Constraining Isocurvature Perturbations with Cosmic Microwave Background Polarization

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The role of cosmic microwave background polarization data in constraining the presence of primordial isocurvature modes is examined. While MAP will be unable to simultaneously constrain isocurvature modes and cosmological parameters, PLANCK will set strong limits on isocurvature modes. If one allows isocurvature modes, the recently obtained BOOMERANG measurement of the curvature of the Universe fails. However, a comparably sensitive polarization measurement on the same angular scales will permit a determination of the curvature without the prior assumption of adiabaticity.

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Measurements of the cosmic microwave background anisotropy may soon allow us to rigorously determine the fundamental character of the primordial cosmological perturbations with a minimum of simplifying hypotheses. Recent data [1,2] already impose formidable constraints on the parameter space of simple inflationary models, and with forthcoming satellite missions [3,4] this situation is expected to improve substantially.

This paper focuses on how to test the assumption of adiabaticity, namely, that all components contributing to the density of the Universe are present in spatially uniform ratios on hypersurfaces of equal cosmic temperature and initially share a common velocity field. The hypothesis of adiabaticity, put forth initially on the basis of simplicity, gained support when it was realized that this was the prediction of the simplest one-field inflationary models [5]. Multifield inflationary models, however, generically excite isocurvature modes as well [6].

Testing the hypothesis of adiabaticity through observation requires studying models where the primordial perturbations are not solely adiabatic (AD) and thus establishing bounds on the allowed admixtures of nonadiabatic modes. Nonadiabatic, or isocurvature, perturbations have already been studied in the literature, but most work focused on evaluating the viability of cosmological models in which the perturbations were entirely isocurvature in character, with no adiabatic component at all [7]. Observational constraints on an uncorrelated admixture of adiabatic and cold dark matter isocurvature (CDMI) perturbations have also been considered [8].

Baryon isocurvature (BI) and CDMI models were studied some time ago [7]. More recently it was realized that two additional isocurvature modes are possible: a neutrino isocurvature density (NID) mode and a neutrino isocurvature velocity (NIV) mode [9,10]. In the NID mode, the neutrino-photon ratio varies spatially. As modes enter the horizon, the photon-baryon fluid begins to oscillate acoustically, whereas the neutrinos free stream. This differential behavior perturbs the total energy density, leading to structure formation via gravitational clustering. The neutrino velocity mode assumes a relative velocity between the photon and neutrino components but zero initial total momentum density. As with the NID mode, differential evolution spoils this cancellation after horizon crossing, again leading to structure formation.

In a universe composed of just photons, baryons, neutrinos, baryons, and a cold dark matter component, these four isocurvature modes and the adiabatic mode exhaust the possible modes nonsingular in the $t \to 0$ limit [9]. The most general Gaussian primordial perturbation in such a cosmology is completely characterized by the matrix valued generalization of the power spectrum,

$$\langle A_a(\mathbf{k}) A_b(\mathbf{k}')\rangle = P_{ab}(k) \cdot \delta^3(\mathbf{k} - \mathbf{k}'),$$

where the indices $(a, b = 0, 1, 2, 3, 4)$ label the modes. When expectation values of observables quadratic in the linearized perturbations [such as the cosmic microwave background (CMB) C_l's] are considered, the assumption of Gaussianity is superfluous. A detailed discussion of how the CMB satellite missions will be able to constrain these modes is given in Ref. [11].

We briefly remark on the possible microphysical origin of these modes. The neutrino isocurvature mode is rapidly damped before neutrino decoupling, and therefore can be plausibly produced only by physics operating after ~ 1 sec. The neutrino isocurvature density mode is likewise damped by electroweak $B + L$ violating anomalous processes (which would convert it mostly into a baryon isocurvature mode) operating at times earlier than 10^{-10} sec. Since both times are well before photon decoupling ($\sim 10^{13}$ sec), we nevertheless think it legitimate to describe the perturbations as "primordial."

The temperature anisotropy spectra associated with the regular perturbation modes and their cross correlations are shown in Fig. 1. An appropriate power law autocorrelation spectrum was assumed for each mode, so that the large scale CMB anisotropy is approximately scale invariant. The cross-correlation power spectra were then taken

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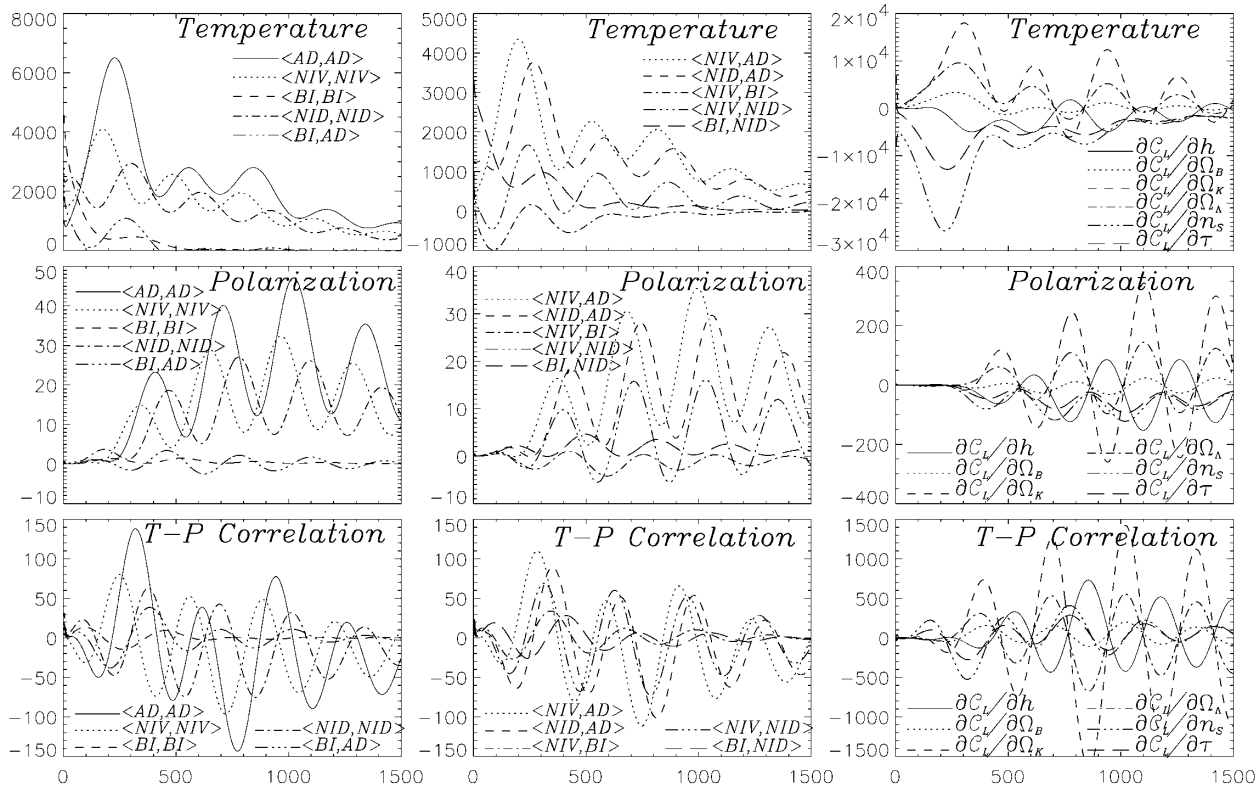

FIG. 1. CMB multipole spectra for the various modes, their cross correlations, and variations in the cosmological parameters: From top to bottom the rows show $l(l+1)C_l/2\pi$ for the temperature, polarization, and temperature-polarization cross correlation, respectively, in $\mu\mathrm{K}^2$. The C_l spectra for the various modes and their cross correlations are shown in the first two columns. The rightmost column shows the derivatives of the spectra with respect to the different cosmological parameters. The modes are indicated as follows: AD, NIV, BI, and NID. A fiducial model with the parameter choices $\Omega_b = 0.06$, $\Omega_\Lambda = 0.69$, $\Omega_{\mathrm{cdm}} = 0.25$, $h = 0.65$, $\tau_{\mathrm{reion}} = 0.1$, and $n_s = 1$ has been assumed. Because the CDMI mode produces a spectrum nearly identical to that of the BI mode, it is not considered separately.

to be proportional to the geometric mean of the two autocorrelation spectra. Clearly there is scope for significant generalization of these assumptions.

The adiabatic C_l temperature spectrum is characterized by a flat Sachs-Wolfe plateau at low l and a series of acoustic peaks, at $l \approx 220 + 310n$ ($n = 0, 1, \ldots$), in a flat universe. Scale invariant baryon and CDMI spectra produce little power at high l. The NID mode exhibits the phase shift characteristic of isocurvature density modes but the NIV mode produces a pattern of peaks more similar to that of the adiabatic mode. The NIV mode acquires a nearly canceling phase shift because the velocity is out of phase with the density. Note in particular how similar the NIV adiabatic-mode cross correlation C_l is to the pure adiabatic mode C_l. The polarization and temperature-polarization cross correlation power spectra associated with the isocurvature modes are indicated in the bottom two rows of Fig. 1.

How feasible will it be to constrain or detect isocurvature modes using CMB measurements? The key question is whether one can distinguish the effect of the isocurvature modes from those of variations in the cosmological parameters (see, e.g., [12]). The derivatives of the C_l power spectra about a fiducial ΛCDM model with respect to cosmological parameters are shown in the rightmost column of Fig. 1. For small admixtures of isocurvature modes, the question is whether linear combinations of these spectra can be distinguished from those of the first two columns of Fig. 1.

For small variations of cosmological parameters and small admixtures of isocurvature modes, one can parametrize the likelihood function as a multivariate Gaussian about a fiducial adiabatic model. By using projected estimates of the instrument noise for the MAP and PLANCK satellite missions, we have computed the estimated errors on the cosmological parameters and isocurvature autocorrelation and cross-correlation amplitudes, assuming that the sky is actually described by a simple adiabatic ΛCDM model. Table I shows that the MAP satellite will be unable to simultaneously constrain isocurvature modes and measure the cosmological parameters. The PLANCK satellite will, but only if it measures the CMB polarization as accurately as currently planned.

We now discuss precisely how the polarization measurement resolves the degeneracy between isocurvature modes and cosmological parameters. In Fig. 2 we illustrate how polarization serves to remove the degeneracy corresponding to the eigenvector pointing in the most uncertain (i.e.,

TABLE I. This table indicates the 1σ percentage errors on cosmological parameters and isocurvature mode amplitudes anticipated for the MAP and PLANCK satellite experiments. In the column headers, T denotes constraints inferred from temperature measurements alone, TP denotes those from the complete temperature and polarization measurements, and $T + P$ denotes those inferred if temperature and polarization information is used separately without including the cross correlation.

	MAP T adia. only	MAP TP adia. only	MAP T all modes	MAP TP all modes	PLANCK T adia. only	PLANCK TP adia. only	PLANCK T all modes	PLANCK $T + P$ all modes	PLANCK TP all modes
$\delta h/h$	12.37	7.42	175.74	20.40	11.50	3.71	94.67	7.75	4.53
$\delta\Omega_b/\Omega_b$	27.76	13.34	325.21	28.57	22.89	7.34	182.01	15.44	8.98
$\delta\Omega_k$	9.79	2.72	75.30	4.55	5.93	1.86	50.28	3.95	2.30
$\delta\Omega_\Lambda/\Omega_\Lambda$	12.92	5.02	123.61	18.53	2.67	1.20	9.79	2.36	1.48
$\delta n_s/n_s$	7.02	1.62	89.87	6.53	0.74	0.37	4.89	0.87	0.70
$\tau_{\rm reion}$	37.39	1.81	104.83	2.23	10.16	0.41	72.53	0.77	0.57
$\langle$NIV, NIV$\rangle$	⋯	⋯	114.34	11.47	⋯	⋯	80.87	1.36	1.14
$\langle$BI, BI$\rangle$	⋯	⋯	573.41	29.71	⋯	⋯	56.72	6.31	4.27
$\langle$NID, NID$\rangle$	⋯	⋯	351.72	29.87	⋯	⋯	42.05	4.73	2.40
$\langle$NIV, AD$\rangle$	⋯	⋯	434.70	44.06	⋯	⋯	212.80	8.19	4.69
$\langle$BI, AD$\rangle$	⋯	⋯	1034.79	59.25	⋯	⋯	94.11	14.97	9.05
$\langle$NID, AD$\rangle$	⋯	⋯	1287.47	67.49	⋯	⋯	179.17	13.68	5.85
$\langle$NIV, BI$\rangle$	⋯	⋯	601.83	32.29	⋯	⋯	79.07	7.63	3.68
$\langle$NIV, NID$\rangle$	⋯	⋯	743.93	46.46	⋯	⋯	133.88	7.42	2.98
$\langle$BI, NID$\rangle$	⋯	⋯	534.33	39.11	⋯	⋯	115.54	7.68	4.70

flattest) direction of the relative likelihood when only temperature information is taken into account.

The dotted curves are the contributions of the various components of the eigenvector. Summing these contributions, one finds very little net contribution to the temperature anisotropy (solid line). This sum is shown *multiplied by ten* for clarity. The lower two panels indicate the corresponding curves for the polarization and temperature-polarization cross-correlation, respectively. For these the delicate cancellation is broken, indicating that the polarization and temperature-polarization cross correlation provide the information required to break the degeneracy. We have examined at which angular scales the degeneracy breaking by PLANCK polarization measurements occurs and found that most of the polarization information resides at $l \lesssim 100$. Figure 2 shows that there is in fact considerable degeneracy breaking at higher l, but this is not detectable with the instrument noise anticipated for PLANCK.

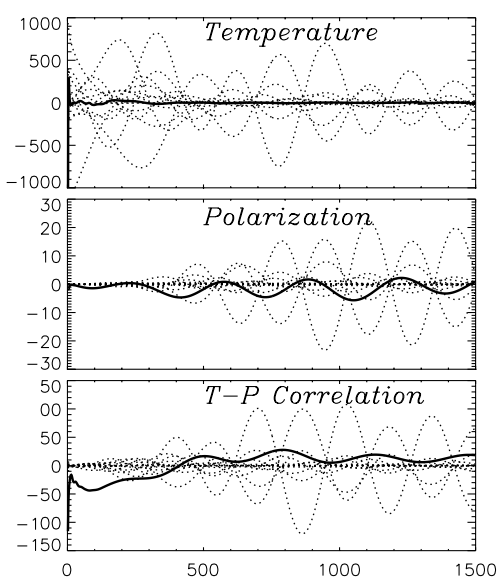

FIG. 2. Breaking the degeneracies with polarization: The top panel indicates the delicate cancellation in the temperature power spectrum between the various components of the most uncertain principal direction (see text). The lower panels show how this cancellation is broken in the polarization and temperature-polarization cross-correlation spectra.

To see why considerable information resides in the range $l \lesssim 100$, note that, on these scales, one directly observes the primordial (superhorizon scale) polarization. This is very different for the adiabatic and isocurvature modes (Fig. 3). In particular, for the NIV mode, since the large scale CMB anisotropy comes mainly from the Doppler effect rather than the Sachs-Wolfe effect, the polarization spectrum is enhanced by a factor of l^{-2} at low l relative to that for the adiabatic mode. The effect at very low l of varying τ_{reion}, is well known [12].

Finally, one can ask to what extent the constraint on the curvature of the Universe, derived using the recent BOOMERANG data, $\delta\Omega_k < 0.12$ at 2σ [1], is affected by the possible presence of isocurvature modes. Adopting the above flat fiducial model, we have computed the errors in $\delta\Omega_k$, allowing for sample variance and using the published values for the instrument noise [1]. We do not account for a calibration uncertainty and fix all cosmological parameters, except Ω_k. We then allow arbitrary amounts of isocurvature and cross-correlation power and attempt to set limits simultaneously on these and on $\delta\Omega_k$. With the assumption of adiabaticity, the data yield a 1σ

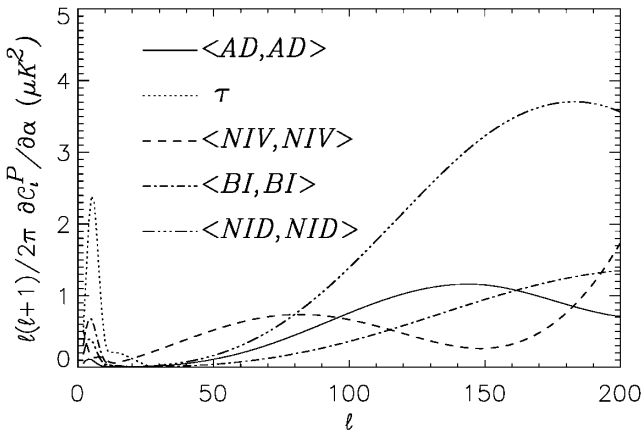

FIG. 3. Polarization signal of the various modes on large angular scales ($l \lesssim 100$). Measurements on these scales are largely responsible for the degeneracy breaking which polarization measurements allow.

error on $\delta\Omega_k$ of 2%. However, with isocurvature modes allowed, this error rises to 577%. Of course, the approximation that the likelihood is Gaussian breaks down at such a large level. Nevertheless, the conclusion that the error in $\delta\Omega_k$ is of order unity is firm. If we assume that it will be possible to make a BOOMERANG-like polarization map with accurate source subtraction and that the polarization error is optimal, $\sigma_P^2 = 2\sigma_{T,\mathrm{BOOM}}^2$, then we find that the additional polarization information allows the constraint on Ω_k to be reduced to 13%, a very significant improvement. This work therefore provides strong motivation for such a polarization measurement.

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[1] P. de Bernardis *et al.,* Nature (London) **404**, 955 (2000).
[2] S. Hanany *et al.,* Astrophys. J. **545**, L5 (2000).
[3] http://map.gsfc.nasa.gov
[4] http://astro.estec.esa.nl/Planck/
[5] V. Mukhanov, H. A. Feldman, and R. H. Brandenberger, Phys. Rep. **215**, 203 (1992).
[6] D. Polarski and A. A. Starobinskii, Phys. Rev. D **50**, 6123 (1994); A. Linde and S. Mukhanov, Phys. Rev. D **56**, 535 (1997); D. Langlois, Phys. Rev. D **59**, 123512 (1999); D. Langlois and A. Riazuelo, Phys. Rev. D **62**, 043504 (2000).
[7] J. R. Bond and G. Efstathiou, Mon. Not. R. Astron. Soc. **22**, 33 (1987); P. J. E. Peebles, Astrophys. J. **315**, L73 (1987); Astrophys. J. **510**, 531 (1999).
[8] K. Enqvist, H. Kurki-Suonio, and J. Valiviita, Phys. Rev. D **62**, 103003 (2000); K. Enqvist and H. Kurki-Suonio, Phys. Rev. D **61**, 043002 (2000); E. Pierpaoli, J. Garciá-Bellido, and S. Borgani, J. High Energy Phys. **9910**, 15 (1999).
[9] M. Bucher, K. Moodley, and N. Turok, Phys. Rev. D **62**, 083508 (2000).
[10] A. Rebhan and D. Schwarz, Phys. Rev. D **50**, 2541 (1994); A. Challinor and A. Lasenby, Astrophys. J. **513**, 1 (1999).
[11] M. Bucher, K. Moodley, and N. Turok, astro-ph/0007360.
[12] M. Zaldarriaga, D. N. Spergel, and U. Seljak, Astrophys. J. **488**, 1 (1997).