomenon [3], is characterized by a bulk scale (the saturation scale) Λ_s, where Λ_s^2 is proportional to the gluon density per unit area in a nucleon or nucleus. A simple saturation model for nucleons with $\Lambda_s^2 = \Lambda_{s0}^2 (x_0/x_{Bj})^\delta$ with $\Lambda_{s0} = 1$ GeV, $x_0 = 3 \times 10^{-4}$, and $\delta = 0.29$ describes well deeply inelastic scattering data at the Hadron Electron Ring Accelerator for $x_{Bj} < 0.01$ and all values Q^2 of the transverse momentum squared from $Q^2 \sim 0$ up to $Q^2 = 450$ GeV^2 [4]. For nuclei, one expects that $\Lambda_s^2 \approx \Lambda_{\rm QCD}^2 A^{1/3}/x^\delta$.

In a heavy-ion collision, the CGC "shatters," producing "on shell" gluons. In this Letter, we obtain nonperturbative expressions relating the energy and number distributions of produced gluons to the saturation scale Λ_s of the CGC. Therefore, in principle, the saturation scale Λ_s may be determined from heavy-ion experiments.

The CGC can be quantified in a classical effective field theory, where Λ_s^2 is the only dimensionful scale [5]. When $\Lambda_s^2 \gg \Lambda_{\rm QCD}^2$ (for high energies and large nuclei), the coupling is weak: $\alpha_s \equiv \alpha_s(\Lambda_s^2) \ll 1$. However, the occupation number is large, $\propto 1/\alpha_s \gg 1$. Thus weak coupling, classical methods are applicable and can be used to compute the classical parton distributions of nuclei [5,6]. Recently, renormalization group methods were developed which systematically incorporate quantum corrections to the effective field theory (EFT) [6,7].

The classical EFT can be applied to nuclear collisions [8–10]. The spectrum of gluons produced when the CGC shatters is described by the solution of the classical Yang-Mills equations in the presence of two light cone sources, one for each nucleus, with initial conditions for the gauge fields given by the gauge fields of the two nuclei before the collision. Analytical expressions for classical gluon production were obtained to lowest order in the parton density [8–10]. However, these are infrared divergent and need to be summed to all orders in the parton density. This was first done numerically by two of us for an SU(2) gauge theory [11], and nonperturbative expressions relating the energy [12] and number [13] distributions of produced gluons to the saturation scale were obtained. Here we extend the work of Refs. [12,13] to an SU(3) gauge theory [14]. Our results can thereby be compared to available and forthcoming data from RHIC.

Simulating the SU(3) theory is technically more difficult than the SU(2) theory. For a comparable set of parameters, the SU(3) case is about an order of magnitude more challenging numerically than the SU(2) one. The lattice formulation of the theory is described in detail in [11]. The numerical techniques we use are well known in lattice gauge theory, with one notable exception. Specifically, a new procedure had to be devised in order to determine the initial condition for the transverse components of the gauge fields. In the SU(2) case, a closed form analytical expression for the transverse components of the gauge field can be obtained, while for the SU(3) case it has to be obtained numerically. The technical details of how this is achieved is outside the scope of this Letter and will be described in detail elsewhere.

In this paper we will determine two observables: the energy and the number distribution of produced gluons. In doing so, we closely follow the procedure developed for the SU(2) case. In the continuum limit the theory contains two-dimensional parameters: Λ_s and the nuclear radius R. Any observable can therefore be expressed as a power of Λ_s, times a function of the dimensionless product $\Lambda_s R$ and of the coupling constant g [16].

For the transverse energy of gluons we obtain

$$\frac{1}{\pi R^2}\frac{dE_T}{d\eta}\bigg|_{\eta=0} = \frac{1}{g^2} f_E(\Lambda_s R)\Lambda_s^3\,, \qquad (1)$$

The function f_E is determined nonperturbatively as follows. In Fig. 1(a), we plot the Hamiltonian density, for a particular fixed value [17] of $\Lambda_s R = 83.7$ (on a 512×512 lattice) in dimensionless units as a function of the proper time in dimensionless units. We note that in the SU(3) case, as in SU(2), $\varepsilon\tau$ converges very rapidly to a constant value. The form of $\varepsilon\tau$ is well parametrized by the functional form $\varepsilon\tau = \alpha + \beta\exp(-\gamma\tau)$. Here $dE_T/d\eta/\pi R^2 = \alpha$ has the proper interpretation of being the energy density of produced gluons, while $\tau_D = 1/\gamma/\Lambda_s$ is the "formation time" of the produced glue.

In Fig. 1(b), the convergence of α to the continuum limit is shown as a function of the lattice spacing in dimensionless units for two values of $\Lambda_s R$. In Ref. [12], this convergence to the continuum limit was studied extensively for very large lattices (up to 1024×1024 sites) and shown to be linear. The trend is the same for the SU(3) results. Thus, despite being farther from the continuum limit for SU(3) (due to the significant increase in computer time), a linear extrapolation is justified. We can therefore extract the continuum value for α. We find $f_E(25) = 0.537$ and $f_E(83.7) = 0.497$. The RHIC value likely lies in this range of $\Lambda_s R$. The formation time $\tau_D = 1/\gamma/\Lambda_s$ is essentially the same for SU(2)—for $\Lambda_s R = 83.7$, $\gamma = 0.362 \pm 0.023$. As discussed in Ref. [12], it is ~0.3 fm for RHIC and ~0.13 fm for CERN Large Hadron Collider (LHC) (taking $\Lambda_s = 2$ and 4 GeV, respectively).

We now combine our expression in Eq. (1) with our nonperturbative expression for the formation time to obtain a nonperturbative formula for the initial energy density,

$$\varepsilon = \frac{0.17}{g^2}\Lambda_s^4\,, \qquad (2)$$

This formula gives a rough estimate [18] of the initial energy density, at a formation time of $\tau_D = 1/\bar{\gamma}/\Lambda_s R$, where we have taken the average value of the slowly varying function γ to be $\bar{\gamma} = 0.34$.

To determine the gluon number per unit rapidity, we first compute the gluon transverse momentum distributions. The procedure followed is identical to that described in Ref. [13]—we compute the number distribution in Coulomb gauge [19], $\nabla_\perp \cdot A_\perp = 0$. In Fig. 2(a), we plot the normalized gluon transverse momentum distributions versus k_T/Λ_s with the value $\Lambda_s R = 83.7$, together with the SU(2) result. Clearly, we see that the normalized result for SU(3) is suppressed relative to the SU(2)

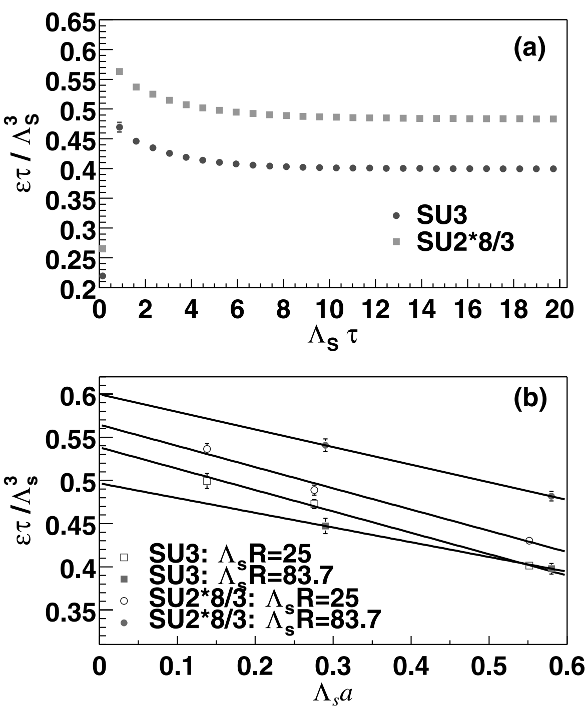

FIG. 1. (a) $\varepsilon\tau/\Lambda_s^3$ as a function of $\tau\Lambda_s$ for $\Lambda_s R = 83.7$. (b) $\varepsilon\tau/\Lambda_s^3$ as a function of $\Lambda_s a$ for $\Lambda_s R = 83.7$ (squares) and 25 (circles), where a is the lattice spacing. Lines are fits of the form $a - bx$.

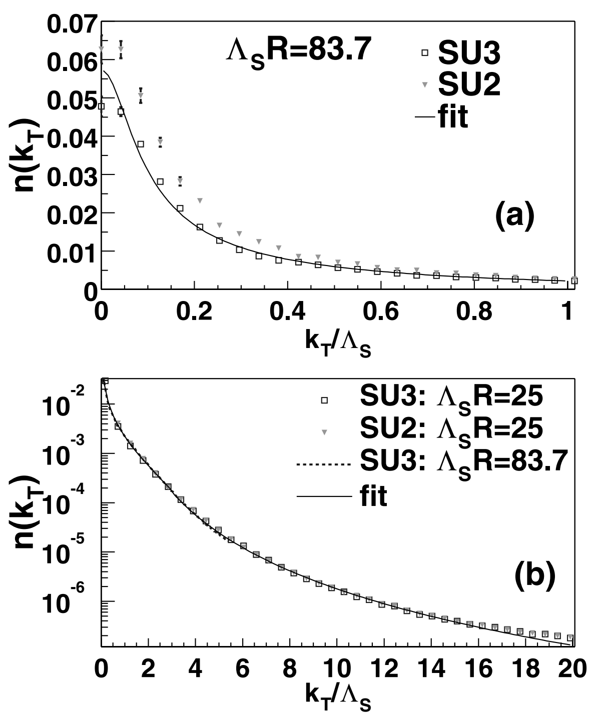

FIG. 2. Transverse momentum distribution of gluons, normalized to the color degrees of freedom, $n(k_T) = \tilde{f}_n/(N_c^2 - 1)$ [see Eq. (3)] as a function of $\Lambda_S R$ for SU(3) (squares) and SU(2) (triangles). (a) For soft momenta; (b) for all momenta. The solid lines correspond to the fit in Eq. (4).

result in the low momentum region. In Fig. 2(b), we plot the same quantity over a wider range in k_T/Λ_s for two values of $\Lambda_s R$. At large transverse momentum, we see that the distributions scale exactly as $N_c^2 - 1$, the number of color degrees of freedom. This is as expected since at large transverse momentum, the modes are nearly those of noninteracting harmonic oscillators. At smaller momenta, the suppression is due to nonlinearities, whose effects, we have confirmed, are greater for larger values of the effective coupling $\Lambda_s R$.

The SU(3) gluon momentum distribution can be fitted by the following function:

$$\frac{1}{\pi R^2}\frac{dN}{d\eta d^2k_T} = \frac{1}{g^2}\tilde{f}_n(k_T/\Lambda_s), \tag{3}$$

where $\tilde{f}_n(k_T/\Lambda_s)$ is

$$\tilde{f}_n = \begin{cases} a_1[\exp(\sqrt{k_T^2 + m^2}/T_{\mathrm{eff}}) - 1]^{-1} & (k_T/\Lambda_s \le 3) \\ a_2\Lambda_s^4 \log(4\pi k_T/\Lambda_s)k_T^{-4} & (k_T/\Lambda_s > 3) \end{cases} \tag{4}$$

with $a_1 = 0.0295$, $m = 0.067\Lambda_s$, $T_{\mathrm{eff}} = 0.93\Lambda_s$, and $a_2 = 0.0343$. At low momenta, the functional form is approximately that of a Bose-Einstein distribution in two dimensions even though the underlying dynamics is that of classical fields. The functional form at high momentum is motivated by the lowest order perturbative calculations [8–10].

Integrating our results over all momenta, we obtain, for the gluon number per unit rapidity, the nonperturbative result,

$$\frac{1}{\pi R^2}\frac{dN}{d\eta}\bigg|_{\eta=0} = \frac{1}{g^2} f_N(\Lambda_s R)\Lambda_s^2. \tag{5}$$

We find that $f_N(83.7) = 0.3$. The results for a wide range of $\Lambda_s R$ vary on the order of 10% in the case of SU(2).

The broad features of the CGC picture have recently been compared to the RHIC data [20,21]. We shall here discuss the phenomenological implications of our specific model in light of the recent RHIC data on multiplicity and energy distributions. The final multiplicity of hadrons [22] is related to the initial gluon multiplicity by the relation $dN^h/d\eta = \kappa_{\mathrm{inel}} dN_i^g/d\eta$. Here κ_{inel} is a factor accounting for $2 \to n$ gluon number changing processes which may occur at late times beyond when the classical approach is applicable [23]. Moreover, if partial or full thermalization does occur [23,24], the initial transverse energy is reduced—both due to inelastic collisions prior to thermalization and, subsequently, due to hydrodynamic expansion—by a factor κ_{work}. We then have

$$\begin{aligned} \frac{dE_T^h}{d\eta}\bigg|_{\eta=0} &= \frac{\pi}{g^2}\frac{1}{\kappa_{\mathrm{work}}} f_E(\Lambda_s R)\Lambda_s(\Lambda_s R)^2, \\ \frac{dN^h}{d\eta}\bigg|_{\eta=0} &= \frac{\pi\kappa_{\mathrm{inel}}}{g^2} f_N(\Lambda_s R)\,(\Lambda_s R)^2. \end{aligned} \tag{6}$$

From the RHIC data at $\sqrt{s_{NN}} = 130$ GeV, we have $dN^h/d\eta|_{\eta=0} \sim 1000$ for central collisions [25–28]. For $g = 2(\alpha_s = 0.33)$, $\pi R^2 = 148$ fm^2, and $f_N = 0.3$, we have $\kappa_{\mathrm{inel}}\Lambda_s^2 = 3.5$ GeV^2. Now, from Eq. (6), the ratio $R^h = dE_T^h/d\eta/dN^h/d\eta$ is, since $f_E/f_N = 1.66$, $R^h = 1.66\Lambda_s/\kappa_{\mathrm{work}}/\kappa_{\mathrm{inel}}$. The experimental value [26] for $\sqrt{s_{NN}} = 130$ GeV is $R^h = 0.5$ GeV. Now, if we assume that there is no work done due to thermalization, $\kappa_{\mathrm{work}} = 1$, we obtain from the two conditions $\Lambda_s = 1.02$ GeV and $\kappa_{\mathrm{inel}} = 3.4$ as the values that give agreement with the data. The latter value is the maximal amount of inelastic gluon production possible. Alternatively, if we assume that hydrodynamic work is done, one obtains $\kappa_{\mathrm{work}} = (\tau_f/\tau_i)^{1/3}$, where τ_f and τ_i are the final and initial times of hydrodynamic expansion, respectively. This gives us $\kappa_{\mathrm{work}} \approx 2$. Following the same analysis as previously, we obtain $\Lambda_s = 1.28$ GeV and $\kappa_{\mathrm{inel}} = 2.13$. Thus, within the CGC approach, we are able to place bounds on both the saturation scale and on the amount of inelastic gluon production at RHIC energies. An independent method to extract Λ_s directly from the data (albeit assuming parton-hadron duality) is to compute the relative event-by-event fluctuations of the gluon number [29].

It is difficult to compare the initial p_t distributions we have computed directly to the data. First, we assumed uniform nuclear matter distributions and periodic boundary conditions. Using realistic nuclear profiles and open boundary conditions will likely modify our results. Work is in progress in this direction and will be reported soon [29]. Second, even if there is no additional rescattering, we note that the spectrum of hadrons is obtained by convolving the spectrum of produced gluons with fragmentation functions. The spectrum of hadrons will therefore be different from the spectrum of the initially produced gluons. Further, if rescattering occurs, one expects the gluon spectrum itself to be modified from the initial gluon spectrum [24,30]. The initial gluon spectrum computed in our approach can, however, be used as the initial condition for transport (parton cascade [31]) calculations of the subsequent evolution of the system.

We thank Shigemi Ohta for a helpful discussion. R. V.'s research was supported by DOE Contract No. DE-AC02-98CH10886. A. K. and R. V. acknowledge support from the Portuguese FCT, under Grants No. CERN/P/FIS/15196/1999 and No. CERN/P/FIS/40108/2000. R. V. and Y. N. thank RIKEN-BNL for support.

[1] For reviews and recent developments, see special issue, *Quark Matter 99* [Nucl. Phys. **A661** (1999)]; special issue, *Quark Matter 2001* [Nucl. Phys. (to be published)].

[2] K. J. Eskola, K. Kajantie, and J. Lindfors, Nucl. Phys. **B323**, 37 (1989); K. J. Eskola, K. Kajantie, P. V. Ruuskanen, and K. Tuominen, Nucl. Phys. **B570**, 379 (2000); K. Geiger, Phys. Rep. **258**, 237 (1995); X-N. Wang,

Phys. Rep. **280**, 287 (1997); N. Hammon, H. Stocker, and W. Greiner, Phys. Rev. C **61**, 014901 (2000).

[3] L. V. Gribov, E. M. Levin, and M. G. Ryskin, Phys. Rep. **100**, 1 (1983); A. H. Mueller and J.-W. Qiu, Nucl. Phys. **B268**, 427 (1986); J. P. Blaizot and A. H. Mueller, Nucl. Phys. **B289**, 847 (1987).

[4] K. Golec-Biernat and M. Wüsthof, Phys. Rev. D **59**, 014017 (1999); Phys. Rev. D **60**, 114023 (1999); A. Stasto, K. Golec-Biernat, and J. Kwiecinski, Phys. Rev. Lett. **86**, 596 (2001).

[5] L. McLerran and R. Venugopalan, Phys. Rev. D **49**, 2233 (1994); **49**, 3352 (1994); **50**, 2225 (1994).

[6] J. Jalilian-Marian, A. Kovner, L. McLerran, and H. Weigert, Phys. Rev. D **55**, 5414 (1997); Y. V. Kovchegov, Phys. Rev. D **54**, 5463 (1996).

[7] J. Jalilian-Marian, A. Kovner, A. Leonidov, and H. Weigert, Nucl. Phys. **B504**, 415 (1997); J. Jalilian-Marian, A. Kovner, and H. Weigert, Phys. Rev. D **59**, 014015 (1999); L. McLerran and R. Venugopalan, Phys. Rev. D **59**, 094002 (1999); E. Iancu, A. Leonidov, and L. McLerran, hep-ph/0011241; Phys. Lett. B **510**, 133 (2001).

[8] M. Gyulassy and L. McLerran, Phys. Rev. C **56**, 2219 (1997).

[9] A. Kovner, L. McLerran, and H. Weigert, Phys. Rev D **52**, 3809 (1995); **52**, 6231 (1995).

[10] Y. V. Kovchegov and D. H. Rischke, Phys. Rev. C **56**, 1084 (1997); S. G. Matinyan, B. Müller, and D. H. Rischke, Phys. Rev. C **56**, 2191 (1997); Phys. Rev. C **57**, 1927 (1998); Xiao-feng Guo, Phys. Rev. D **59**, 094017 (1999).

[11] A. Krasnitz and R. Venugopalan, hep-ph/9706329; hep-ph/9808332; Nucl. Phys. **B557**, 237 (1999).

[12] A. Krasnitz and R. Venugopalan, Phys. Rev. Lett. **84**, 4309 (2000).

[13] A. Krasnitz and R. Venugopalan, Phys. Rev. Lett. **86**, 1717 (2001).

[14] For recent analytical work, see Y. V. Kovchegov, hep-ph/0011252.

[15] A. H. Mueller, Nucl. Phys. **B572**, 227 (2000).

[16] The quantity Λ_s in this work is identical to $g^2\mu$ in Refs. [11–13]. It is approximately twice the scale Q_s introduced by Mueller [15].

[17] For a gold nucleus, $R = 6.87$ fm, so this value of $\Lambda_s R$ corresponds to a $\Lambda_s \sim 2.44$ GeV, somewhat larger than most estimates for RHIC.

[18] The number $0.17 = f_E \gamma$. The function f_E is a slowly varying function of $\Lambda_s R$, varying in the SU(2) case in the entire RHIC-LHC range by 20%. The function γ was shown to be constant within error bars for the RHIC-LHC range. Therefore, in the regime of interest, we estimate the systematic uncertainty of $f_E \gamma$ to be $\sim$10%.

[19] Although there are potentially gauge artifacts in this method, it was shown empirically in Ref. [13] that the results for the integrated number per unit rapidity in this method agree well with results from a gauge invariant cooling method.

[20] D. Kharzeev and M. Nardi, Phys. Lett. B **507**, 121 (2001); D. Kharzeev and E. Levin, nucl-th/0108006.

[21] L. McLerran and J. Schaffner-Bielich, Phys. Lett. B **514**, 29 (2001); J. Schaffner-Bielich, D. Kharzeev, L. McLerran, and R. Venugopalan, nucl-th/0108048.

[22] We assume that hadronization is isentropic, in accordance with the "parton-hadron duality" seen empirically.

[23] L. Xiong and E. Shuryak, Phys. Rev. C **49**, 2241 (1994); T. S. Biro, E. van Doorn, B. Müller M. H. Thoma, and X.-N Wang, Phys. Rev. C **48**, 1275 (1993); S. M. H. Wong, Phys. Rev. C **54**, 2588 (1996); D. M. Elliott and D. H. Rischke, Nucl. Phys. **A671**, 583 (2000); R. Baier, A. H. Mueller, D. Schiff, and D. T. Son, Phys. Lett. B **502**, 51 (2001).

[24] A. H. Mueller, Phys. Lett. B **475**, 220 (2000); J. Bjoraker and R. Venugopalan, Phys. Rev. C **63**, 024609 (2001); A. Dumitru and M. Gyulassy, Phys. Lett. B **494**, 215 (2000); G. C. Nayak, A. Dumitru, L. McLerran, and W. Greiner, Nucl. Phys. **A687**, 457 (2001).

[25] PHOBOS Collaboration, B. B. Back *et al.,* Phys. Rev. Lett. **85**, 3100 (2000).

[26] PHENIX Collaboration, K. Adcox *et al.,* Phys. Rev. Lett. **87**, 052301 (2001).

[27] STAR Collaboration, C. Adler *et al.,* nucl-ex/0106004.

[28] BRAHMS Collaboration, F. Videbæk *et al.,* special issue, *Quark Matter 2001* [Nucl. Phys. (to be published)].

[29] A. Krasnitz, Y. Nara, and R. Venugopalan (to be published).

[30] X. Wang and M. Gyulassy, Phys. Rev. Lett. **68**, 1480 (1992); M. Gyulassy, P. Levai, and I. Vitev, Phys. Rev. Lett. **85**, 5535 (2000); U. A. Wiedemann, Nucl. Phys. **A690**, 731 (2001); R. Baier, D. Schiff, and B. G. Zakharov, Annu. Rev. Nucl. Part. Sci. **50**, 37 (2000).

[31] S. A. Bass, nucl-th/0104040; D. Molnar and M. Gyulassy, Phys. Rev. C **62**, 054907 (2000); Y. Nara, S. E. Vance, and P. Csizmadia, nucl-th/0109018.